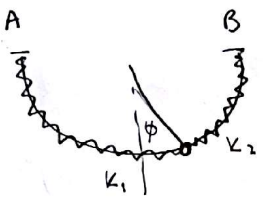
Examen mecânica

P3]



(a) $L = \frac{mR^2\dot{\phi}^2}{2} - k_1\left(R(\pi/2+\phi) - \frac{R\pi}{2}\right)^2 - k_2\left(R(\pi/2-\phi) - \frac{R\pi}{2}\right)^2 + mgR\cos\phi$

ϕ]

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}}\right) = mR^2\ddot{\phi} = -2k_1R^2\phi - 2k_2R^2\phi + mgR\,\mathrm{sen}\,\phi$$

$$\Rightarrow \ddot{\phi} = \frac{-2\phi(k_1+k_2)}{m} + \frac{g}{R}\,\mathrm{sen}\,\phi \;/\; \dot{\phi}$$

(b) $$\Rightarrow \frac{\dot{\phi}^2}{2} = -\frac{\phi^2}{m}(k_1+k_2) - \frac{g}{R}\cos\phi$$

(c)

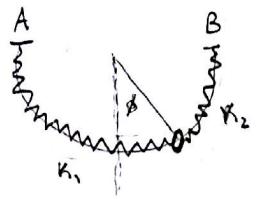
$$U = k_1(R^2\phi^2) + k_2(R^2\phi^2) - mgR\cos\phi$$

$$\frac{dU}{d\phi} = +2\phi(k_1+k_2)R^2 - mgR\,\mathrm{sen}\,\phi = 0 \Rightarrow \phi = 0$$

$$\Rightarrow \frac{d^2U}{d\phi^2} = 2R^2(k_1+k_2) - mgR\underbrace{\cos\phi}_{1}\Big|_{\phi=0} = 2R^2(k_1+k_2) - mgR$$

$$\Rightarrow \omega_{p_0}^2 = \frac{U''|_{\phi=0}}{mR^2} = \frac{2(k_1+k_2)}{m} - \frac{g}{R}$$

P3)



$\downarrow \vec{g}$ $[\pi R/2]$

(a) $T = \frac{m}{2} R^2 \dot{\phi}^2$ $\quad U = \frac{k_1}{2}(\pi R/2 - R\phi)^2 + \frac{k_2}{2}(\pi R/2 + R\phi)^2 - mg\cos\phi R$

$$L = \frac{mR^2\dot{\phi}^2}{2} - \frac{k_1}{2}(\pi R/2 - R\phi)^2 - \frac{k_2}{2}(\pi R/2 + R\phi)^2 + mg\cos\phi R$$

ϕ) $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}}\right) = mR^2\ddot{\phi}$ $\quad \frac{\partial L}{\partial \phi} = +Rk_1(\pi R/2 - R\phi) - Rk_2(\pi R/2 + R\phi) - mg\,\mathrm{sen}\phi R$

$$\Rightarrow \quad \ddot{\phi} = \frac{k_1}{m}(\pi/2 - \phi) - \frac{k_2}{m}(\pi/2 + \phi) - \frac{g}{R}\mathrm{sen}\phi \;/\; \dot{\phi} \qquad \ddot{\phi} = \frac{d\dot{\phi}}{d\phi}\frac{d\phi}{dt}$$

$$\pi/2m(k_1 - k_2) + \phi(k_1 + k_2)$$

$$\Rightarrow \quad \frac{\dot{\phi}^2}{2} = \frac{\pi}{2m}(k_1 - k_2)\phi - \frac{\phi^2}{2}(k_1 + k_2) + \frac{g}{R}\cos\phi + C \longrightarrow \text{condiciones iniciales tal que sea 0.}$$