P1) a)

$$\int \frac{dx}{x(4+7\ln^2(x))} = \int \frac{dx}{x\left(2^2+(\sqrt{7}\ln(x))^2\right)}$$

$u = \sqrt{7}\ln(x)$

$du = \frac{\sqrt{7}}{x}\,dx$

$$= \frac{1}{\sqrt{7}}\int \frac{\cancel{x}\cdot du}{\cancel{x}(2^2+u^2)} = \frac{1}{\sqrt{7}}\cdot\frac{1}{2}\arctan\left(\frac{u}{2}\right) + C = \frac{1}{2\sqrt{7}}\arctan\left(\frac{\sqrt{7}\ln(x)}{2}\right) + C$$

b)

$$\int \frac{x^2-2x+3}{(x-1)^2(x^2+1)}\,dx$$

Fracciones parciales

$$\frac{x^2-2x+3}{(x-1)^2(x^2+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+1}$$

$$x^2-2x+3 = A(x-1)(x^2+1) + B(x^2+1) + (Cx+D)(x-1)^2$$

$$x^2-2x+3 = A(x^3+x-x^2-1) + B(x^2+1) + C(x^3-2x^2+c) + D(x^2-2x+1)$$

$$\left.\begin{array}{ll} x^3 & A+C=0 \\ x^2 & -A+B-2C+D=1 \\ x & A+C-2D=-2 \\ 1 & -A+B+D=3 \end{array}\right\} \quad \begin{array}{l} A=-1 \\ B=1 \\ C=1 \\ D=1 \end{array}$$

$$= \int \frac{-1}{x-1} + \frac{1}{(x-1)^2} + \underbrace{\frac{x+1}{x^2+1}}\,dx$$

$\downarrow$ $-\ln(x-1)$

$\downarrow$ $-\frac{1}{(x-1)}$

$$= \int \frac{x}{x^2+1} + \frac{1}{x^2+1}\,dx$$

$\frac{1}{x^2+1} \to \arctan(x)$

$\downarrow$

$u = x^2+1$

$du = 2x\,dx$

$$= \int \frac{x}{2x\,u}\,du = \frac{1}{2}\ln(u)$$

$$= \frac{1}{2}\ln(x^2+1)$$

$$= -\ln(x-1) - \frac{1}{(x-1)} + \frac{1}{2}\ln(x^2+1) + \arctan(x) + C \;//.$$

c) $\displaystyle\int \frac{1}{\sqrt{x^2+2x+2}}\,dx = \int \frac{1}{\sqrt{(x+1)^2+1}}\,dx$ | $\tan(u) = x+1$; $\sec^2(u)\,du = dx$ (porque $\sec^2(x) = 1+\tan^2(x)$)

$$= \int \frac{1}{\sqrt{\tan^2(u)+1}}\cdot \sec^2(u)\,du = \int \frac{\sec^2(u)\,du}{\sec(u)} = \int \frac{1}{\cos(u)}\,du$$

$t = \tan\left(\frac{u}{2}\right)$

$dt = \frac{\sec^2\left(\frac{u}{2}\right)}{2}\,du = \frac{\left(1+\tan^2\left(\frac{u}{2}\right)\right)}{2}\,du$

$dt = \frac{1+t^2}{2}\,du$

$\cos(u) = \frac{1-t^2}{1+t^2}$

$$= \int \frac{\frac{2\,dt}{\cancel{1+t^2}}}{\frac{1-t^2}{\cancel{1+t^2}}} = \int \frac{2\,dt}{1-t^2} = 2\int \frac{dt}{(1+t)(1-t)}$$

Fracciones parciales: $\frac{1}{(t+1)(1-t)} = \frac{A}{1+t} + \frac{B}{1-t}$

$1 = A + At + B - Bt$

$\left.\begin{array}{l} A - B = 0 \\ A + B = 1 \end{array}\right\}\quad A = B = \frac{1}{2}$

$$= \cancel{2}\int \frac{1}{\cancel{2}(1+t)} + \frac{1}{\cancel{2}(1-t)}\,dt = \ln(1+t) - \ln(1-t) + C$$

$$= \ln\left(\frac{1+t}{1-t}\right) + C$$

$$= \ln\left(\frac{1+\tan\left(\frac{u}{2}\right)}{1-\tan\left(\frac{u}{2}\right)}\right) + C = \ln\left(\frac{1+\tan\left(\frac{\arctan(x+1)}{2}\right)}{1-\tan\left(\frac{\arctan(x+1)}{2}\right)}\right) + C$$

//

d) $\int \frac{\sqrt{a^2-x^2}}{x^2}\,dx$ $\qquad x = a\,sen(u)$ $\quad$ (porque $1-sen^2(x) = cos^2(x)$)

$dx = a\cos(u)\,du$

$$= \int \frac{\sqrt{a^2 - a^2 sen^2(u)}}{a^2 sen^2(u)} \cdot a\cos(u)\,du = \int \frac{\cancel{a}\sqrt{1-sen^2(u)}}{\cancel{a^2}\, sen^2(u)} \cdot \cancel{a}\cos(u)\,du = \int \frac{\cos^2(u)}{sen^2(u)}\,du$$

$$= \int \frac{1-sen^2(u)}{sen^2(u)}\,du = \int \frac{1}{sen^2(u)} - 1\,du = -\cotan(u) - u + C$$

$$= -\cotan\left(arcsen\left(\frac{x}{a}\right)\right) - arcsen\left(\frac{x}{a}\right) + C \quad //$$

P2)

$$\lim_{x\to 1} \frac{\int_1^x (x-1)\,sen(t^2)\,dt}{\int_{x^2}^{x^3} sen(t^2-1)\,dt} \quad \left(=\frac{0}{0}\right) \overset{L'H}{=} \lim_{x\to 1} \frac{(x-1)\,sen(x^2) + \int_1^x sen(t^2)}{sen(x^6-1)\cdot 3x^2 - sen(x^4-1)\cdot 2x}$$

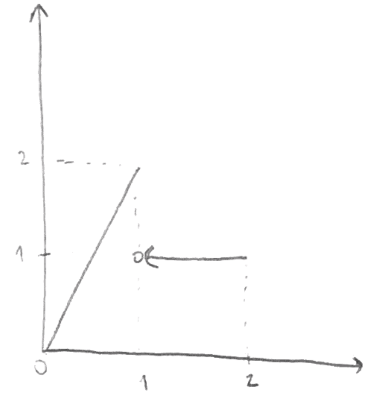
(regla del producto y TFC)

(regla de la cadena y TFC)

$$\left(=\frac{0}{0}\right) = \lim_{x\to 1} \frac{\overset{0}{\cancel{(x-1)}}\cos(x^2)\cdot 2x + sen(x^2) + sen(x^2)}{3x^2 \underset{1}{\cancel{\cos(x^6-1)}}\cdot 6x^5 + \underset{5}{\cancel{sen(x^6-1)\cdot 6x}} - \left[\underset{1}{\cancel{\cos(x^4-1)}}\cdot 4x^3\cdot 2x + 2\,\underset{0}{\cancel{sen(x^4-1)}}\right]}$$

$$= \frac{2\,sen(1)}{18-8} = \frac{sen(1)}{5} \quad //$$

P3)



a) $$s(f,P) = (2-1)\cdot \inf_{x\in(1,2)} f(x) + (1-0)\cdot \inf_{x\in(0,1)} f(x) = 1\cdot 1 + 1\cdot 0 = 1 //$$

$$S(f,P) = (2-1) \sup_{x\in(1,2)} f(x) + (1-0)\cdot \sup_{x\in(0,1)} f(x) = 1\cdot 2 + 1\cdot 2 = 4 //.$$

b) si $x \in [0,1]$

$$\Delta x = \frac{1}{n} \Rightarrow x_i = \frac{i}{n}$$

$$f(x) = 2x$$

como $f(x)=2x$ es estrictamente creciente, siempre se cumplirá que

$$\inf_{x\in(x_{i-1},x_i)} f(x) = f(x_{i-1}) \qquad \sup_{x\in(x_{i-1},x_i)} f(x) = x_i$$

Luego $s(f,P) = \sum_{i=1}^{n} \frac{1}{n} \inf_{x\in(x_{i-1},x_i)} f(x) = \frac{1}{n}\sum_{i=1}^{n} f(x_{i-1}) = \frac{1}{n}\sum_{i=1}^{n} 2x_{i-1} = \frac{1}{n}\sum_{i=1}^{n} 2\frac{(i-1)}{n} = \frac{2}{n^2}\sum_{i=1}^{n}(i-1)$

$$= \frac{2}{n^2}\sum_{i=0}^{n-1} i = \frac{\cancel{2}}{n^{\cancel{2}}}\frac{(n-1)\cancel{n}}{\cancel{2}} = \frac{n-1}{n}$$

análogamente, $S(f,P) = \frac{2}{n^2}\sum_{i=1}^{n} i = \frac{\cancel{2}}{n^{\cancel{2}}}\frac{(\cancel{n})(n+1)}{\cancel{2}} = \frac{n+1}{n}$

Si $x\in[1,2]$

$$s(f,P) = (2-(1+\delta))\cdot 1 + (1+\delta-1)\cdot 1 = 1-\delta+\delta = 1$$

$$S(f,P) = (2-(1+\delta))\, 1 + (1+\delta-1)\cdot 2 = 1-\delta+2\delta = 1+\delta$$

entonces:

$$s(f,P) = \frac{(n-1)}{n} + 1 = 2 - \frac{1}{n} \qquad \text{y así} \quad S(f,P) - s(f,P) = \frac{2}{n} + \delta \ //$$

$$S(f,P) = \frac{(n+1)}{n} + 1 + \delta = 2 + \delta + \frac{1}{n}$$

c) Condición de Riemman:

$$\forall \varepsilon > 0 \ \exists P \in \mathcal{P}_{[0,2]} \ \text{tq}$$

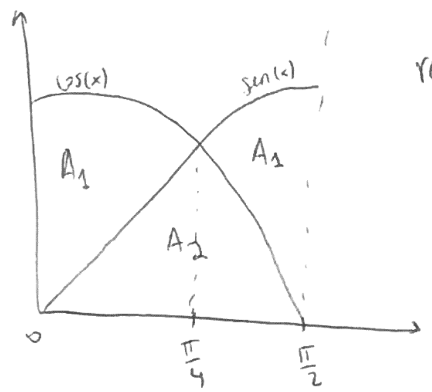
$$S(f,P) - s(f,P) < \varepsilon$$

ya que $S(f,P) - s(f,P) = \frac{2}{n} + \delta$, queremos que $\frac{2}{n} + \delta < \varepsilon$

Basta tomar P con $\delta = \frac{\varepsilon}{2}$ (por ejemplo) (asumiendo $\varepsilon < 1$)

//

P4)



recordemos $V_{ox} = \pi \int_a^b (f(x))^2$

$$V_{oy} = 2\pi \int_a^b x f(x)$$

a) $\pi \int_0^{\frac{\pi}{4}} \cos^2(x) - \operatorname{sen}^2(x) + \pi \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \operatorname{sen}^2(x) - \cos^2(x) \, dx$

$$= \pi \int_0^{\frac{\pi}{4}} \cos(2x) \, dx - \pi \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos(2x) \, dx$$

$$= \frac{\pi}{2} \operatorname{sen}(2x) \Big|_0^{\frac{\pi}{4}} - \frac{\pi}{2} \operatorname{sen}(2x) \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2} \left[1 - 0 \right] - \frac{\pi}{2} \left[0 - 1 \right] = \pi \ //$$

b) $2\pi \int_0^{\frac{\pi}{4}} x \operatorname{sen}(x) + 2\pi \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} x \cos(x)$

Integramos por partes:

$$= 2\pi \left[-x\cos(x) + \operatorname{sen}(x) \right] \Big|_0^{\pi/4} + 2\pi \left[x\operatorname{sen}(x) + \cos(x) \right] \Big|_{\pi/4}^{\pi/2}$$

$$= 2\pi \left[-\frac{\pi}{4} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - (0 + 0) \right] + 2\pi \left[\frac{\pi}{2} + 0 - \left(\frac{\pi}{4} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \right) \right]$$

$$= \pi^2 \left(1 - \frac{\sqrt{2}}{2} \right) \ //$$

P5

$$P'(h) = e^{1+(h-5)^8} \ln\left(\frac{1}{(h-5)^4+1}\right) 4(h-5)^3 \qquad (TFC)$$

$$P'(h) = 0 \quad \Rightarrow h = 5$$

$$e^{1+(h-5)^8} > 0 \quad \forall h$$

$$\ln\left(\frac{1}{(h-5)^4+1}\right) \leq 0 \quad \forall h$$

$\Rightarrow P'(h) > 0$ si $h \in (-\infty, -5) \Rightarrow P(h) \nearrow$ creciente $h \in (-\infty, -5)$

$P'(h) < 0$ si $h \in (-5, +\infty) \Rightarrow P(h) \searrow$ decreciente $h \in (-5, +\infty)$

$\therefore h = 5$ es un máximo

$\therefore$ Claudia debería acostarse a las 2 a.m.

P6)

$$I_n = \int \sqrt{x+b}\,(x+a)^n\,dx$$

integramos por partes:

$f(x) = (x+a)^n \qquad f'(x) = n(x+a)^{n-1}$

$g'(x) = \sqrt{x+b} \qquad g(x) = \frac{2}{3}(x+b)^{2/3}$

$$I_n = \frac{2}{3}(x-b)^{2/3}(x+a)^n - \frac{2n}{3}\int (x+b)^{2/3}(x+a)^{n-1}\,dx$$

$$I_n = (x+a)^n\frac{2}{3}(x-b)^{2/3} - \frac{2n}{3}\int (x+b)\sqrt{x+b}\,(x+a)^{n-1}\,dx$$

usamos indicación

$$= (x+a)^n\frac{2}{3}(x-b)^{2/3} - \frac{2n}{3}\left[\underbrace{\int (x+a)\sqrt{x+b}\,(x+a)^{n-1}\,dx}_{I_n} + \underbrace{\int (b-a)\sqrt{x+b}\,(x+a)^{n-1}\,dx}_{(b-a)I_{n-1}}\right]$$

$$I_n = (x+a)^n\frac{2}{3}(x-b)^{2/3} - \frac{2n}{3}\left[I_n + (b-a)I_{n-1}\right]$$

$$\Rightarrow I_n = \frac{2}{2n+3}(x+a)^n(x+b)^{2/3} - \frac{2n}{2n+3}(b-a)I_{n-1}$$