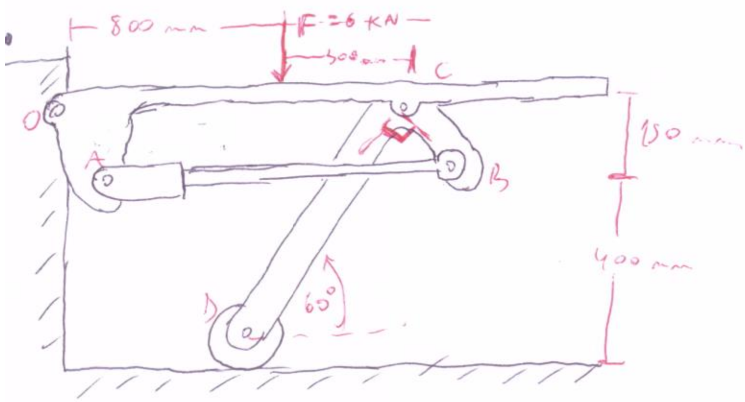
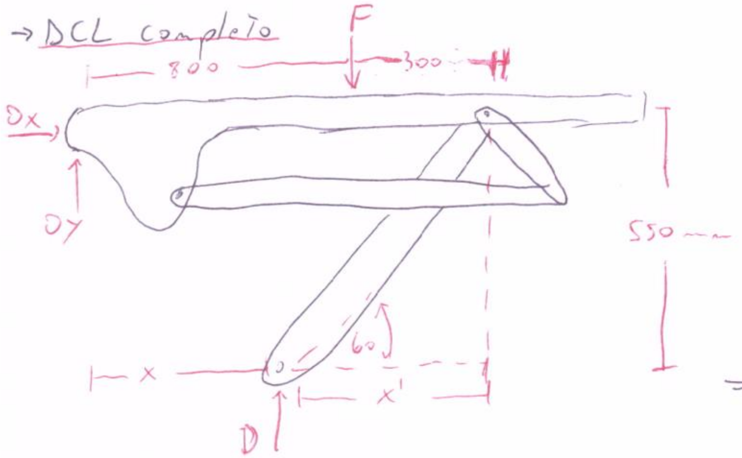


$L_{AB} = 1000\ mm$

$E = 190\ GPa$

$F.S = 2,5$

→ DCL completo



$\cdot \sum_O M_z = 0 \rightsquigarrow x \cdot D - 800F = 0$

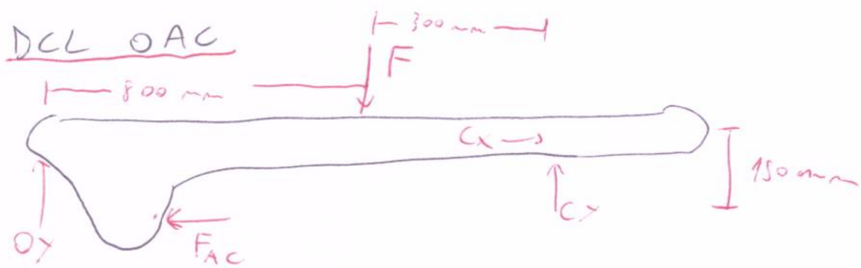
$* \tan 60 = \frac{550}{x'} \Rightarrow x' = \frac{550}{\tan 60} = 317\ mm$

$\boxed{x = 1100 - x' = 782,46\ m}$

$\Rightarrow \boxed{D = \frac{800}{x} F = 6,13\ kN}$

$\cdot \sum F_x = 0 \rightsquigarrow O_x = 0 \ \wedge \ \cdot \sum F_y = 0 \rightsquigarrow \boxed{O_y = F - D = -0,13\ kN}$

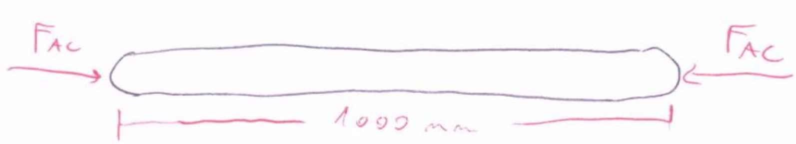
→ DCL OAC



$\cdot \sum_C M_z = 0 \rightarrow -150 F_{AC} - 1100 O_y + 300 F = 0$

$\boxed{\therefore F_{AC} = 12,953\ kN}$

→ DCL F_{AC}



→ Análisis de pandeo

$$\rightarrow \hat{y}(x) = C_1 \cdot \operatorname{sen}\left(\underbrace{\sqrt{\frac{P}{EI_z}}}_{\lambda} x\right) + C_2 \cos\left(\underbrace{\sqrt{\frac{P}{EI_z}}}_{\lambda} x\right) + C_3 x + C_4$$

$$\rightarrow \frac{d\hat{y}}{dx}(x) = C_1 \lambda \cos(\lambda x) - C_2 \lambda \operatorname{sen}(\lambda x) + C_3$$

$$\rightarrow \frac{d^2\hat{y}}{dx^2}(x) = -C_1 \lambda^2 \operatorname{sen}(\lambda x) - C_2 \lambda^2 \cos(\lambda x)$$

→ C.B.

- $\frac{d^2\hat{y}}{dx^2}(0) = 0 \Rightarrow C_2 = 0$
- $\hat{y}(0) = 0 \Rightarrow C_4 = 0$
- $\frac{d^2\hat{y}}{dx^2}(L) = 0 \Rightarrow -C_1 \lambda^2 \operatorname{sen}(\lambda L) = 0 \Rightarrow \boxed{C_1 = 0 \quad \vee \quad \operatorname{sen}(\lambda L)}$ (*)
- $\hat{y}(L) = 0 \Rightarrow C_1 \cdot \operatorname{sen}(\lambda L) + C_3 L = 0$

→ Si tomamos $C_1 = 0 \rightsquigarrow C_3 = 0 \Rightarrow \hat{y}(x) = 0$ (Trivial)

→ Si tomamos $\operatorname{sen}(\lambda L) = 0 \Rightarrow \lambda L = n\pi \quad , \forall n = 1, 2, \ldots$

$$\sqrt{\frac{P}{EI_z}} L = n \cdot \pi$$

$$\Rightarrow P_{cr}^{(n)} = \frac{n^2 \pi^2}{L^2} E I_z = \frac{n^2\pi^2}{L^2} E \frac{\pi d^4}{64} \Rightarrow \boxed{d^{(n)} = \sqrt[4]{\frac{64 L^2 P_{cr}^{(n)}}{\pi E n^2 \pi^2}}}$$

→ Analizamos el 1º modo de pandeo : $P_{cr}^{(n)} = P_{cr}^{(1)} = P_{ac} \wedge n = 1$

$$\boxed{\therefore d = \sqrt[4]{\frac{64 L^2 P_{ac}}{\pi^3 E}} = 0{,}0184 \text{ m}}$$

Pauta Aux. 7 ME3204

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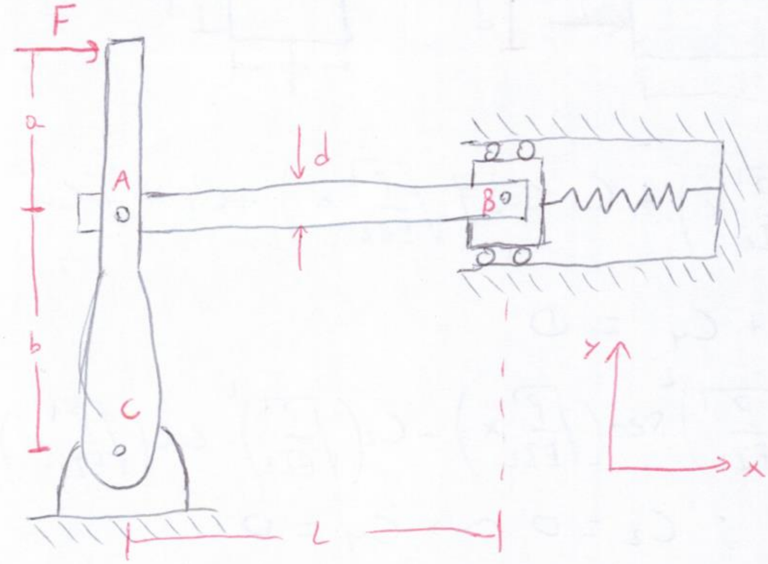
P1



$\sigma_0 = 350\ MPa$

$E = 190\ GPa$

$L = 20\ cm$

$b = 8\ cm$

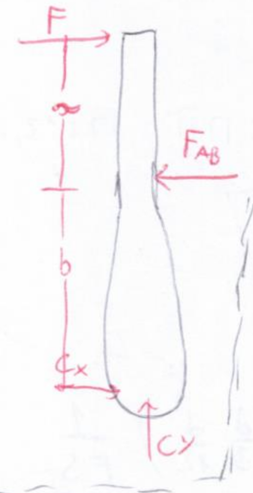
$a = 10\ cm$

$d = 8\ mm$

$F.S = 2$

- Existen 3 fuentes de falla:
  - Falla por plasticidad en compresión
  - Falla por pandeo en el plano x-y
  - Falla por pandeo en el plano x-z

DCL (Para encontrar fuerza de compresión en AB)



$$\sum_c M_z = 0 \Rightarrow b \cdot F_{AB} - (a+b) \cdot F = 0$$

$$\boxed{\therefore F_{AB} = \frac{(a+b)}{b} F}$$

↳ Falla por plasticidad: Encontremos σ_x

↳ $\sigma_x = \frac{F_{AB}}{A}$, pero σ_x debe cumplir factor de seguridad

$$\Rightarrow \sigma_x = \frac{\sigma_0}{F.S} \Rightarrow \frac{(a+b)}{b} \cdot \frac{4F}{\pi \cdot \frac{d^2}{3}} = \frac{\sigma_0}{F.S}$$

$$\boxed{\therefore F = \frac{\sigma_0 b \cdot d^2}{3 F.S (a+b)} = 1659{,}26\ N}$$

ii) Falla por pandeo en el plano $x-y$

Sdn | $y(x) = C_1 \cdot \operatorname{Sen}\left(\sqrt{\frac{P}{EI_z}}\,x\right) + C_2 \cos\left(\sqrt{\frac{P}{EI_z}}\,x\right) + C_3 x + C_4$

C.b | • $y(0) = 0 \sim C_2 + C_4 = 0$

• $\frac{d^2y}{dx^2}(0) = 0 \sim -C_1\left(\sqrt{\frac{P}{EI_z}}\right)^2 \operatorname{Sen}\left(\sqrt{\frac{P}{EI_z}}\,x\right) - C_2\left(\sqrt{\frac{P}{EI_z}}\right)^2 \cdot \cos\left(\sqrt{\frac{P}{EI_z}}\,x\right) = 0$

$\therefore C_2 = 0 \sim C_4 = 0$

• $\frac{dy}{dx}(L) = 0 \sim -C_1 \frac{P}{EI_z} \operatorname{Sen}\left(\sqrt{\frac{P}{EI_z}}\,L\right) = 0$ (*)

• $y(L) = 0 \sim C_1 \cdot \operatorname{Sen}\left(\sqrt{\frac{P}{EI_z}}\,L\right) + C_3 L = 0$

↳ De (*) $\Rightarrow C_3 = 0$

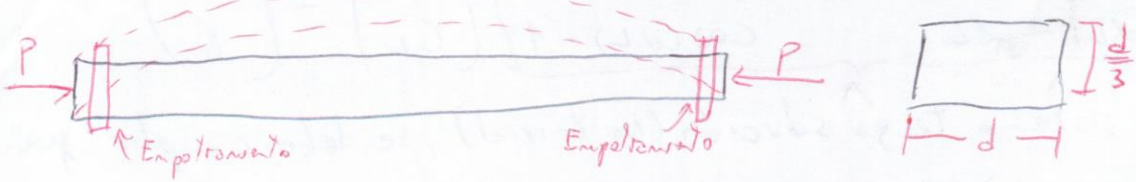
* Tomando (*) $\sim \operatorname{Sen}\left(\sqrt{\frac{P}{EI_z}}\,L\right) = 0 \sim \sqrt{\frac{P}{EI_z}}\,L = n\pi \;, n = 1, 2, \dots$

$$\boxed{\therefore \underset{\text{(Primera Carga crítica)}}{P_{cr}^{(1)}} = \left(\frac{n\pi}{L}\right)^2 E I_z = \left(\frac{\pi}{L}\right)^2 E I_z}$$

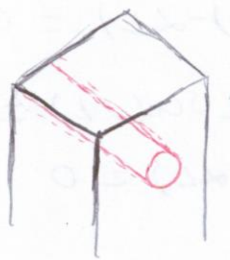
Condición de falla | $F_{AB} = \frac{P_{cr}^{(1)}}{F.S} \sim \frac{(a+b)}{b} F = \left(\frac{\pi}{L}\right)^2 E \cdot \overbrace{\left(\frac{d}{3} \cdot \frac{d^3}{12}\right)}^{I_z} \cdot \frac{1}{F.S}$

$$\boxed{\therefore F = \left(\frac{\pi}{L}\right)^2 \cdot \frac{b\, d^4 E}{36(a+b) \cdot F.S} = 1185{,}33\ N}$$

iii) Falla por pandeo en el plano x-z



Soln)
$$z(x) = C_1 \cdot \text{Sen}\left(\sqrt{\frac{P}{EI_y}}\,x\right) + C_2 \cos\left(\sqrt{\frac{P}{EI_y}}\,x\right) + C_3 x + C_4$$



El pasador permite el giro libre en una dirección (plano x-y), sin embargo, impide completamente el movimiento en la otra (plano x-z), por lo tanto, en ese plano se considera como un empotramiento (y se toman las cond. de borde correspondientes)

C.B)

- $z(0) = 0 \rightsquigarrow \boxed{C_2 + C_4 = 0}$ (1)
- $\frac{dz}{dx}(0) = 0 \rightsquigarrow \boxed{C_1\sqrt{\frac{P}{EI_y}} + C_3 = 0}$ (2)
- $\frac{dz}{dx}(L) = 0 \rightsquigarrow C_1\sqrt{\frac{P}{EI_y}}\cos\left(\sqrt{\frac{P}{EI_y}}\,L\right) - C_2 \,\text{Sen}\left(\sqrt{\frac{P}{EI_y}}\,L\right) + C_3 = 0$

* Reemplazando C_3 desde (2): $\left(\alpha = \sqrt{\frac{P}{EI_y}}\right)$

$$\boxed{C_1\left(\alpha \cos\left(\underbrace{\sqrt{\frac{P}{EI_y}}}_{\alpha}\,L\right) - \alpha\right) - C_2\,\text{Sen}\left(\underbrace{\sqrt{\frac{P}{EI_y}}}_{\alpha}\,L\right)\alpha = 0} \quad (3)$$

- $z(L) = 0 \rightsquigarrow C_1 \,\text{Sen}(\alpha L) + C_2 \cos(\alpha L) + C_3 L + C_4 = 0$

* Reemplazando C_3 desde (2) y C_4 desde (1):

$$\boxed{C_1\left(\text{Sen}(\alpha L) - \alpha L\right) + C_2\left(\cos(\alpha L) - 1\right) = 0} \quad (4)$$

- Es posible expresar las ecs. (3) y (4) como un sistema matricial:

$$\underbrace{\begin{pmatrix} \alpha\cos(\alpha L)-\alpha & -\alpha \operatorname{sen}(\alpha L) \\ \operatorname{sen}(\alpha L)-\alpha L & \cos(\alpha L)-1 \end{pmatrix}}_{A} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

- Para que este sistema tenga solución (No trivial), se debe cumplir que:

$$\det(A) = 0$$

$$\Rightarrow (\alpha\cos(\alpha L)-\alpha)(\cos(\alpha L)-1) + \alpha \operatorname{sen}(\alpha L)(\operatorname{sen}(\alpha L)-\alpha L) = 0$$

$$\alpha\cos^2(\alpha L) - \alpha\cos(\alpha L) - \alpha\cos(\alpha L) + \alpha + \alpha\operatorname{sen}^2(\alpha L) - \alpha L \operatorname{sen}(\alpha L) = 0$$

$$\alpha(\underbrace{\operatorname{sen}^2(\alpha L) + \cos^2(\alpha L)}_{1}) - 2\alpha\cos(\alpha L) + \alpha - \alpha L \operatorname{sen}(\alpha L) = 0$$

$$2\alpha - 2\alpha\cos(\alpha L) - \alpha L \operatorname{sen}(\alpha L) = 0$$

$$\boxed{\therefore 2 - 2\cos(\alpha L) - L\operatorname{sen}(\alpha L) = 0}$$

- Ec. se cumple si $\alpha L = 2n\pi$, $n = 1, 2, 3, \ldots$

$$\Rightarrow \sqrt{\frac{P}{EI_y}}\, L = 2\pi\pi \rightsquigarrow \boxed{P_{cr}^{(1)} = 4\left(\frac{\pi}{L}\right)^2 EI_y}$$

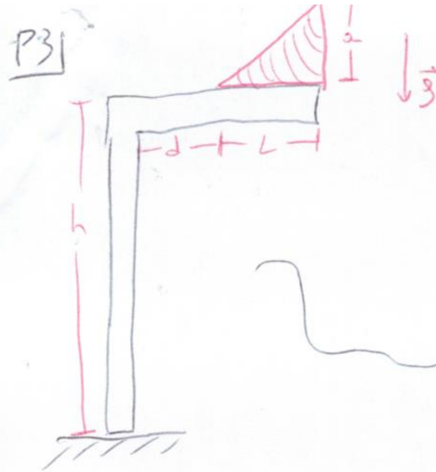
Condición de falla | $F_{AB} = \dfrac{P_{cr}^{(1)}}{F.S} \rightsquigarrow \dfrac{(a+b)}{b} F = \dfrac{4}{F.S}\left(\dfrac{\pi}{L}\right)^2 \cdot E I_y$

$$* \; I_y = \frac{d\cdot\left(\frac{d}{3}\right)^3}{12} = \frac{d^4}{324} \rightsquigarrow \therefore F = \left(\frac{\pi}{L}\right)^2 \frac{4Eb\cdot d^4}{324(a+b)\cdot F.S}$$

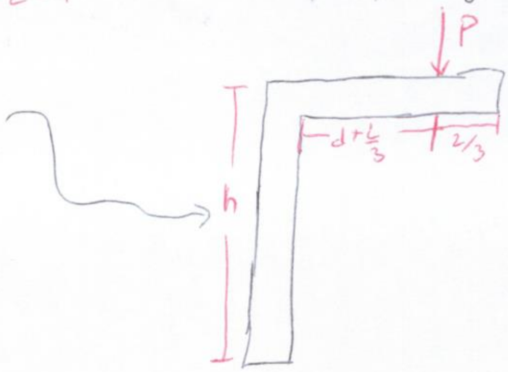
$$\boxed{F = 526{,}81\ N}$$

$$\boxed{\therefore F_{crítico} = 526{,}81\ N \text{ (Falla por pandeo en el plano } x\text{-}z \text{ primero)}}$$

P3)



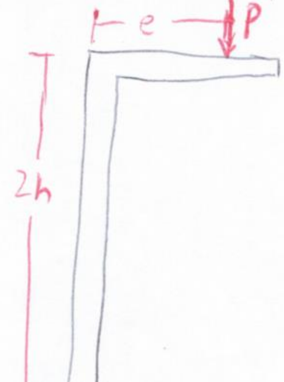
* La veleta se comporta como una fza distribuida lineal, luego



$$P = \rho \cdot V \cdot g$$

$$\boxed{P = \frac{\rho \, a \cdot L \cdot e \cdot g}{2}} \quad (*)$$

* El problema se asemeja a la mitad de un problema de pandeo con excentricidad, luego:



Soln)

$$y(x) = C_1 \operatorname{Sen}\left(\sqrt{\frac{P}{EI_z}}\,x\right) + C_2 \operatorname{Cos}\left(\sqrt{\frac{P}{EI_z}}\,x\right) - e$$

C.B) (Sólo dos)

- $y(0) = 0 \sim C_2 = e$
- $y(2h) = 0 \sim C_1 \operatorname{Sen}\left(\sqrt{\frac{P}{EI_z}}\,2h\right) + e\cos\left(\sqrt{\frac{P}{EI_z}}\,L\right) - e = 0$

$$\boxed{\therefore C_1 = \frac{e\left(1 - \cos\left(\sqrt{\frac{P}{EI_z}}\,L\right)\right)}{\operatorname{Sen}\left(\sqrt{\frac{P}{EI_z}}\,2h\right)}}$$

* En C_1 se puede dar inestabilidad si $\operatorname{Sen}\left(\sqrt{\frac{P}{EI_z}}\,2h\right) = 0$

$$\Rightarrow \sqrt{\frac{P}{EI_z}}\cdot 2h = n\pi \Rightarrow \boxed{P_{cr}^{(n)} = \frac{1}{4}\left(\frac{n\pi}{h}\right)^2 EI_z} \quad (n=1)$$

* $I_z = \frac{\pi}{64}\left(\O_e^4 - \O_i^4\right) = 3{,}68 \cdot 10^{-8}\ m^4$

* Reemplazando ec (*), se tiene $\sim \boxed{a = \frac{EI_z}{2\rho L e g}\left(\frac{\pi}{h}\right)^2 = 0{,}274\ m}$