Pauta Ej 7.

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a) $\mathcal{U}(r) = E_0 \ln(r/r_0)$

$|v| = \sqrt{\dot{r}^2 + r^2\dot{\phi}^2}$

$$E_T = \frac{1}{2} m \dot{r}^2 + \underbrace{\frac{l_0^2}{2mr^2} + E_0 \ln(r/r_0)}_{\mathcal{U}_{eff}}$$ 0.5

orb circunferencial.

$$\frac{\partial \mathcal{U}_{eff}}{\partial r} = 0 = \frac{E_0}{r} - \frac{l_0^2}{mr^3} \Rightarrow r^2 = \frac{l_0^2}{mE_0} \Rightarrow r_c = l_0\sqrt{\frac{1}{mE_0}}.$$

0.5 1.0.

b) $\omega_{p.o} = \sqrt{\frac{\mathcal{U}''_{eff}(r_c)}{\alpha}}$ ~~#1~~ ; $\mathcal{U}_{eff}'' = -\frac{E_0}{r^2} + \frac{3l_0^2}{mr^4}$ 0.5.

$$\mathcal{U}''_{eff}(r_c) = \frac{3\cancel{l_0^2}}{\cancel{m}\frac{l_0^{\cancel{4}2}}{m^{\cancel{2}}E_0^2}} - \frac{E_0}{\frac{l_0^2}{mE_0}} = \frac{3E_0^2 m}{l_0^2} - \frac{E_0^2 m}{l_0^2}$$

$$\mathcal{U}''_{eff}(r_c) = \frac{2E_0^2 m}{l_0^2}$$ 0.5

$$\Rightarrow \omega_{p.o} = \sqrt{\frac{2E_0^2}{l_0^2}} = \frac{E_0}{l_0}\sqrt{2}$$ 1.0.

c) $r_c^2 \omega_{circ} m = l_0. \Rightarrow \frac{l_0^{\cancel{2}}}{\cancel{m}E_0} \cdot \omega_{cir} \cancel{m} = \cancel{l_0}. \Rightarrow \omega_{circ} = \frac{E_0}{l_0}.$ 0.5

$$\therefore \frac{\omega_{p.o}}{\omega_{circ}} = \frac{\sqrt{2}\,E_0/l_0}{E_0/l_0} = \sqrt{2}.$$ 0.5

Como $\omega_{p.o}/\omega_{cir} \notin \mathbb{Q}$ la órbita no es cerrada. Por lo demás, dado a que ~~las demás~~ no depende de E_0, r_0 ni l_0, nunca lo será.

1.0.