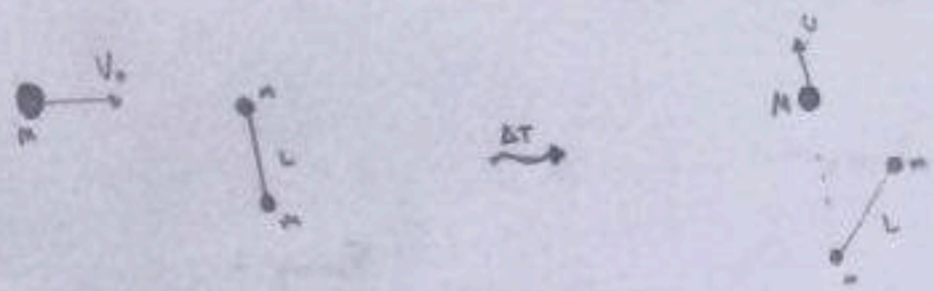


(a) * Conserv. momentum lineal

$$P_{ix} = MV_0 \qquad P_{iy} = 0$$

$$P_{fx} = P_{fb}\sin\theta \qquad P_{fy} = MU + P_{fb}\cos\theta$$

$$\Rightarrow \quad P_i = P_{fb}\sin\theta \ (1) \qquad P_U = -P_{fb}\cos\theta \ (2)$$

2 ptos

* Conserv. Energía cinética $\left(v^2 = \frac{p^2}{m^2}\right)$

$$\frac{P_i^2}{2M} = \frac{P_U^2}{2M} + \frac{P_{fb}^2}{4m} \Rightarrow \frac{P_i^2}{M} = \frac{P_U^2}{M} + \frac{P_{fb}^2}{2m}$$

$$(1)^2 + (2)^2 : \ P_{fb}^2 = P_i^2 + P_U^2 \Rightarrow \frac{P_i^2}{M} = \frac{P_U^2}{M} + \frac{P_i^2}{2m} + \frac{P_U^2}{2m}$$

$$\Rightarrow P_U^2 \underbrace{\left(\frac{1}{M} + \frac{1}{2m}\right)}_{\frac{2m+M}{2mM}} = P_i^2 \underbrace{\left(\frac{1}{M} - \frac{1}{2m}\right)}_{\frac{2m-M}{2mM}} \Rightarrow M^2U^2 = M^2V_0^2\,\frac{(2m-M)}{(2m+M)}$$

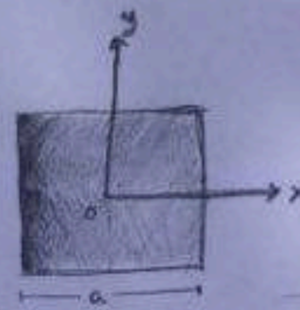
$$\Rightarrow U^2 = V_0^2\,\frac{(2m-M)}{(2m+M)}$$

2ptos

(b) $U > 0 \qquad 2m - M > 0 \Rightarrow M > 2m$

Pauta P1 C3 Mecánica

P1] (a) $I = \begin{pmatrix} \int (y^2+z^2) & -\int xy & -\int zx \\ -\int xy & \int (z^2+x^2) & -\int yz \\ -\int zx & -\int yz & \int (x^2+y^2) \end{pmatrix}$



$\rho = \frac{m}{a^2}$

$\Rightarrow I_{xx} = \int (y^2+z^2)\rho \, dx\, dy = \int_{-a/2}^{a/2}\int_{-a/2}^{a/2} y^2 \rho \, dx\, dy = a \int_{-a/2}^{a/2} \rho y^2 dy$

$= \rho a \cdot \underbrace{\frac{y^3}{3}\Big|_{-a/2}^{a/2}}_{\frac{1}{3}\left(\frac{a^3}{8}+\frac{a^3}{8}\right)} = \rho \frac{a^4}{12} = \frac{m}{a^2}\frac{a^4}{12} = \frac{ma^2}{12}$

análogo para I_{yy} , $I_{zz} = I_{xx} + I_{yy} = \frac{ma^2}{6}$ 2.5 ptos

$\Rightarrow I_{xy} = -\int (xy)\rho \, dx\, dy = -\rho \int_{-a/2}^{a/2} x\, dx \int_{-a/2}^{a/2} y\, dy = -\rho \underbrace{\frac{x^2}{2}\Big|_{-a/2}^{a/2}}_{\left(\frac{a^2}{8}+\frac{a^2}{8}\right)} \frac{y^2}{2}\Big|_{-a/2}^{a/2}$

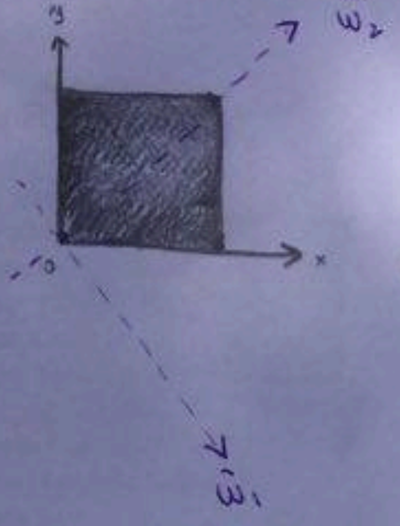
$= -\rho \frac{a^2}{4}\frac{a^2}{4} = -\frac{m}{a^2}\frac{a^4}{4} = -\frac{ma^2}{4}$

análogo para I_{yx}

0.5 ptos

$I_{zx} = I_{zy} = 0$ (válido para los términos simétricos)

(b)



ejes principales: porque los elementos $\int xy$ se anulan, para cada x hay un -x.
Es simétrico respecto a estos ejes.
Puede rotar respecto a estos ejes.

0.5 ptos.

· Podemos trasladarnos a I_o usando teorema ejes paralelos.

Se encuentran a una distancia $d = \frac{\sqrt{2}}{2} a \hat{\omega}_2$ por teor. pitágoras

$\hat{\omega}_2 = \cos 45° \hat{x} + \sin 45° \hat{y} = \frac{\sqrt{2}}{2}\hat{x} + \frac{\sqrt{2}}{2}\hat{y}$ 1 pto.

$\Rightarrow I_o = I_{cm} + m\left(\frac{a^2}{4}\hat{x} + \frac{a^2}{4}\hat{y}\right)$

$\Rightarrow$ los elementos fuera de la diagonal se hacen 0
lo que es consistente con que sean los ejes ppales
y el resto $I_{xx} = I_{yy} = \frac{1}{3}ma^2$ $I_{zz} = \frac{2}{3}ma^2$

1.5 ptos.

* Esta pauta es solo una referencia.
El desarrollo en cualquier orden será considerado válido.

P2] Otras alternativas.

Forma 1

$\overset{\Delta t}{\leadsto}$

(sigue sobre P)

(a)

Cons. Energía $E_0 = E_f$

$$\frac{M}{2} V_0^2 = \frac{MU^2}{2} + \frac{mv^2}{2}$$

$$\Rightarrow \quad U^2 = V_0^2 - \frac{m}{M} v^2$$ 1.5 ptos

Cons momentum lineal

$$M V_0 = m v \operatorname{sen}\theta \ \hat{x} \qquad \left(v = \frac{MU}{m \cos\theta} \Rightarrow \tan\theta = \frac{V_0}{U} \right)$$

* determina el ángulo

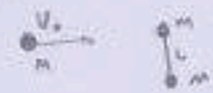
$$\Rightarrow \quad v = \frac{M V_0}{m \operatorname{sen}\theta} \Rightarrow v^2 = \frac{M^2 V_0^2}{m^2 \operatorname{sen}\theta^2}$$ 1.5 ptos

$$\Rightarrow \quad U^2 = V_0^2 - \frac{M V_0^2}{m \operatorname{sen}\theta^2} = V_0^2\left(1 - \frac{M}{m \operatorname{sen}\theta^2}\right)$$

(b)

$$U > 0 \quad \Rightarrow \quad m > m \operatorname{sen}\theta$$ 1.5 ptos

Forma 2.



$\overset{\Delta t}{\leadsto}$

(gira sobre sí mismo)

(a) Cons. Energía $E_0 = E_f$

$$\frac{M V_0^2}{2} = \frac{MU^2}{2} + \frac{I\dot{\theta}^2}{2}$$

$$\Rightarrow \quad U^2 = V_0^2 - \frac{I\dot{\theta}^2}{M}$$ 1.5 p

$$I_G = mL^2 \qquad I_{cm} = \frac{mL^2}{2}$$

Cons. momentum angular

$$\vec{v}_m = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

$$\vec{\ell} = \vec{r} \times \vec{p} = L\hat{r} \times m(L\dot{\theta}\hat{\theta}) = mL^2\dot{\theta}\hat{k}$$

$$\tau = r \times \vec{F} = L\, mg \cos\theta$$

$$\left(mg(\operatorname{sen}\theta\,\hat{\theta} - \cos\theta\,\hat{r}) \right)$$

$$\Rightarrow \quad \dot{\theta} = \frac{g}{L} \operatorname{sen}\theta$$

1.5

$$U^2 = V_0^2 - \frac{mL^2}{2M}\frac{g^2}{L^2}\operatorname{sen}^2\theta$$

$$U^2 = V_0^2 - \frac{g\,m \operatorname{sen}^2\theta}{2M}$$

(b) $U > 0 \qquad M > \frac{g\,m}{2V_0^2}$

1.5 ptos