Symon. Sección 2. $P_1 \to P_5$ // $P_2 \to P_{21}$

P1 m, $t=0$ reposo (P_2 tarea)

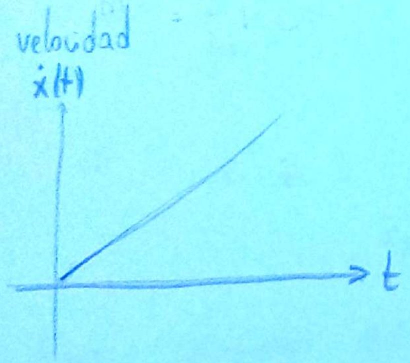
(a) $m\ddot{x} = F_0 \operatorname{sen}^2 \omega t$

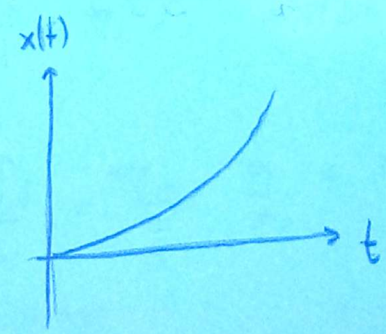
$$m\dot{x} = \int F_0 \operatorname{sen}^2 \omega t = \int F_0 \left(\frac{1}{2} - \frac{\cos(2\omega t)}{2}\right) dt = \frac{F_0 t}{2} - \frac{F_0 \operatorname{sen}(2\omega t)}{4\omega}$$

$$\Rightarrow \dot{x}(t) = \frac{F_0 t}{2m} - \frac{F_0 \operatorname{sen}(2\omega t)}{4m\omega} + \frac{c}{m} \Big/ \int dt \; ; \quad \left.\dot{x}(t=0) = 0 = \frac{c}{m}\right\} \text{condición inicial}$$

$$\Rightarrow x(t) = \frac{F_0}{4m} t^2 - \frac{F_0}{4m\omega} \cdot \left(-\frac{\cos(2\omega t)}{2\omega}\right) + x_0$$

$$\Rightarrow x(t) = \frac{F_0}{4m} t^2 + \frac{F_0}{8m\omega^2} \cos(2\omega t) + x_0$$





P2] Potencial

(a) Dado un potencial $U(x) = -Cx^n e^{(-\alpha x)}$ $\boxed{n, \alpha > 0}$

calcule la Fuerza

$$F = -\nabla U = -\frac{\partial U}{\partial x} = -\frac{\partial}{\partial x}\left(-Cx^n e^{(-\alpha x)}\right)$$

$$= -\left(-nCx^{n-1} e^{-\alpha x} + Cx^n \alpha e^{(-\alpha x)}\right)$$

$$= nCx^{n-1}e^{-\alpha x} - Cx^n\alpha e^{(-\alpha x)}$$

(b) Energía mínima. En el mínimo

$$\Rightarrow \frac{\partial U}{\partial x} = 0 \Rightarrow \cancel{C}x^n\alpha\, \cancel{e^{(-\alpha x)}} - n\cancel{C}x^{n-1}\cancel{e^{-\alpha x}} = 0$$

$$\Rightarrow x^n\alpha - nx^{n-1} = 0 \Rightarrow \frac{\alpha}{n} = x^{-1} \Rightarrow \boxed{x_{eq} = \frac{n}{\alpha}}$$

Para verificar que es mínimo $\left.\frac{\partial^2 U}{\partial x^2}\right|_{x_{eq}} > 0$

$$\left.\frac{\partial^2 U}{\partial x^2}\right|_{x=\frac{n}{\alpha}} = nCx^{n-1}\alpha e^{-\alpha x} - Cx^n\alpha^2 e^{-\alpha x} - n(n-1)Cx^{n-2}e^{-\alpha x}$$
$$+ nCx^{n-1}\alpha e^{-\alpha x} = 0$$

$$= Ce^{-\alpha x}\left(\alpha n x^{n-1} - x^n\alpha^2 - n(n-1)x^{n-2} + n\alpha x^{n-1}\right)$$

$$= Ce^{-n}\left(\cancel{n}\left(\frac{n}{\alpha}\right)^n \frac{\alpha^2}{\cancel{\alpha}} - \left(\frac{n}{\alpha}\right)^n\alpha^2 - \underbrace{n(n-1)}_{n^2-n}\left(\frac{n}{\alpha}\right)^n\frac{\alpha^2}{n^2} + \cancel{n}\alpha\left(\frac{n}{\alpha}\right)^n\frac{\alpha}{\cancel{\alpha}}\right)$$

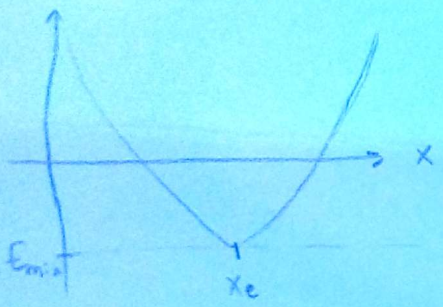
$$= Ce^{-n}\left(n^n\cancel{\alpha^{2-n}} - n^n\cancel{\alpha^{2-n}} - n^n\cancel{\alpha^{2-n}} + n^{n-1}\alpha^{2-n} + n^n\cancel{\alpha^{2-n}}\right)$$

$$\Rightarrow \left.\frac{\partial U^2}{\partial x^2}\right|_{x=x_{eq}} = \frac{c\, e^{-n}\, n^{n-1}}{\alpha^{n-2}} > 0 \quad \Rightarrow \quad x = \frac{n}{\alpha} \text{ mínimo}$$

$$\Rightarrow \quad U(x_{min}) = -C\left(\frac{n}{\alpha}\right)^n e^{(-\alpha n/\alpha)} = -C\left(\frac{n}{\alpha}\right)^n e^{-n} \text{ energía mínima}$$

(c) Comente como se mueve la partícula:

- debe ser mayor a E_{min} para que se mueva. si es mucho mayor se dispara al ∞, si no, hay oscilaciones.



(d) Frec. pequeñas oscilaciones.

$$\omega_{p.o}^2 = \frac{1}{m}\frac{\partial^2 U_{eq}}{\partial^2 x} = \frac{c\, e^{-n}\, n^{n-1}}{m.\alpha^{n-2}}$$

$\hat{\rho} = \hat{\imath}\cos\theta + \hat{\jmath}\,\mathrm{sen}\,\theta$ $\qquad$ $\dot{\hat{\rho}} = \dot{\theta}\,\hat{\theta}$

$\hat{\theta} = -\hat{\imath}\,\mathrm{sen}\,\theta + \hat{\jmath}\cos\theta$ $\qquad$ $\dot{\hat{\theta}} = -\dot{\theta}\,\hat{\rho}$

$\hat{k} = \hat{k}$

P3) Cilíndricas

trayectoria de una partícula dada por $\rho = A\exp(k\theta)$ $\quad z = h\rho$

(a) $\vec{r}$ en función de θ, A, k, h y V_0

$$\vec{r} = \rho\hat{\rho} + z\hat{k} = Ae^{k\theta}\hat{\rho} + hAe^{k\theta}\hat{k}$$

$$\dot{\vec{r}} = Ake^{k\theta}\dot{\theta}\hat{\rho} + Ae^{k\theta}\underbrace{\dot{\theta}\hat{\theta}}_{\dot{\hat{\rho}}} + hAke^{k\theta}\dot{\theta}\hat{k}$$

$$= Ae^{k\theta}\dot{\theta}\left(k\hat{\rho} + \hat{\theta} + hk\hat{k}\right)$$, no sabemos $\dot{\theta}$

pero sabemos $\underbrace{\|\dot{\vec{r}}\|}_{\sqrt{\dot{r}^2}} = V_0 \Rightarrow Ae^{k\theta}\dot{\theta}\sqrt{k^2+1+h^2k^2} = V_0 \Rightarrow \dot{\theta} = \dfrac{V_0}{Ae^{k\theta}\sqrt{k^2+1+h^2k^2}}$

$$\Rightarrow \quad \dot{\vec{r}} = \frac{V_0\left(k\hat{\rho} + \hat{\theta} + hk\hat{k}\right)}{\sqrt{k^2+1+h^2k^2}}$$

(b) aceleración

$$\ddot{\vec{r}} = \frac{V_0}{\sqrt{}}\left(k\dot{\theta}\hat{\theta} - \dot{\theta}\hat{\rho} + 0\right) = \frac{V_0\dot{\theta}}{\sqrt{k^2+1+h^2k^2}}\left(k\hat{\theta} - \hat{\rho}\right)$$

$$\ddot{\vec{r}} = \frac{V_0^2\left(k\hat{\theta} - \hat{\rho}\right)}{Ae^{k\theta}\left(k^2+1+h^2k^2\right)}$$

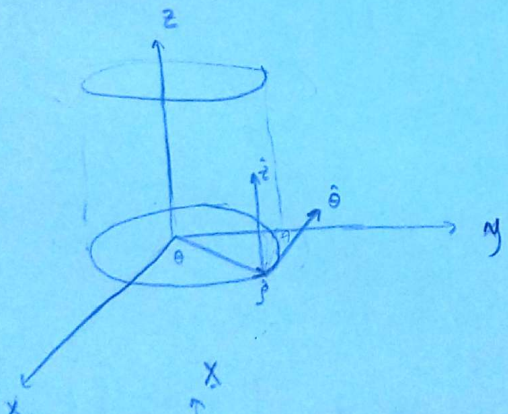
(c) Encuentre una expresión para $\theta(t)$

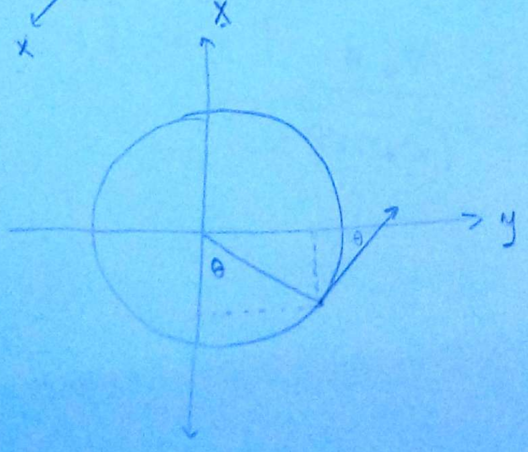
$$\dot{\theta} = \frac{d\theta}{dt} = \frac{V_0}{A e^{k\theta}\sqrt{k^2+1+h^2k^2}} \quad \Big/ \int \quad \Rightarrow \int e^{k\theta}\, d\theta = \int \frac{V_0}{\sqrt{k^2+1+h^2k^2}\, A}\, dt$$

$$\Rightarrow \quad \frac{e^{k\theta}}{k} = \frac{V_0}{\sqrt{k^2+1+h^2k^2}\, A}\, t + C \quad \Big/ \cdot k,\ \ln(\,)$$

$$k\theta = \ln\left(\frac{V_0 k}{\sqrt{k^2+1+h^2k^2}\, A}\, t + kC\right)$$

$$\Rightarrow \quad \theta(t) = \frac{1}{k} \ln\left(\frac{V_0 k}{A\sqrt{k^2+1+h^2k^2}}\, t + kC\right)$$





$$\hat{\rho} = +\hat{i}\cos\theta + \hat{j}\operatorname{sen}\theta$$

$$\hat{\theta} = -\hat{i}\operatorname{sen}\theta + \hat{j}\cos\theta$$