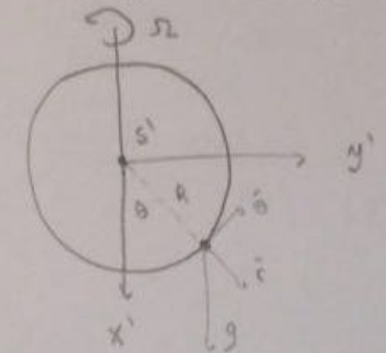
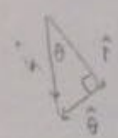
Pauta Ejercicio # 8



$\vec{g} = g(\hat{r}\cos\theta - \hat{\theta}\sin\theta)$

$\vec{\Omega} = \Omega(\hat{\theta}\sin\theta - \hat{r}\cos\theta) \quad \vec{\Omega} = \Omega\hat{k}$

• En S' $\quad r' = R\hat{r} \Rightarrow v' = R\dot{\theta}\hat{\theta} \Rightarrow \vec{a}' = R(\ddot{\theta}\hat{\theta} - \dot{\theta}^2\hat{r})$



$\cos\theta = \frac{\hat{r}}{\hat{g}} \Rightarrow \hat{r} = g\cos\theta$

$\sin\theta = \frac{\hat{\theta}}{\hat{g}} \Rightarrow \hat{\theta} = g\sin\theta$

(a)

$\hat{\theta}]$ $\quad mR\ddot{\theta} = -mg\sin\theta$

• Peso: $-mg\hat{j} = -mg\sin\theta\,\hat{\theta} - mg\cos\theta\,\hat{r}$

• Normal $= N\hat{r}$ $\quad$ 0.5 ptos.

• Pseudo Fuerzas

→ centrífuga: $-m\vec{\Omega}(\hat{\theta}\sin\theta - \hat{r}\cos\theta) \times \underbrace{(\Omega(\hat{\theta}\sin\theta - \hat{r}\cos\theta) \times R\hat{r})}_{-\hat{k}\sin\theta}$

$= -m\Omega^2 R(-\sin^2\theta\,\hat{r} - \cos\theta\sin\theta\,\hat{\theta})$

1 ptos.

$= m\Omega^2 R(\sin^2\theta\,\hat{r} + \cos\theta\sin\theta\,\hat{\theta})$

→ Coriolis $\quad -2m\Omega(\hat{\theta}\sin\theta - \hat{r}\cos\theta) \times R\dot{\theta}\hat{\theta}$

$= +2m\Omega R\dot{\theta}\cos\theta\,\hat{k}$

→ transversal $= 0$

(b)

⇒ ecuación completa $\hat{\theta}]$ $\quad mR\ddot{\theta} = -mg\sin\theta + m\Omega^2 R\cos\theta\sin\theta$ $\quad$ 1.5 ptos.

(c)

$F = -\nabla U \Rightarrow U = \int F\,dr = -\int \left(m\Omega^2 R\underbrace{\cos\theta\sin\theta}_{\frac{\sin(2\theta)}{2}} - mg\sin\theta\right) R\,d\theta$ $\quad (\sin(2\theta) = 2\cos\theta\sin\theta)$

$\Rightarrow U = mRg\left(\frac{R\Omega^2}{4g}\cos(2\theta) - \cos(\theta)\right)$ $\quad$ 1.5 ptos.

(d)

• entonces para que sea punto de equilibrio $F = 0 \Rightarrow \cos\theta\sin\theta = \frac{\cancel{m}g\cancel{\sin\theta}}{mR\Omega^2}$

$\Rightarrow \cos\theta = \frac{g}{R\Omega^2}$ y $\frac{g}{R\Omega^2} < 1 \Rightarrow \Omega^2 \geq \frac{g}{R}$

1.5 ptos.

* $\frac{\partial U}{\partial\theta} = -m\Omega^2 R\sin^2\theta + m\Omega^2 R\cos^2\theta + mg\cos\theta \quad \theta = 0 \Rightarrow m\Omega^2 R + mg > 0 \Rightarrow \Omega^2 > \frac{g}{R}$