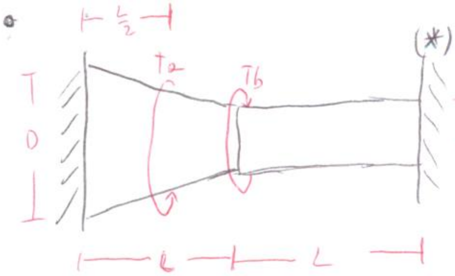


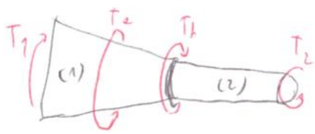
$D = 10\ cm$ $\quad T_a = 500\ Nm$ $\quad G_1 = 50\ GPa$

$d = 6\ cm$ $\quad T_b = 300\ Nm$ $\quad G_2 = 60\ GPa$

$L = 40\ cm$

* Cuando existen torques en medio de barras se requiere realizar cortes imaginarios antes y después de cada torque para observar qué momento produce qué torsión en la barra.

* DCL completo



$$\sum T = 0 \Rightarrow T_1 + T_b = T_a + T_2$$

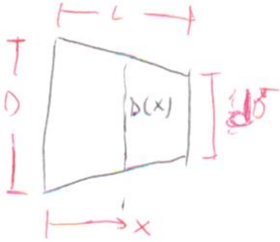
$$\boxed{\therefore T_1 = T_a + T_2 - T_b} \quad ★$$

* Cortes imaginarios:

(Antes y después de Ta) (Antes y después de Tb)

* Cada trozo sufrirá una torsión diferente. Lo que se hará es calcular el ángulo de torsión de cada trozo de izq. a derecha, e imponer que el ángulo de torsión en (*) es cero.

* Torsión sección cónica (a)



→ D varía con la distancia de manera lineal

$x = 0 \rightarrow D(0) = D$

$x = L \rightarrow D(L) = d$

$$\boxed{D(x) = D - (D-d)\frac{x}{L}}$$

$\cancel{d}^* = \frac{1}{2}(D+d)$

* Tomando un elemento diferencial:

$J(x) = \frac{\pi D(x)^4}{32}$

$$\rightarrow T = \frac{GJ\theta}{\cancel{L}} \rightarrow \boxed{d\theta = \frac{T}{GJ(x)}dx} \quad (\bullet)$$

* Integrando (∘) en L $(T = T_1)$

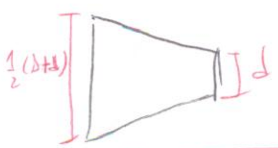
$$\int d\theta_a = \frac{T_1}{G_1}\int_0^{L/2}\frac{dx}{J(x)} = \frac{32T}{\pi G_1}\int_0^{L/2}\frac{dx}{\left[D-(D-d)\frac{x}{L}\right]^4}$$

* Sabiendo que: $\displaystyle\int \frac{1}{(\alpha x+\beta)^n}\,dx = \frac{-1}{(n-1)\alpha}\cdot\frac{1}{(\alpha x+\beta)^{n-1}}$

$$\Rightarrow \Delta\theta_a = \frac{32\,T_1 L}{3\pi G_1\left(D-\frac{1}{2}(D+d)\right)}\left(\frac{1}{\left[D-(D-d)\frac{x}{L}\right]^3}\right)\Bigg|_0^{L/2}$$

$$\boxed{\therefore \Delta\theta_a = \frac{64\,TL}{3\pi G_1(D-d)}\left(\frac{8}{(D+d)^3}-\frac{1}{D^3}\right) = 6{,}472\cdot10^{-7}\,\underline{T_1}}$$

* Para torsión sección cónica (b) $(T = T_b - T_2)$



$$\Delta\theta_b = \frac{32\,(T_b-T_2)L}{3\pi G_1(D-d)}\left(\frac{1}{\left[D-(D-d)\frac{x}{L}\right]^3}\right)\Bigg|_{L/2}^{L}$$

$$\boxed{\therefore \Delta\theta_b = \frac{32\,(T_b-T_2)L}{3\pi G_1(D-d)}\left(\frac{1}{d^3}-\frac{8}{(D+d)^3}\right) = 1{,}818\cdot10^{-6}(T_b-T_2)}$$

* Torsión sección cilíndrica (c) $(T = T_2)$

$$\Delta\theta_c = \frac{T_2 L}{G_2 J} \quad\wedge\quad J = \frac{\pi d^4}{32} \quad\wedge\quad \Rightarrow \boxed{\Delta\theta_c = 5{,}24\cdot10^{-6}\,T_2}$$

* Imponiendo que el ángulo de torsión en (*) es cero:

$$\Delta\theta_a + \Delta\theta_c - \Delta\theta_b = 0$$

$$\Rightarrow 6{,}472\cdot10^{-7}T_1 + 5{,}24\cdot10^{-6}T_2 - 1{,}818\cdot10^{-6}T_b - 1{,}818\cdot10^{-6}T_2 = 0$$

* Desde el DCL ✪, reemplazando en la ec. anterior, es posible encontrar un valor para T_2 y T_1

$$\boxed{T_2 = 54\ N\cdot m \quad \wedge \quad T_1 = 254\ N\cdot m}$$

* Para determinar los máximos esf. de corte por torsión:

$$\tau = \frac{Tr}{J} \rightsquigarrow \tau_{max} = \frac{T\, r_{max}}{J} \quad / \; r_{max} = \frac{d}{2}$$

$$a)\ \tau_{max} = \frac{T_1 \cdot \frac{D(x)}{2}}{\frac{\pi \cdot D(x)^4}{32}} = \frac{16}{\pi}\frac{T_1}{D(x)^3}$$

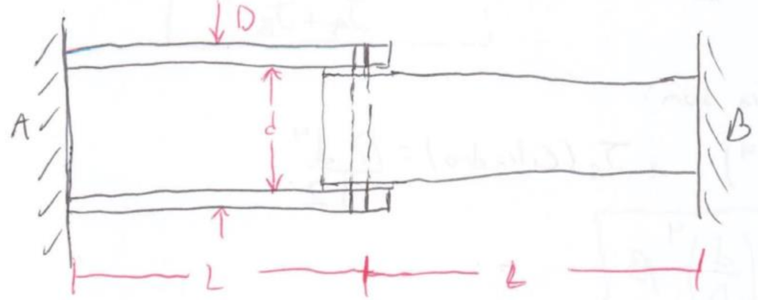
/ ~~D(x) max = D~~
Máximo en D(x) min
$D(x) = \frac{D+d}{2} \quad (x = \frac{L}{2})$

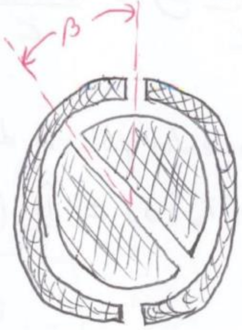
$$\boxed{\therefore \tau_{max} = 2{,}527\ MPa}$$

$$b)\ \tau_{max} = \frac{(T_b - T_2)}{D(x)^3} \cdot \frac{16}{\pi} \quad / \; max \text{ en } x = L \rightsquigarrow D(L) = d$$

$$\boxed{\therefore \tau_{max} = 5{,}8003\ MPa}$$

$$c)\ \boxed{\tau_{max} = \frac{16}{\pi}\frac{T_2}{d^3} = 1{,}2732\ MPa}$$

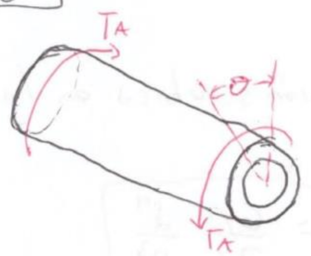




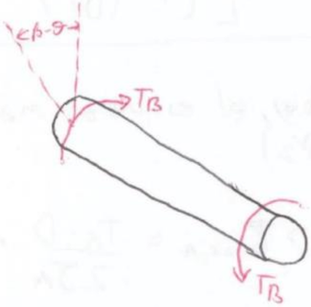
*La barra B se debe torcer un ángulo β para que los agujeros coincidan. Al soltar la barra B, ésta se devuelve cierto ángulo θ, mientras que la barra A se tuerce el mismo ángulo θ:

DCL)

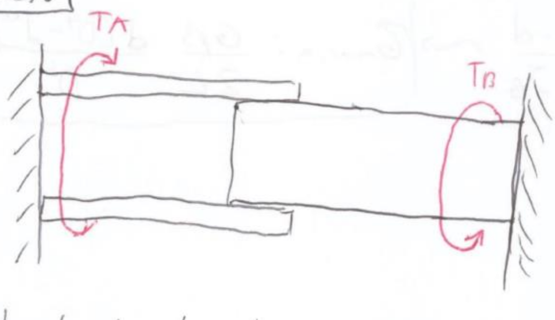
A)



B)



Completo)



$$T_A = T_B$$

*Calculando las torsiones:

$$\rightarrow T_A = \frac{G \cdot J_A}{L}\theta_A = \frac{G J_A}{2}\theta \quad \wedge \quad T_B = \frac{G J_B}{2}\theta_B = \frac{G J_B}{L}(\beta-\theta)$$

*Como $T_A = T_B$; calculamos θ:

$$\frac{GJ_A}{L}\theta = \frac{GJ_B}{L}\beta - \frac{GJ_B}{L}\theta \Rightarrow \boxed{\theta = \frac{J_B}{J_A + J_B}\beta}$$

*Los momentos polares de inercia son:

$$J_A(\text{Tubo}) = \frac{\pi}{32}(D^4 - d^4) \quad \wedge \quad J_B(\text{Cilindro}) = \frac{\pi d^4}{32}$$

$$\boxed{\therefore \theta = \left(\frac{d}{D}\right)^4 \beta}$$

*Reemplazando en las torsiones:

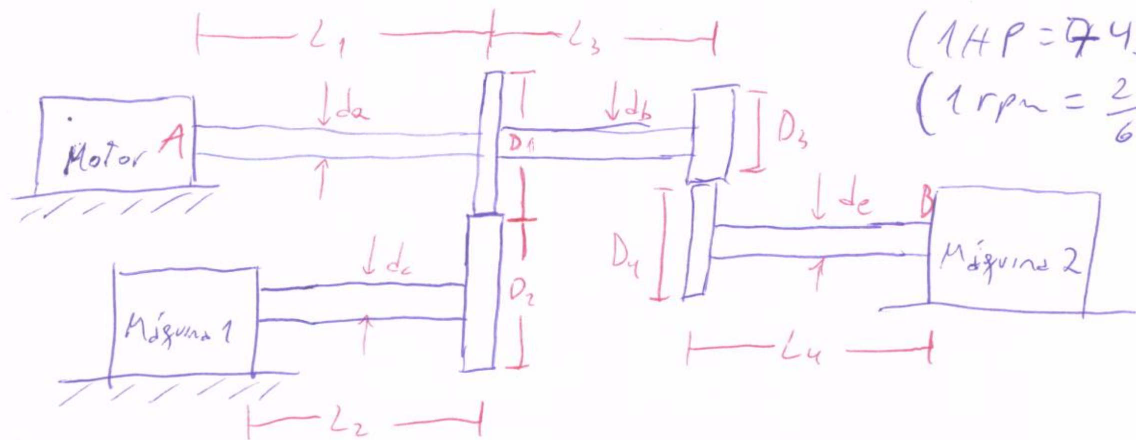
$$\boxed{T_A = \frac{G}{L}\left(\frac{d}{D}\right)^4 J_A \beta}$$

$$\boxed{T_B = \frac{G}{L}\left(1 - \left(\frac{d}{D}\right)^4\right) J_B \beta}$$

* En cilindros y tubos, el esfuerzo máximo de torsión se ubica en la superficie ($r = D/2$)

$$\sim \tau = \frac{Tr}{J} \Rightarrow \tau_{max,A} = \frac{T_A \cdot D}{2J_A} \sim \boxed{\tau_{max,A} = \frac{G\beta}{2L}\frac{d^4}{D^3}}$$

$$\Rightarrow \tau_{max,B} = \frac{T_B \cdot d}{2J_B} \sim \boxed{\tau_{max,B} = \frac{G\beta}{2L}\frac{d(D^4 - d^4)}{D^4}}$$



$(1HP = 745,69\ W)$

$(1\ rpm = \frac{2\pi}{60}\ rad/s)$

(a)

* Potencia en cada máquina

$$P_{mot} = P_{maq1} + P_{maq2} \Rightarrow P_{maq2} = P_{mot} - P_{maq1}$$

$$\boxed{P_{maq2} = 450\ HP = 33{,}56\ kW}$$

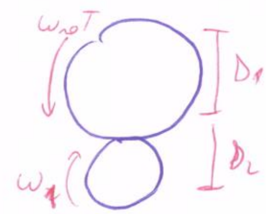
* Velocidad angular en cada máquina

$\rightsquigarrow$ Motor: $P = T \cdot \omega \Rightarrow \boxed{T_{mot} = \frac{P_{mot}}{\omega_{mot}} = 11{,}393\ kN \cdot m}$

$$\theta_1 \cdot D_1 = \theta_2 \cdot D_2 \ \wedge \ \omega_1 \cdot D_1 = \omega_2 D_2$$

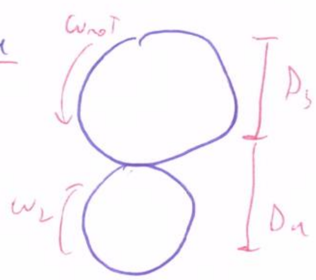
Notar que si el motor gira a vel. angular ω, todas sus partes giran a la misma velocidad (es, los engranajes)

En engranajes $D_1 - D_2$



$$\boxed{\omega_1 = \omega_{mot} \cdot \frac{D_1}{D_2}}$$

En engranajes $D_3 - D_4$



$$\boxed{\omega_2 = \omega_{mot} \cdot \frac{D_3}{D_4}}$$

~> Máquina 1: $T_{maq1} = \frac{P_{maq1}}{\omega_1} = \frac{P_{maq1}}{\omega_{mot}} \cdot \frac{D_2}{D_1} = 3,74 \text{ kNm}$

~> Máquina 2: $T_{maq2} = \frac{P_{maq2}}{\omega_{mot}} \cdot \frac{D_4}{D_3} = 0,513 \text{ kNm}$

• Esf. máximo admisible $\left(\tau = \frac{Tr}{J}\right)$

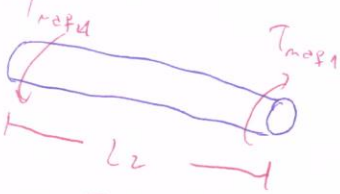
(a)

$\Rightarrow \tau_{adm} = \frac{T_{mot} \cdot \frac{d_a}{2}}{J_a}$

$*\ J_a = \frac{\pi d_a^4}{32} \Rightarrow \tau_{adm} = \frac{T_{mot} \cdot d_a \cdot 16}{\pi d_a^4} = \frac{16\, T_{mot}}{\pi d_a^3}$

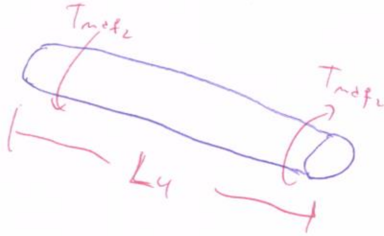
$$\boxed{\therefore\ d_a = \sqrt[3]{\frac{16\, T_{mot}}{\pi\, \tau_{adm}}}}$$

(c)



$$\boxed{d_c = \sqrt[3]{\frac{16\, T_{maq1}}{\pi\, \tau_{adm}}}}$$

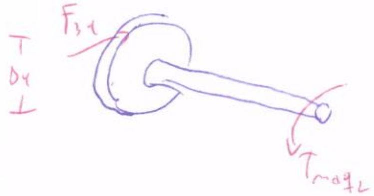
(e)



$$\boxed{d_e = \sqrt[3]{\frac{16\, T_{maq2}}{\pi\, \tau_{adm}}}}$$

* Para estimar d_b, se necesita saber el torque asociado a ese segmento

* Desde Máquina 2



$$T_{motz} = F_{34} \cdot \frac{D_4}{2} \Rightarrow \boxed{F_{34} = \frac{2\,T_{motz}}{D_4}}$$

· En eje (b)

$$T_b = F_{34} \cdot \frac{D_3}{2} = \frac{\cancel{2}\,T_{motz}}{\cancel{2}} \cdot \frac{D_3}{D_4} = \frac{D_3}{D_4} \cdot \frac{P_{motz}}{\omega_{mot}} \cdot \frac{\cancel{D_4}}{\cancel{D_3}}$$

$$\boxed{\therefore T_b = \frac{P_{motz}}{\omega_{mot}} = 0{,}641 \text{ kNm}}$$

$$\boxed{\therefore d_b = \sqrt[3]{\frac{16\,T_b}{\pi\,\tau_{adm}}}}$$

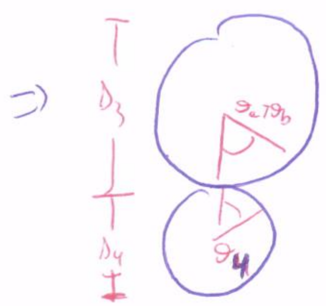
$$\Rightarrow \boxed{d_a = 0{,}0774\text{ m}\ ;\ d_b = 0{,}0297\text{ m}\ ;\ d_c = 0{,}0534\text{ m}\ ;\ d_e = 0{,}0275\text{ m}}$$

(b) Ángulos de torsión $\sim$ $T = \frac{GJ\theta}{L} \Rightarrow \theta = \frac{TL}{GJ}$

· Eje (a) $\Rightarrow$ $\theta_a = \frac{T_{mot} \cdot L_1}{G\,J_a}$

· Eje (b) $\Rightarrow$ $\theta_b = \frac{T_b \cdot L_3}{G \cdot J_b}$

El eje A-D_1-D_3 gira en total $\theta_a + \theta_b$ (En D_3)



$$\Rightarrow \theta_4 = (\theta_a + \theta_b) \cdot \frac{D_3}{D_4}$$

• Eje (e)

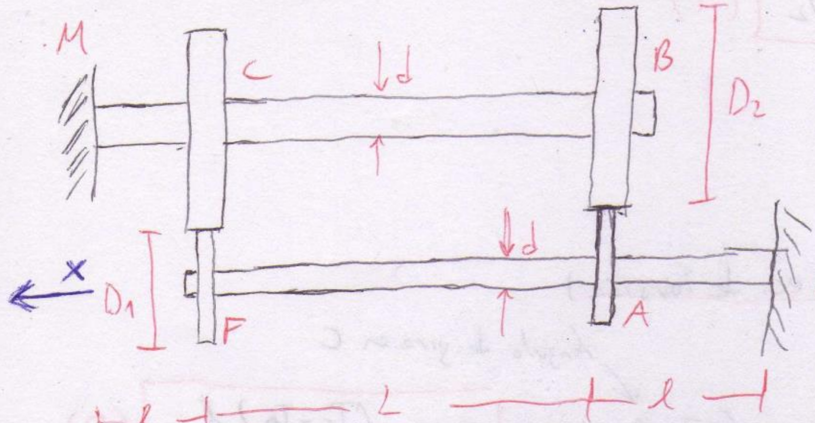
$$\theta_e = \frac{T_{max2} \cdot L_4}{G J_e}$$

∴ El ángulo total de torsión entre A y B es

$$\theta_{AB} = \theta_4 + \theta_3$$

$$= (\theta_a + \theta_b)\frac{D_3}{D_4} + \frac{T_{max2} \cdot L_4}{G J_e}$$

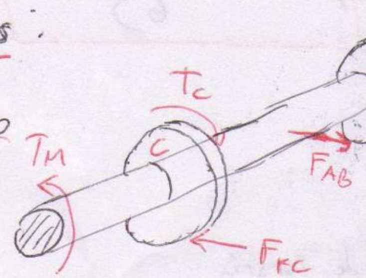
$$\boxed{\theta_{AB} = 0{,}0652 \text{ rad}}$$



$L = 1\,m$
$l = 30\,cm$
$d = 5\,cm$
$D_1 = 15\,cm$
$D_2 = 40\,cm$
$\theta = 30°$
$G = 90\,GPa$

· DCL's:

~ MCB

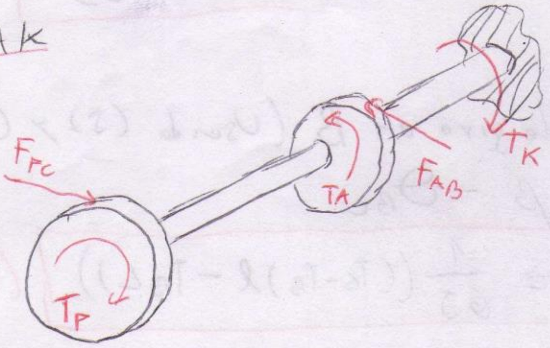


· $\sum T_x = 0$

$$\boxed{T_M = T_C - T_B} \quad (1)$$

donde
$$T_C = F_{FC} \cdot \frac{D_2}{2}$$
$$T_B = F_{AB} \cdot \frac{D_2}{2}$$

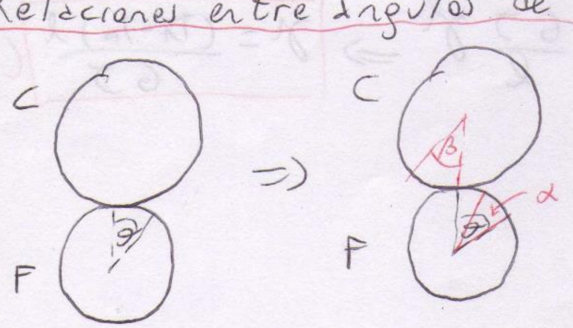
~ FAK



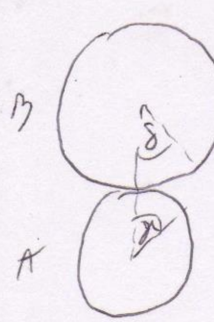
· $\sum T_x = 0$

$$T_K = T_A - T_F$$
$$T_A = F_{AB} \cdot \frac{D_1}{2} \quad (2)$$
$$T_F = F_{FC} \cdot \frac{D_1}{2}$$

· Relaciones entre ángulos de engranajes:



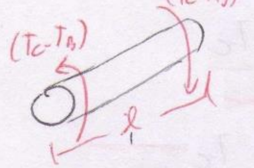
$$\boxed{\therefore \alpha D_1 = \beta D_2} \quad (3)$$



$$\boxed{\therefore\ \gamma D_1 = \delta D_2} \quad (4)$$

· Cortes en los ejes (Utilizando ec. de torsión)

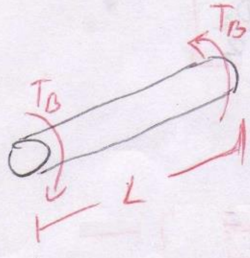
⇝ Segmento MCB



Ángulo de giro en C

$$(T_C - T_B) = \frac{GJ}{l}\beta \Rightarrow \boxed{\beta = \frac{(T_C - T_B)\,l}{GJ}} \quad (5)$$

⇝ Segmento CB



Ángulo de torsión del segmento CB

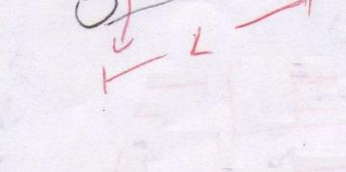
$$T_B = \frac{GJ}{L}\vartheta_{BC} \Rightarrow \boxed{\vartheta_{BC} = \frac{T_B L}{GJ}} \quad (*)$$

∴ Ángulo total de giro en B (Usando (5) y (*))

$$\hookrightarrow \delta = \beta - \vartheta_{BC}$$

$$\boxed{\therefore\ \delta = \frac{1}{GJ}\left((T_C - T_B)\,l - T_B L\right)} \quad (6)$$

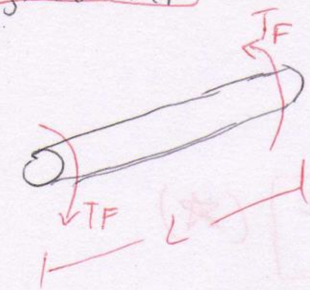
⇝ Segmento KA



Ángulo de giro en A

$$(T_A - T_F) = \frac{GJ}{l}\gamma \Rightarrow \boxed{\gamma = \frac{(T_A - T_F)\,l}{GJ}} \quad (7)$$

→ Segmento AF



Ángulo de torsión del segmento AF

$$T_F = \frac{GJ}{L}\vartheta_{AF}$$

$$\hookrightarrow \boxed{\vartheta_{AF} = \frac{T_F L}{GJ}} \quad (**)$$

∴ Ángulo total de giro en $F = \gamma - \vartheta_{AF}$
(Usando (7) y (**))

• El ángulo total de giro en F es lo que giró antes (ϑ) menos lo que se restituyó elásticamente (α)

$$\Rightarrow \boxed{\vartheta - \alpha = \frac{1}{GJ}\left((T_A - T_F)\ell - T_F L\right)} \quad (8)$$

• Ahora iremos reemplazando ecuaciones hasta encontrar las fuerzas de contacto F_{FC} y F_{AB}:

$$* (7) \text{ en } (4) \Rightarrow \frac{(T_A - T_F)\ell}{GJ} D_1 = \delta D_2 \Rightarrow \boxed{\delta = \frac{D_1}{D_2}\frac{(T_A - T_F)\ell}{GJ}} \quad (9)$$

$$* (9) \text{ en } (6) \Rightarrow \frac{D_1}{D_2}\frac{(T_A - T_F)\ell}{\cancel{GJ}} = \frac{1}{\cancel{GJ}}\left((T_C - T_B)\ell - T_B L\right)$$

$$\boxed{\therefore (T_A - T_F)\,\ell\cdot\frac{D_1}{D_2} = \left(T_C\,\ell - (\ell + L)T_B\right)} \quad (A)$$

$$* (5) \text{ en } (3) \Rightarrow \alpha D_1 = \frac{(T_C - T_B)\ell}{GJ} D_2 \Rightarrow \boxed{\alpha = \frac{D_2}{D_1}\frac{(T_C - T_B)\ell}{GJ}} \quad (10)$$

$$* (10) \text{ en } (8) \Rightarrow \vartheta - \frac{D_2}{D_1}\frac{(T_C - T_B)\ell}{GJ} = \frac{1}{GJ}\left((T_A - T_F)\ell - T_F L\right)$$

$$\cancel{(T_C - T_B) = \frac{\vartheta GJ}{\ell}\frac{D_1}{D_2}}$$

$$\boxed{(T_C - T_B)\,\ell\cdot\frac{D_2}{D_1} = \vartheta GJ - \left(T_A\,\ell - (\ell + L)T_F\right)} \quad (B)$$

• Reemplazando (1) y (2) en (A) y (B)

$$(A)\left(F_{AB}\cdot\frac{D_1}{2}-F_{FC}\cdot\frac{D_1}{2}\right)\frac{l D_1}{D_2}=F_{FC}\cdot\frac{D_2 l}{2}-F_{AB}\frac{D_2}{2}(l+L)$$

$$\boxed{(F_{AB}-F_{FC})\,l\,D_1^2=(F_{FC}\,l-F_{AB}(l+L))D_2^2}\quad(\star)$$

$$(B)\left(F_{FC}\cdot\frac{D_2}{2}-F_{AB}\cdot\frac{D_2}{2}\right)\frac{l D_2}{D_1}=\theta G J-\left(F_{AB}\cdot\frac{D_1}{2}l-F_{FC}\cdot\frac{D_1}{2}(l+L)\right)$$

$$\boxed{(F_{FC}-F_{AB})\,l\,D_2^2=2\theta G J-(F_{AB}\,l-F_{FC}(l+L))D_1^2}\quad(\star\star)$$

• Finalmente, se resuelve el sistema de ecuaciones para ($\star$) y ($\star\star$)

$$F_{FC}=\frac{2D_1 G J\,[D_2^2(l+L)-D_1^2 l]\,\theta}{L\,[l(D_2^4-D_1^4)+D_1^2D_2^2(2l+L)]}$$

$$F_{AB}=\frac{2D_1 G J\,(D_2^2-D_1^2)\,l\,\theta}{L\,[l(D_2^4-D_1^4)+D_1^2D_2^2(2l+L)]}$$

$$\wedge\quad J=\frac{\pi d^4}{32}=6{,}136\cdot10^{-7}\,[m^4]$$

• Reemplazando valores: $F_{FC}=26275{,}1\,[N]\ \wedge\ F_{AB}=5385{,}58\,[N]$

• Los valores de las torsiones son: $T_A=403{,}92\,[N\cdot m]$; $T_B=1077{,}12\,[N\cdot m]$

$T_C=5255{,}02\,[N\cdot m]$, $T_F=1970{,}63\,[N\cdot m]$

• Finalmente calculando los esf. máximos:

Eje MCB

Segmento MC: $\tau_{max}=\dfrac{(T_C-T_B)\cdot\frac{d}{2}}{J}$

$$\boxed{\tau_{max}=170{,}22\,[MPa]}$$

Segmento CB: $\tau_{max}=\dfrac{T_B\frac{d}{2}}{J}$

$$\boxed{\tau_{max}=43{,}885\,[MPa]}$$

Eje FAK

Segmento KA: $\tau_{max}=\dfrac{(T_A-T_F)\frac{d}{2}}{J}$

$$\boxed{\tau_{max}=-63{,}833\,[MPa]}$$

Segmento AF: $\tau_{max}=\dfrac{T_F\frac{d}{2}}{J}$

$$\boxed{\tau_{max}=80{,}289\,[MPa]}$$