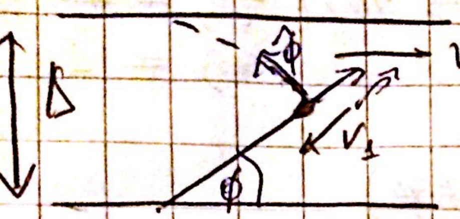
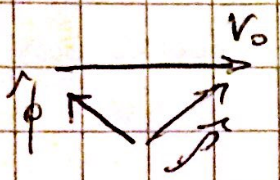
Profesor: Patricio Cordero.

Pauta P1 C1



$$\vec r = r\hat\rho \quad , \quad \vec v = \dot r\hat\rho + r\dot\phi\hat\phi \qquad (1)$$

a) Descompondremos las velocidades del bote y del río en las coordenadas polares:



$$\Rightarrow \vec{v_0} = v_0(\cos\phi\,\hat\rho - \sin\phi\,\hat\phi) \qquad 0.5$$

$$\vec{v_1} = -v_1\hat\rho$$

$$\Rightarrow \vec v = (v_0\cos\phi - v_1)\hat\rho - v_0\sin\phi\,\hat\phi \qquad (2)$$

$$(1) \text{ y } (2) \longrightarrow \dot r = v_0\cos\phi - v_1 \quad ; \quad r\dot\phi = -v_0\sin\phi$$

$$\Rightarrow \boxed{\dot r = v_0\cos\phi - v_1} \; 0.5 \qquad \boxed{\dot\phi = \frac{-v_0\sin\phi}{r}} \; 0.5$$

b) $$\frac{dr}{dt} = v_0\cos\phi - v_1 \Rightarrow dt = \frac{dr}{(v_0\cos\phi - v_1)}$$

$$\frac{d\phi}{dt} = -\frac{v_0\sin\phi}{r} \Rightarrow dt = \frac{-r\,d\phi}{v_0\sin\phi}$$

$$\Rightarrow \boxed{\frac{dr}{d\phi} = \frac{(v_1 - v_0\cos\phi)\,r}{v_0\sin\phi}} \qquad 1.0$$

c) $$\frac{dr}{r} = \left(\frac{v_1}{v_0}\cdot\frac{1}{\sin\phi} - \frac{1}{\tan\phi}\right)d\phi \quad \Big/ \int$$

$$\Rightarrow \ln\left(\frac{r}{r_0}\right) = \frac{v_1}{v_0}\ln\left(\frac{\tan(\phi/2)}{\tan(\phi_0/2)}\right) - \ln\left(\frac{\sin\phi}{\sin\phi_0}\right) \qquad 0.5$$

Por condiciones iniciales, $r_0 = \Delta$

$\theta_0 = \pi/2 \Rightarrow \tan\left(\frac{\theta_0}{2}\right) = 1$

$\sin(\theta_0) = 1.$

$$\Rightarrow \ln\left(\frac{r}{\Delta}\right) = \ln\left(\tan\left(\frac{\phi}{2}\right)^{\frac{v_1}{v_0}}\right) - \ln(\sin\phi)$$

$$\Rightarrow \ln\left(\frac{r}{\Delta}\right) = \ln\left(\frac{\tan\left(\frac{\phi}{2}\right)^{v_1/v_0}}{\sin\phi}\right) \quad / \exp() \quad 0.5$$

$$\boxed{\frac{r}{\Delta} = \left(\frac{\sin\left(\frac{\phi}{2}\right)}{\cos\left(\frac{\phi}{2}\right)}\right)^{v_1/v_0} \cdot \frac{1}{\sin\phi}.} \quad 1.0$$

d) Se debe cumplir que $r = 0$. ✓0.5

$$\Rightarrow \frac{\sin\left(\frac{\phi}{2}\right)^{v_1/v_0}}{\cos\left(\frac{\phi}{2}\right)^{v_1/v_0}} \cdot \frac{1}{\sin\phi} = 0$$

como $2\cos\theta \cdot \sin\theta = \sin(2\theta)$

$$\Rightarrow \frac{\sin\left(\frac{\phi}{2}\right)^{v_1/v_0}}{\cos\left(\frac{\phi}{2}\right)^{v_1/v_0}} \cdot \frac{1}{2\cos\left(\frac{\phi}{2}\right)\sin\left(\frac{\phi}{2}\right)} = 0.$$

$$\Rightarrow \sin\left(\frac{\phi}{2}\right)^{\frac{v_1}{v_0}-1} \cdot \cos\left(\frac{\phi}{2}\right)^{-\left(1+\frac{v_1}{v_0}\right)} = 0.$$

✓0.5

El peor de los casos corresponde a $\phi = 0$, donde se tiene:

$$\sin(0)^{\frac{v_1}{v_0}-1} \cdot \cancel{\cos(0)^{-\left(1+\frac{v_1}{v_0}\right)}} = 0$$

$\cancel{\cos(0)^{-\left(1+\frac{v_1}{v_0}\right)}} \Rightarrow 1$

$$\Rightarrow 0^{\frac{v_1}{v_0}-1} = 0. \quad ✓0.5$$

$\therefore \frac{v_1}{v_0} - 1 > 0 \Rightarrow v_1 - v_0 > 0 \Rightarrow \boxed{v_1 > v_0}$

Pauta P2C2

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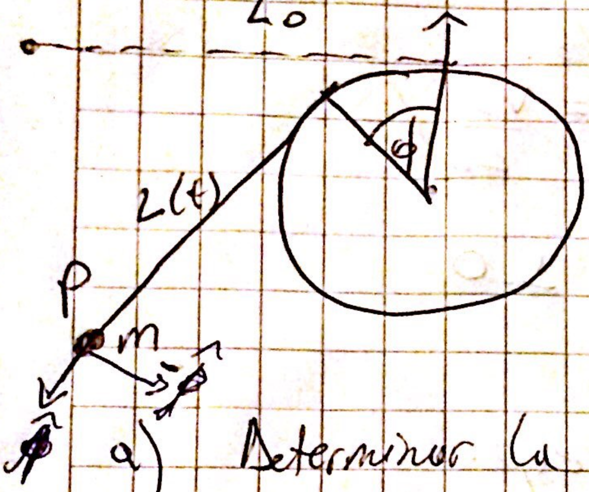


CI: $L(0) = L_0 \quad \phi(0) = 0$

$\|\vec{v}(0)\| = v_0$

NO hay gravedad

$$L(t) = L_0 - R\phi = L(\phi) \quad 0.5$$

a) Determinar la ecuación de movimiento de P.

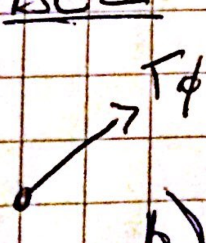
En polares: $\vec{r} = R\hat{\rho} + L(\phi)\hat{\phi} = R\hat{\rho} + (L_0 - R\phi)\hat{\phi}$

$$\vec{v} = R\dot{\phi}\hat{\phi} - R\dot{\phi}\hat{\phi} + R\phi\dot{\phi}\hat{\rho} - L_0\dot{\phi}\hat{\rho}$$

$$\vec{v} = (R\phi - L_0)\dot{\phi}\hat{\rho}$$

$$\vec{a} = (R\dot{\phi})\dot{\phi}\hat{\rho} + (R\phi - L_0)\ddot{\phi}\hat{\rho} + (R\phi - L_0)\dot{\phi}^2\hat{\phi} \quad 0.5$$

DCL $\quad \sum F = -T_\phi \hat{\phi} = m\vec{a}$



$\hat{\rho}$) $\boxed{0 = mR\dot{\phi}^2 + m(R\phi - L_0)\ddot{\phi}}$ $\quad 1.0$

b) 0.5 $\Rightarrow \ddot{\phi}(L_0 - R\phi) = R\dot{\phi}^2$

$$\Rightarrow \frac{\ddot{\phi}}{\dot{\phi}} = \frac{R\dot{\phi}}{L_0 - R\phi} \Big/ \int dt$$

$$\Rightarrow \ln(\dot{\phi})\Big|_{\dot{\phi}(0)}^{\dot{\phi}} = -\ln(L_0 - R\phi)\Big|_{\phi(0)}^{\phi}$$

Por condiciones iniciales, tenemos que

$$\|\vec{V}(0)\| = V_0 \Rightarrow (\cancel{R\phi}^{\,0} - L_0)\dot{\phi}(0) = V_0.$$

$$\Rightarrow \boxed{\dot{\phi}(0) = \frac{V_0}{L_0}}$$

$$\Rightarrow \ln\left(\frac{\dot{\phi}}{\frac{V_0}{L_0}}\right) = -\ln\left(\frac{L_0 - R\phi}{L_0}\right) \quad 1.0$$

$$\Rightarrow \ln\left(\frac{\dot{\phi} L_0}{V_0}\right) = \ln\left(\frac{L_0}{L_0 - R\phi}\right) \quad / \exp.$$

$$\dot{\phi}\frac{L_0}{V_0} = \frac{L_0}{L_0 - R\phi} \Rightarrow \boxed{\dot{\phi} = \frac{V_0}{L_0 - R\phi}} \quad 0.5$$

c) De $\hat{\phi}$ $\quad -T = m\dot{\phi}^2(R\phi - L_0) \quad 0.5$

$$\dot{\phi}^2 = \frac{V_0^2}{(L_0 - R\phi)^2}, \quad T = m\dot{\phi}^2(L_0 - R\phi)$$

$$\Rightarrow T = m\frac{V_0^2}{(L_0 - R\phi)^{\cancel{2}}}\cancel{(L_0 - R\phi)}$$

$$\Rightarrow T = \frac{m V_0^2}{L_0 - R\phi} \quad 1.0$$

El hilo se romperá para $T = T_{max}$

$$\Rightarrow L_0 - R\phi_{max} = \frac{m V_0^2}{T_{max}} \Rightarrow \boxed{\phi_{max} = \frac{L_0}{R} - \frac{m V_0^2}{R T_{max}}} \quad 0.5$$

PROARTE