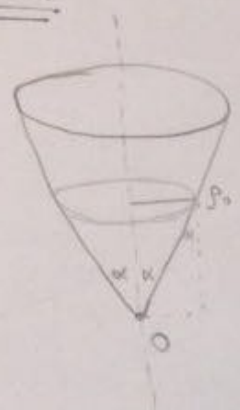
Pauta

P3]



$\downarrow \vec g$

$H = \frac{\rho}{\tan\alpha}$

(a) $v_{masita} = \dot\rho\,\hat\rho + \rho\dot\phi\,\hat\phi$ → $\vec v$ angular

$$E = -mg\frac{\rho}{\tan\alpha} + \underbrace{\frac{m}{2}\dot\rho^2}_{\text{cinética}} + \frac{m}{2}\rho^2\dot\phi^2$$

2 ptos plantear ecuaciones para resolver (E o DCL)

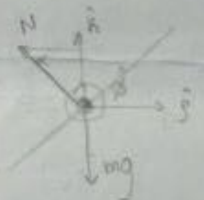
1.5 ptos impone condiciones para pto equilibrio.

$$\Rightarrow E = U_{eff} + K \Rightarrow -\frac{mg\rho}{\tan\alpha} + \frac{m}{2}\rho^2\dot\phi^2 = U_{eff}$$

Impongo que para $\rho = \rho_0$ $U_{eff}' \overset{!}{=} 0$ pto. equilibrio. $\left(\frac{\partial U}{\partial \rho}\right)$

$$U_{eff}' = -\frac{mg}{\tan\alpha} + m\rho\dot\phi^2 = 0 \Rightarrow \dot\phi^2 = \frac{g}{\rho_0\tan\alpha} \Rightarrow \omega = \sqrt{\frac{g}{\rho_0\tan\alpha}}$$

(a) Con DCL



$\Rightarrow$ $\hat\rho]$ $-N\cos\alpha = m(\ddot\rho - \rho\dot\phi^2)$

$\hat k]$ $N\sin\alpha - mg = m\ddot z$

queremos $\rho = \rho_0 \Rightarrow \dot\rho = 0 = \ddot\rho$

$\dot\phi = \omega \Rightarrow \ddot\phi = 0$

$\ddot z = 0$

$\hat\phi]$ $\frac{d}{dt}(\rho^2\dot\phi) = 0 \Rightarrow \rho^2\dot\phi = h = \rho_0^2\omega$

$$\Rightarrow \omega^2 = \frac{N\cos\alpha}{m\rho_0} = \frac{mg\cos\alpha}{\sin\alpha\, m\rho_0} = \frac{g}{\rho_0\tan\alpha}$$

* En esféricas $\rho_0 = r_0\sin\alpha$ $\Rightarrow \omega^2 = \frac{g}{r_0\sin\alpha\tan\alpha}$.

0.5 ptos argumentos sobre la perturbación

(b) La perturbación en $\hat r$, abordamos en esféricas: con $\ddot r \neq 0$

$\hat r]$ $-mg\cos\alpha = m(\ddot r - r\dot\phi^2\sin^2\alpha) \Rightarrow \ddot r - r\dot\phi^2\sin^2\alpha + g\cos\alpha = 0$ $\quad \dot\phi^2 = \frac{h^2}{r^4\sin^4\alpha}$

si r_0 es pto. eq $\Rightarrow \ddot r = 0 \Rightarrow \frac{h^2}{r_0^3\sin^2\alpha} = g\cos\alpha \Rightarrow h^2 = g r_0^3\cos\alpha\sin^2\alpha$ 2 ptos. desarrollo

$\Rightarrow \ddot r - \frac{g r_0^3\cos\alpha}{r^3} + g\cos\alpha = 0$ buscamos MAS: $\frac{1}{r^3} \approx \frac{1}{r_0^3} - \frac{3(r-r_0)}{r_0^4}$ (Taylor) $\Rightarrow \ddot r + \frac{3g\cos\alpha}{r_0}(r-r_0) = 0$

C.V $v = r - r_0$, $\ddot v = \ddot r$ $\Rightarrow \ddot v + \frac{3g\cos\alpha}{r_0}v = 0 \Rightarrow \omega_{po}^2 = 3g\cos\alpha/r_0 \Rightarrow \omega_{po} \overset{!}{=} \omega \Rightarrow \sin\alpha = 1/\sqrt{3} \Rightarrow \alpha =$