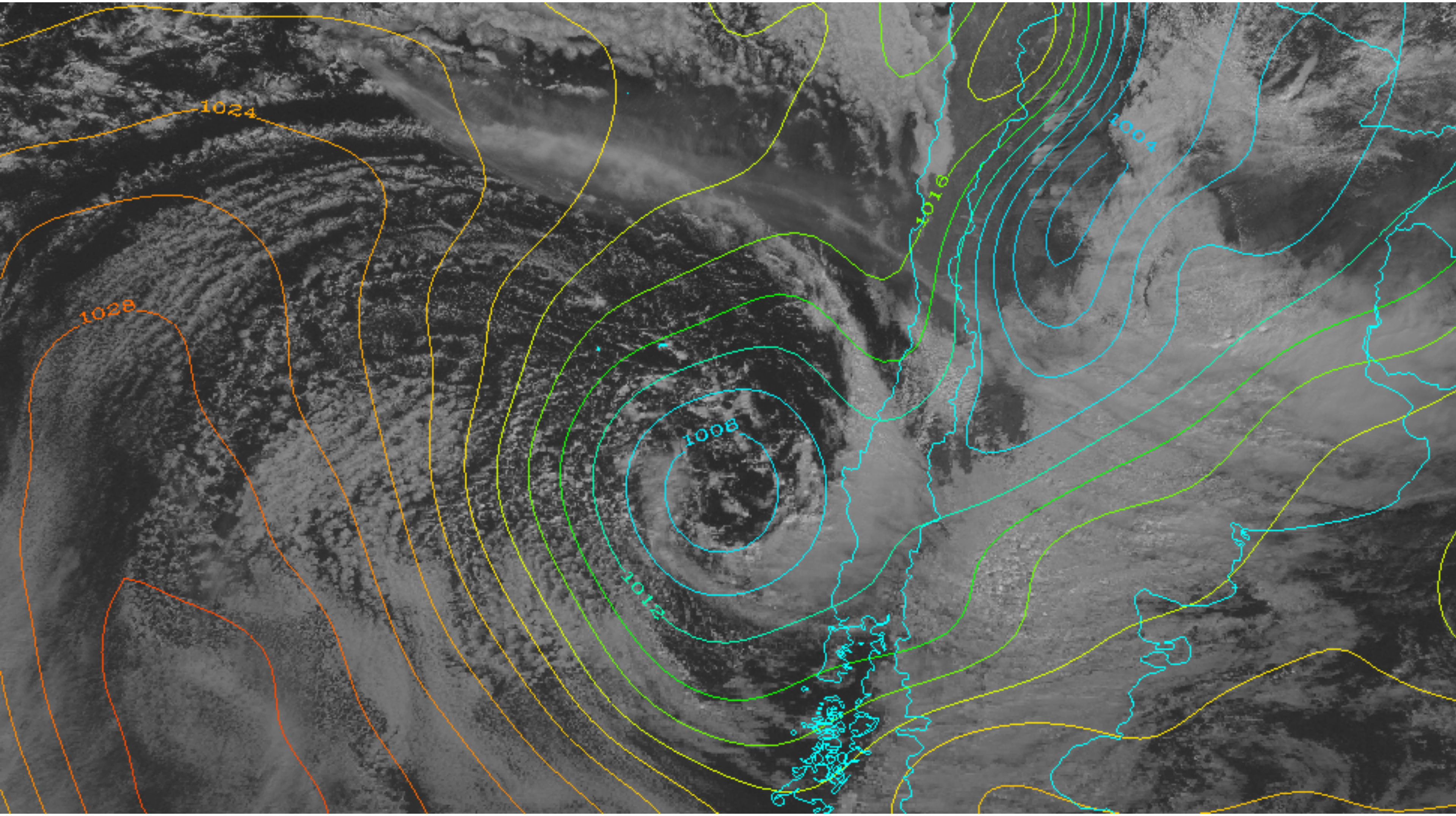
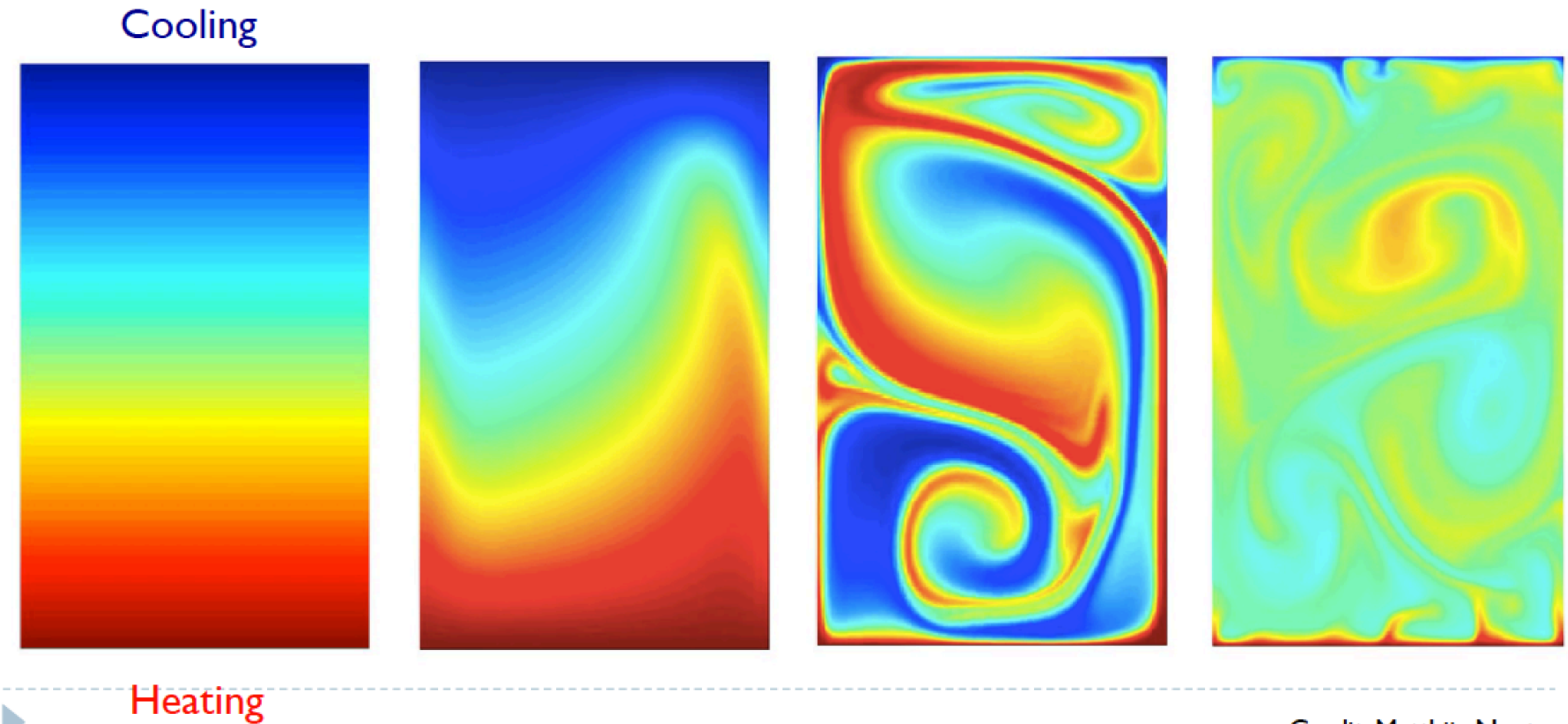
Inestabilidad Baroclinica

Sistemas Sinópticos



Inestabilidad



Credit: Matthijs Neut

Inestabilidad Hidrodinámica

- Se dice que un flujo medio es inestable si una pequeña perturbación en el flujo crece espontáneamente , pasando Eº desde el flujo medio.
- Inestabilidad Barotrópica
- Inestabilidad Baroclínica

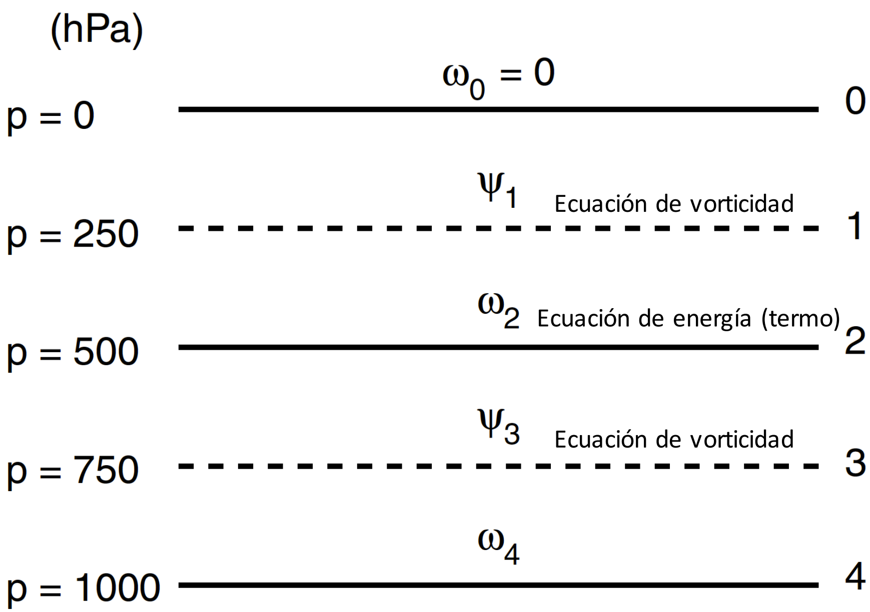
¿Como saber si un flujo es inestable?

- Se aplica el método de las perturbaciones.
- Se introduce una perturbación sinusoidal en el flujo
- Si en la relación de dispersión se obtiene una velocidad de propagación que tiene incorporado una numero imaginario, entonces el flujo es inestable.

Modelo de inestabilidad: 2 capas

$$\textit{geostrophic streamfunction}, \psi \equiv \Phi / f_0$$

$$\mathbf{V}_\psi = \mathbf{k} \times \boldsymbol{\nabla}\psi, \quad \zeta_\mathbf{g} = \nabla^2 \psi$$



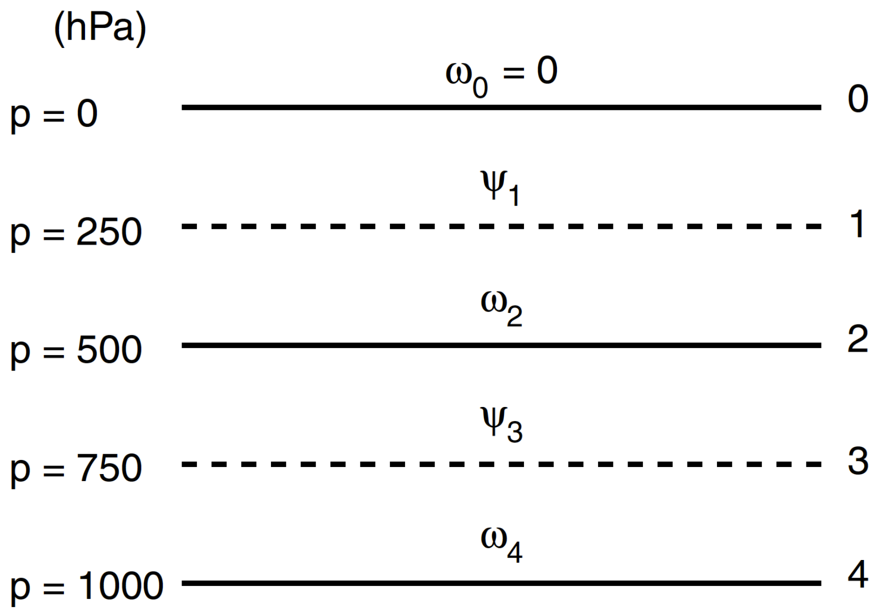
$$\frac{\partial \zeta_g}{\partial t} = -\mathbf{V}_g \cdot \boldsymbol{\nabla}\left(\zeta_g + f\right) + f_0 \frac{\partial \omega}{\partial p}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{V}_g \cdot \boldsymbol{\nabla}\right)\left(-\frac{\partial \Phi}{\partial p}\right) - \sigma\omega = \frac{\kappa J}{p}$$

$$\frac{\partial \zeta_g}{\partial t} = -\mathbf{V}_g \cdot \boldsymbol{\nabla}\left(\zeta_g + f\right) + f_0 \frac{\partial \omega}{\partial p}$$

Fig. 8.2 Arrangement of variables in the vertical for the two-level baroclinic model.

Ecuaciones transformadas por la función de corriente geostrofica



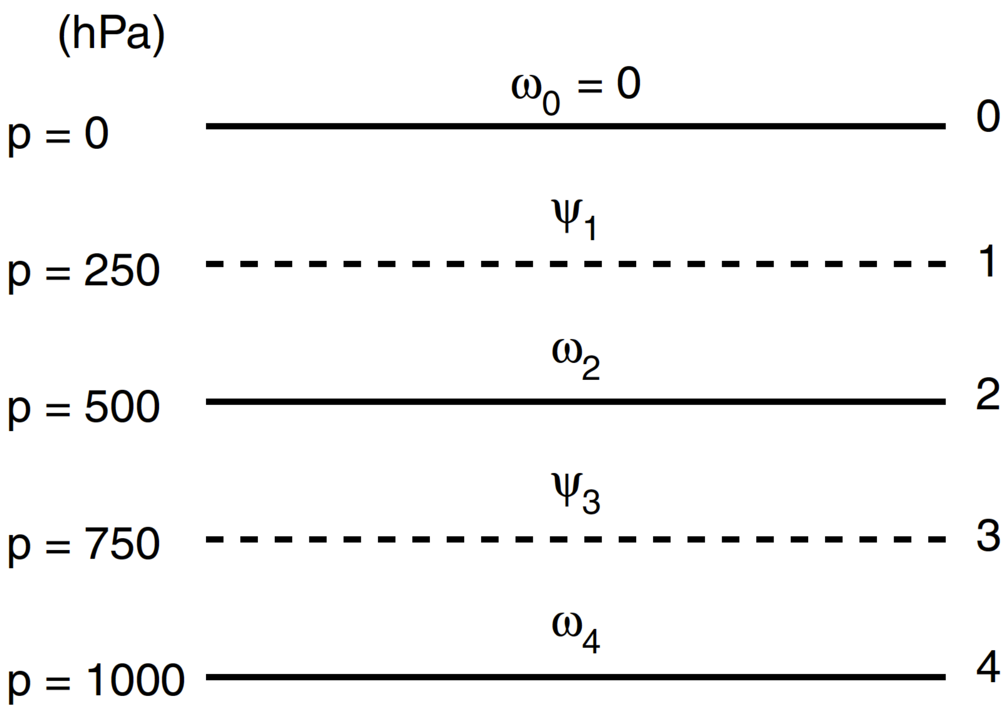
$$\frac{\partial}{\partial t}\nabla^2\psi + \mathbf{V}_\psi \bullet \boldsymbol{\nabla}\left(\nabla^2\psi\right) + \beta\frac{\partial \psi}{\partial x} = f_0\frac{\partial \omega}{\partial p}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial \psi}{\partial p}\right) = -\mathbf{V}_\psi \bullet \boldsymbol{\nabla}\left(\frac{\partial \psi}{\partial p}\right) - \frac{\sigma}{f_0}\omega$$

$$\frac{\partial}{\partial t}\nabla^2\psi + \mathbf{V}_\psi \bullet \boldsymbol{\nabla}\left(\nabla^2\psi\right) + \beta\frac{\partial \psi}{\partial x} = f_0\frac{\partial \omega}{\partial p}$$

Fig. 8.2 Arrangement of variables in the vertical for the two-level baroclinic model.

Aplicando las ecuaciones en los niveles



$$1\quad \frac{\partial}{\partial t}\nabla^2\psi_1 + \mathbf{V}_1\bullet\boldsymbol{\nabla}\left(\nabla^2\psi_1\right)+\beta\frac{\partial\psi_1}{\partial x} = \frac{f_0}{\delta p}\omega_2$$

$$2\quad \frac{\partial}{\partial t}\left(\psi_1-\psi_3\right) = -\mathbf{V}_2\bullet\boldsymbol{\nabla}\left(\psi_1-\psi_3\right)+\frac{\sigma\,\delta p}{f_0}\omega_2$$

$$3\quad \frac{\partial}{\partial t}\nabla^2\psi_3 + \mathbf{V}_3\bullet\boldsymbol{\nabla}\left(\nabla^2\psi_3\right)+\beta\frac{\partial\psi_3}{\partial x} = -\frac{f_0}{\delta p}\omega_2$$

Metodo de las perturbaciones

$$\psi_1 = -U_1 y + \psi_1'(x, t)$$
$$\psi_3 = -U_3 y + \psi_3'(x, t)$$
$$\omega_2 = \omega_2'(x, t)$$

$$\left(\frac{\partial}{\partial t} + U_1 \frac{\partial}{\partial x}\right) \frac{\partial^2 \psi_1'}{\partial x^2} + \beta \frac{\partial \psi_1'}{\partial x} = \frac{f_0}{\delta p} \omega_2'$$

$$\left(\frac{\partial}{\partial t} + U_3 \frac{\partial}{\partial x}\right) \frac{\partial^2 \psi_3'}{\partial x^2} + \beta \frac{\partial \psi_3'}{\partial x} = -\frac{f_0}{\delta p} \omega_2'$$

$$\left(\frac{\partial}{\partial t} + U_m \frac{\partial}{\partial x}\right) \left(\psi_1' - \psi_3'\right) - U_T \frac{\partial}{\partial x} \left(\psi_1' + \psi_3'\right) = \frac{\sigma \delta p}{f_0} \omega_2'$$

Viento zonal Viento térmico

$$U_m \equiv (U_1 + U_3)/2, \quad U_T \equiv (U_1 - U_3)/2$$

$$\left[\frac{\partial}{\partial t} + (U_m + U_T)\frac{\partial}{\partial x}\right]\frac{\partial^2 \psi_1'}{\partial x^2} + \beta\frac{\partial \psi_1'}{\partial x} = \frac{f_0}{\delta p}\omega_2'$$

$$\left[\frac{\partial}{\partial t} + (U_m - U_T)\frac{\partial}{\partial x}\right]\frac{\partial^2 \psi_3'}{\partial x^2} + \beta\frac{\partial \psi_3'}{\partial x} = -\frac{f_0}{\delta p}\omega_2'$$

$$\psi_m \equiv \left(\psi_1' + \psi_3'\right)/2; \qquad \psi_T \equiv \left(\psi_1' - \psi_3'\right)/2$$

Perturbación barotropica

Perturbación baroclinica

suma

$$\left[\frac{\partial}{\partial t} + U_m \frac{\partial}{\partial x}\right] \frac{\partial^2 \psi_m}{\partial x^2} + \beta \frac{\partial \psi_m}{\partial x} + U_T \frac{\partial}{\partial x}\left(\frac{\partial^2 \psi_T}{\partial x^2}\right) = 0$$

Resta y utiliza la ecuaciónón termodinámica

$$\left[\frac{\partial}{\partial t} + U_m \frac{\partial}{\partial x}\right]\left(\frac{\partial^2 \psi_T}{\partial x^2} - 2\lambda^2 \psi_T\right) + \beta \frac{\partial \psi_T}{\partial x} + U_T \frac{\partial}{\partial x}\left(\frac{\partial^2 \psi_m}{\partial x^2} + 2\lambda^2 \psi_m\right) = 0$$

Solución

$$\psi_m = A\, e^{ik(x-ct)},\ \psi_{\mathrm{T}} = B\, e^{ik(x-ct)}$$

$$ik\left[(c-U_m)k^2+\beta\right]A - ik^3 U_T B = 0$$

$$ik\left[(c-U_m)\left(k^2+2\lambda^2\right)+\beta\right]B - ikU_T\left(k^2-2\lambda^2\right)A = 0$$

$$\begin{vmatrix} (c-U_m)\,k^2+\beta & -k^2 U_T \\ -U_T\left(k^2-2\lambda^2\right) & (c-U_m)\left(k^2+2\lambda^2\right)+\beta \end{vmatrix} = 0$$

$$\left(c-U_m\right)^2 k^2\left(k^2+2\lambda^2\right)+2\left(c-U_m\right)\beta\left(k^2+\lambda^2\right)+\left[\beta^2+U_T^2 k^2\left(2\lambda^2-k^2\right)\right]=0$$

$$c=U_m-\frac{\beta\left(k^2+\lambda^2\right)}{k^2\left(k^2+2\lambda^2\right)}\pm\delta^{1/2}$$

where

$$\delta\equiv\frac{\beta^2\lambda^4}{k^4\left(k^2+2\lambda^2\right)^2}-\frac{U_T^2\left(2\lambda^2-k^2\right)}{\left(k^2+2\lambda^2\right)}$$

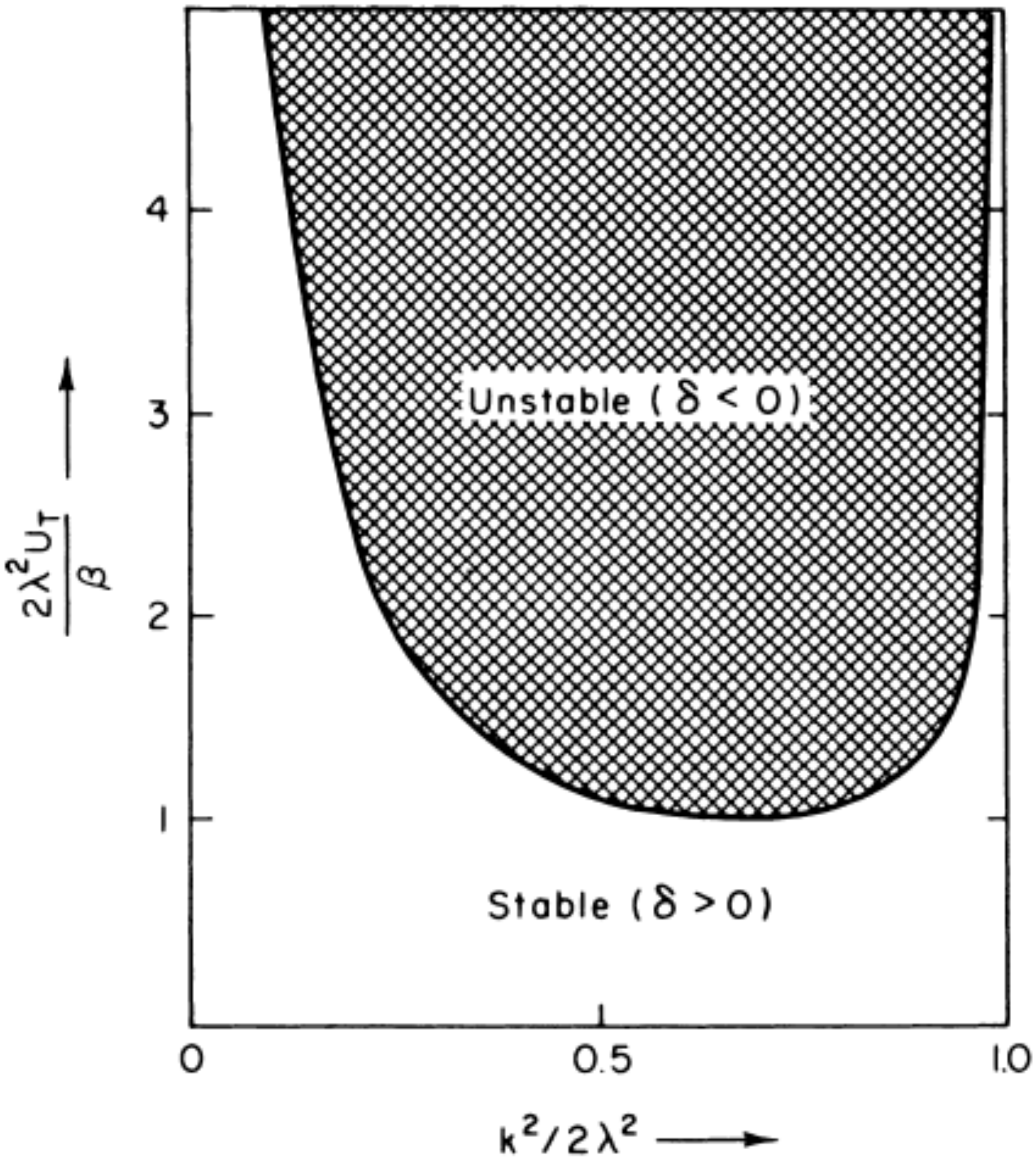
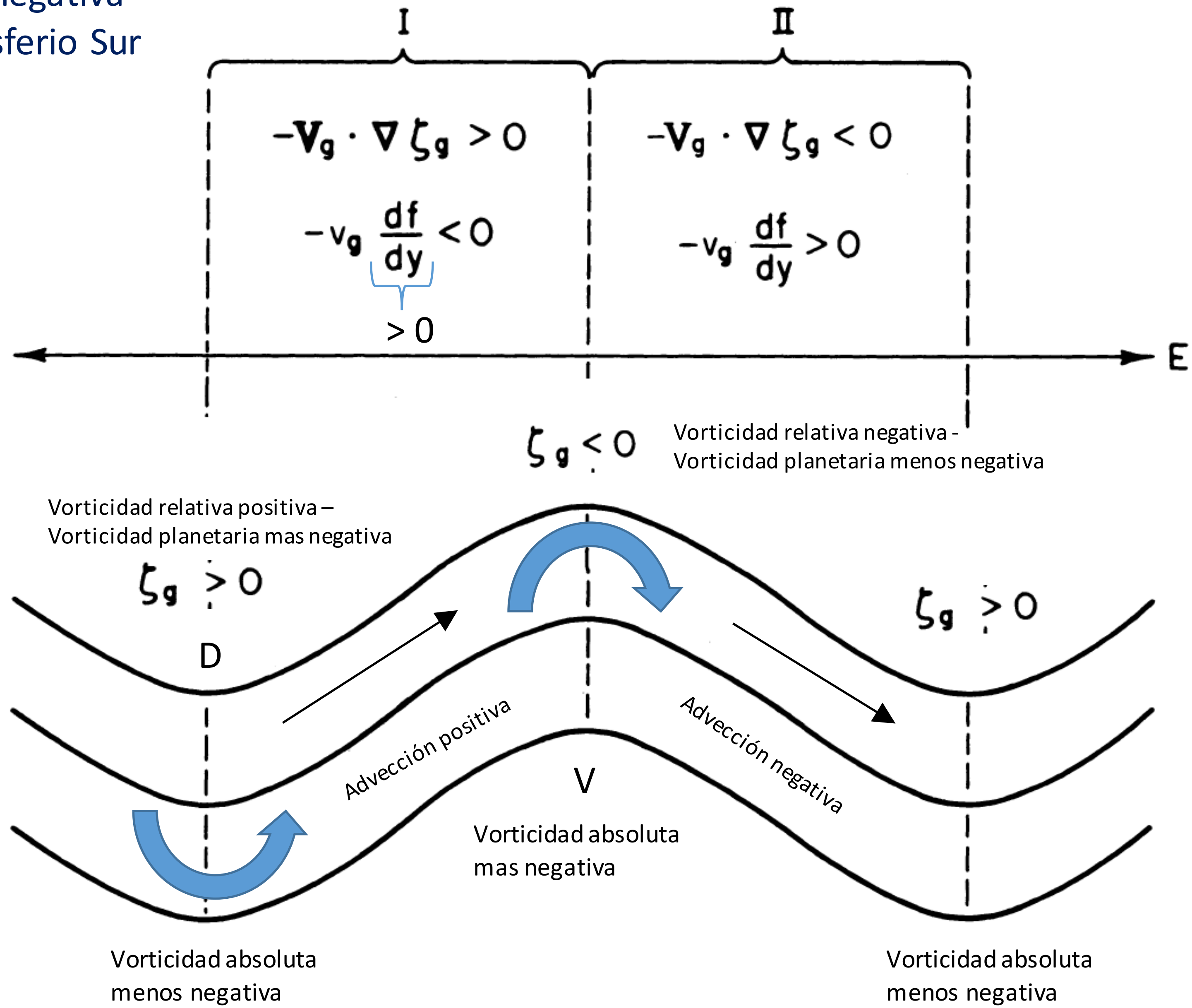


Fig. 8.3 Neutral stability curve for the two-level baroclinic model.

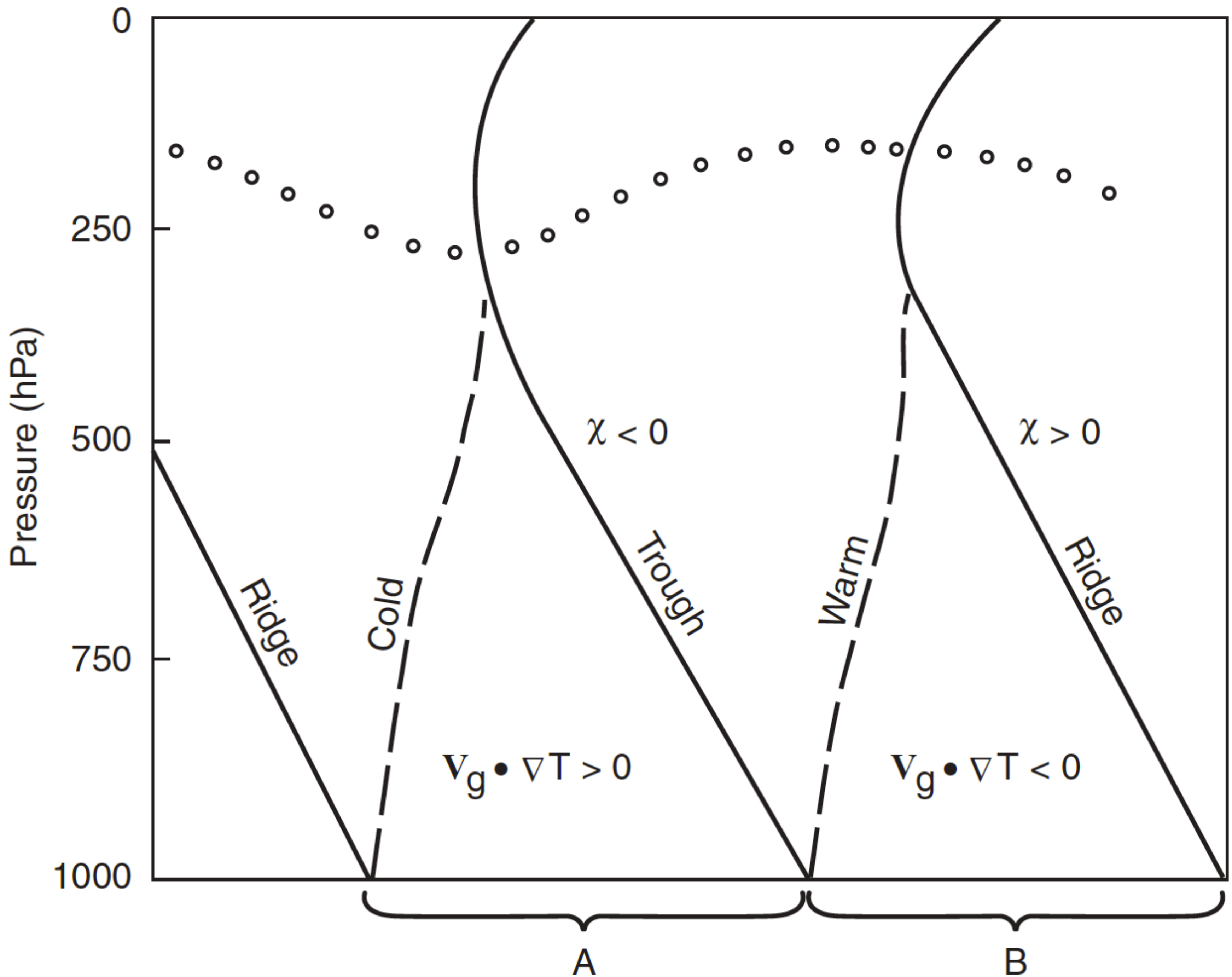
F vorticidad planetaria negativa en el Hemisferio Sur

$$-\mathbf{V}_g \bullet \nabla (\zeta_g + f) = -\mathbf{V}_g \bullet \nabla \zeta_g - \beta v_g$$



-Vorticidad relativa tiende a mover hacia el Este

- Vorticidad planetaria tiende a mover hacia el Oeste



0
250
500
750
1000
Pressure (hPa)
Ridge
Cold
Trough
Warm
Ridge
χ < 0
χ > 0
$\mathbf{V}_g \bullet \nabla T > 0$
$\mathbf{V}_g \bullet \nabla T < 0$
A
B

Movimiento vertical

$$\sigma \boldsymbol{\nabla}^2 \omega + f_0^2 \frac{\partial^2 \omega}{\partial p^2} = -2 \boldsymbol{\nabla} \bullet \mathbf{Q} + f_0 \beta \frac{\partial v_g}{\partial p} - \frac{\kappa}{p} \boldsymbol{\nabla}^2 J$$

$$\sigma \left(\nabla^2 - 2\lambda^2 \right) \omega_2 = -2 \boldsymbol{\nabla} \bullet \mathbf{Q}$$

$$\mathbf{Q} \equiv (Q_1, Q_2) = \left(-\frac{R}{p} \frac{\partial \mathbf{V}_g}{\partial x} \bullet \boldsymbol{\nabla} T, -\frac{R}{p} \frac{\partial \mathbf{V}_g}{\partial y} \bullet \boldsymbol{\nabla} T \right)$$

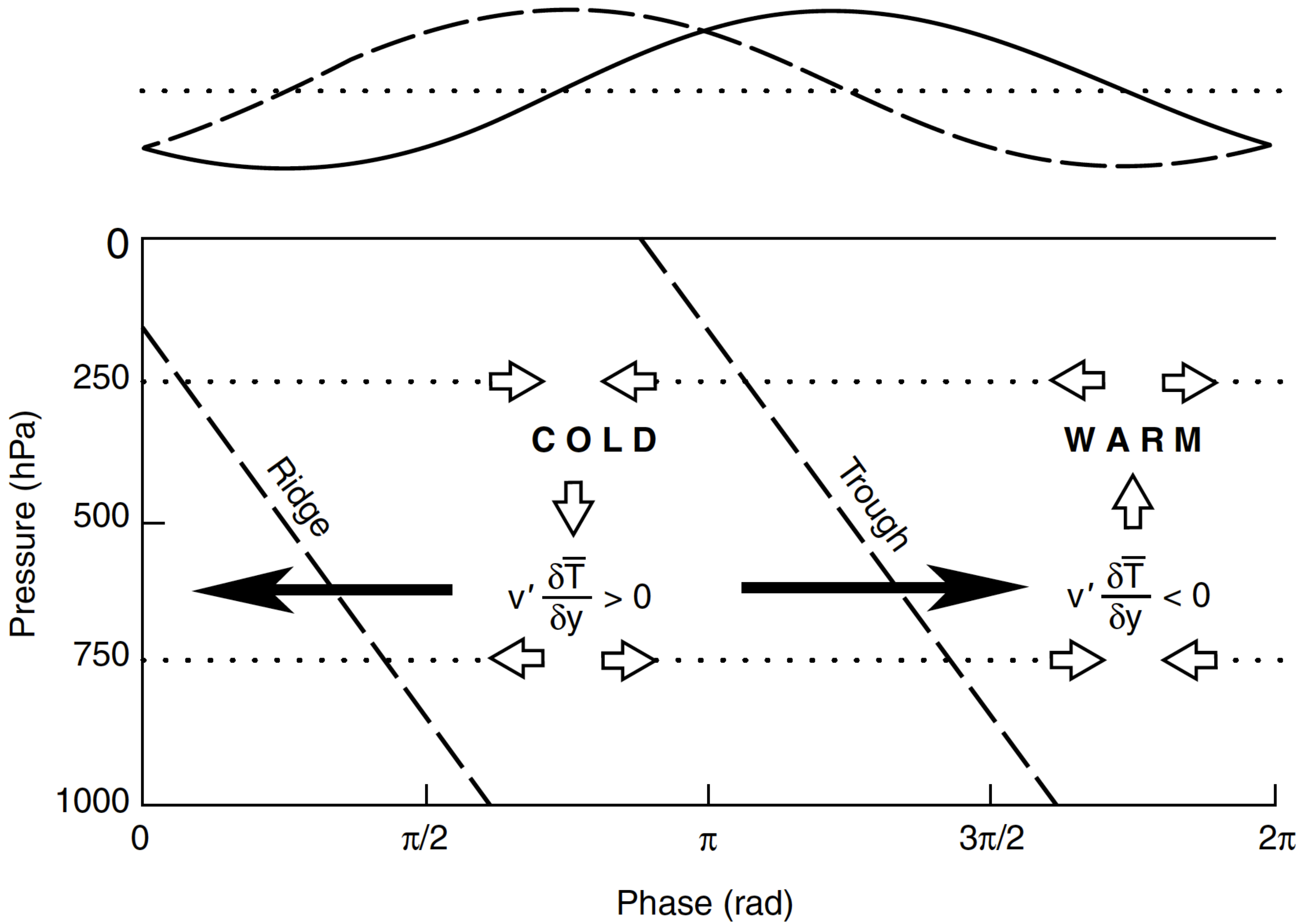
$$\mathbf{Q} = \frac{f_0}{\delta p} \left[-\frac{\partial \mathbf{V_2}}{\partial x} \bullet \boldsymbol{\nabla} (\psi_1 - \psi_3), \; -\frac{\partial \mathbf{V_2}}{\partial y} \bullet \boldsymbol{\nabla} (\psi_1 - \psi_3) \right]$$

Linealiza

$$\left(\frac{\partial^2}{\partial x^2} - 2\lambda^2\right)\omega_2' = -\frac{4 f_0}{\sigma\,\delta p} U_T \frac{\partial \zeta_2'}{\partial x}$$

$$\left(\frac{\partial^2}{\partial x^2} - 2\lambda^2\right)\omega_2' \propto -\omega_2'$$

$$w_2' \propto -\omega_2' \propto -U_T \frac{\partial \zeta_2'}{\partial x} \propto -v_2' \frac{\partial \overline{T}}{\partial y}$$



0
250
500
750
1000
Pressure (hPa)
Ridge
Trough
COLD
WARM
$v' \frac{\delta \overline{T}}{\delta y} > 0$
$v' \frac{\delta \overline{T}}{\delta y} < 0$
0
π/2
π
3π/2
2π
Phase (rad)