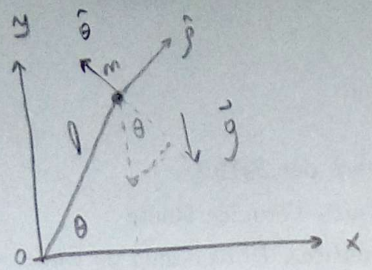
Pauta C₁ Mecánica

P₁)



1 pto. (por las fuerzas)

L cte

(a) $\vec{r} = \rho\hat{\rho} \Rightarrow \dot{\vec{r}} = \dot{\rho}\hat{\rho} + \rho\dot{\theta}\hat{\theta} \Rightarrow \ddot{\vec{r}} = \rho\ddot{\theta}\hat{\theta} - \rho\dot{\theta}^2\hat{\rho}$ 1 pto. (entender coord.)

$\Rightarrow$ $\hat{\theta})$ $mL\ddot{\theta} = -mg\cos\theta$ $\hat{\rho})$ $-mL\dot{\theta}^2 = T - mg\operatorname{sen}\theta$

T positiva para $\hat{\rho}$ pos

1.5pto. (Juntarlas en las ecuaciones)

(b)

$$\ddot{\theta} = -\frac{g}{l}\cos\theta \;/\; \dot{\theta}$$

1.5 pto. (integrar)

$$\frac{\dot{\theta}^2}{2} = -\frac{g}{l}\operatorname{sen}\theta \Rightarrow \dot{\theta}^2 = -\frac{2g}{l}\operatorname{sen}\theta$$

$\Rightarrow$ $T = 2gm\operatorname{sen}\theta + mg\operatorname{sen}\theta = 3mg\operatorname{sen}\theta$ 1 pto. (despejar T)

$\operatorname{sen}\theta > 0 \Rightarrow T > 0 \Rightarrow$ tensión

$\operatorname{sen}\theta < 0 \Rightarrow T < 0 \Rightarrow$ compresión 1 pto. (plantear relación con θ)

Pauta C1 Mecánica

P2) $y^2 = 4f_0^2 - 4f_0 x$

0.5 entender restricción

(a) $\vec{r} = x\hat{x} + y\hat{y}$, pero con la restricción $y = \sqrt{4f_0^2 - 4f_0 x}$

$\vec{r} = x\hat{x} + \sqrt{4f_0^2 - 4f_0 x}\,\hat{y}$

→ velocidad: $\frac{d\vec{r}}{dt} = \dot{x} - \frac{2f_0\dot{x}}{(4f_0^2 - 4f_0 x)^{1/2}} = \dot{x}\left(1 - \frac{2f_0}{(4f_0^2-4f_0x)^{1/2}}\right)$ 1 velocidad (derivar)

→ aceleración: $\frac{d^2\vec{r}}{dt^2} = \ddot{x} - \left(\frac{2f_0\ddot{x}(4f_0^2-4f_0x)^{1/2} - \frac{2f_0\dot{x}\,4f_0\dot{x}}{2(4f_0^2-4f_0x)^{1/2}}}{(4f_0^2-4f_0x)}\right)$

$\frac{d^2\vec{r}}{dt^2} = \ddot{x} - \left(\frac{\ddot{x}\,2f_0}{(4f_0^2-4f_0x)^{1/2}} - \frac{4f_0\dot{x}^2}{(4f_0^2-4f_0x)^{3/2}}\right)$ 1 aceleración (derivar)

(b) $r\cos^2(\theta/2) = f_0$; En coordenadas polares $x = r\cos\theta$ $y = r\,\text{sen}\,\theta$

$r^2\text{sen}^2\theta = 4f_0^2 - 4f_0 r\cos\theta \Rightarrow 4f_0^2 - 4f_0 r\cos\theta - r^2\text{sen}^2\theta = 0$

$\Rightarrow f_0 = \frac{4r\cos\theta \pm \sqrt{16r^2\cos^2\theta + 16r^2\text{sen}^2\theta}}{8} = \frac{r\cos\theta}{2} \pm \frac{r}{2} = r\left(\frac{\cos\theta \pm 1}{2}\right)$

1.5 (demostrar)

$\Rightarrow f_0 = r\cos^2(\theta/2)$

(c) $\vec{r} = r\hat{r}$, → velocidad $\dot{\vec{r}} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$ { pero $r = \frac{f_0}{\cos^2(\theta/2)}$ y $\dot{r} = \frac{f_0}{2\cos(\theta/2)}\,\text{sen}(\theta/2)\,\frac{\dot{\theta}}{2} = \frac{f_0}{4}\tan(\theta/2)\,\dot{\theta}$ }

$\Rightarrow \dot{\vec{r}} = \frac{f_0}{4}\tan(\theta/2)\,\dot{\theta}\,\hat{r} + \frac{f_0}{\cos^2(\theta/2)}\,\dot{\theta}\,\hat{\theta}$

$= f_0\dot{\theta}\left(\frac{\tan(\theta/2)}{4}\hat{r} + \frac{1}{\cos^2(\theta/2)}\hat{\theta}\right)$ 0.5 ($\dot{v}$ en polares)

$$\Rightarrow \ddot{\vec{r}} = \ddot{r}\hat{r} + \dot{r}\dot{\theta}\hat{\theta} + \dot{r}\dot{\theta}\hat{\theta} + r\ddot{\theta}\hat{\theta} - r\dot{\theta}^2\hat{r}$$

$$= (\ddot{r} - r\dot{\theta}^2)\hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta}$$

0.5 aceleracion polares

$$\Rightarrow \ddot{\vec{r}} = \left(\frac{f_0 \sec^2(\theta/2)}{8}\dot{\theta}^2 - \frac{f_0}{\cos^2(\theta/2)}\dot{\theta}^2\right)\hat{r}$$

$$+ \left(\frac{f_0 \tan(\theta/2)}{2}\dot{\theta}^2 + \frac{f_0}{\cos^2(\theta/2)}\ddot{\theta}\right)\hat{\theta}$$

$$r = \frac{f_0}{\cos^2(\theta/2)}$$

$$\dot{r} = \frac{f_0 \tan(\theta/2)}{4}\dot{\theta}$$

$$\ddot{r} = \frac{f_0}{8}\sec^2(\theta/2)\dot{\theta}^2 + \frac{f_0}{4}\tan(\theta/2)\ddot{\theta}$$

1 pto. relacionar $r(\theta)$

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