P2 Aux 24, Ecs de Euler

- Considerando las relaciones entre momentos de inercia:

$$I_1 = I_2 \cos(2\alpha)$$

$$I_2 = I_2 \leftarrow \text{variable libre}$$

$$I_3 = I_2 + I_1 = I_2(1+\cos(2\alpha)) = 2I_2\cos^2\alpha$$

También tenemos $I_1 - I_2 = -I_2(1-\cos 2\alpha) = -I_2 \cdot 2\operatorname{sen}^2\alpha$

• Las ecuaciones de euler para un sistema sin fuerzas externas son:

$$Z]\quad (I_1 - I_2)\,\omega_1\omega_2 - I_3\dot{\omega}_3 = 0$$

$$X]\quad (I_2 - I_3)\,\omega_2\omega_3 - I_1\dot{\omega}_1 = 0$$

$$Y]\quad (I_3 - I_1)\,\omega_3\omega_1 - I_2\dot{\omega}_2 = 0$$

Donde:

$$I_1 - I_2 = -2I_2\operatorname{sen}^2\alpha$$

$$I_2 - I_3 = -I_1 = -I_2\cos(2\alpha)$$

$$I_3 - I_1 = I_2$$

Reemplazando dejando todo en función de nuestra variable libre:

$$(-\cancel{2I_2}\operatorname{sen}^2\alpha)\,\omega_1\omega_2 - \cancel{2I_2}\cos^2\alpha\,\dot{\omega}_3 = 0$$

$$(-\cancel{I_2}\cos(2\alpha))\,\omega_2\omega_3 + \cancel{I_2}\cos(2\alpha)\,\dot{\omega}_1 = 0$$

$$\cancel{I_2}\,\omega_3\omega_1 - \cancel{I_2}\,\dot{\omega}_2 = 0$$

Arreglando términos

$$\Rightarrow \quad \operatorname{tg}^2\alpha\,\omega_1\omega_2 + \dot{\omega}_3 = 0 \quad (1)$$

$$\dot{\omega}_1 + \omega_2\omega_3 = 0 \quad (2)$$

$$\dot{\omega}_2 - \omega_1\omega_3 = 0 \quad (3)$$

Notemos que de nuestras 3 ecuaciones podemos multiplicar por algún valor conveniente para obtener lo siguiente:

$$\underbrace{w_1 \cdot w_2 w_3 = -\dot{w}_3 w_3 \cot g^2\alpha = -w_1 \dot{w}_1 = w_2 \dot{w}_3}$$

Tenemos 2 ecuaciones que podemos integrar

- $$+w_1 \dot{w}_1 = +\dot{w}_3 w_3 \cot g^2\alpha \quad // \int_0^t$$

$$= \cancel{\tfrac{1}{2}}\left(w_1^2 - w_1^2(0)\right) = \cot g^2\alpha \cdot \cancel{\tfrac{1}{2}}\left(w_3^2 - w_3^2(0)\right)$$

$$= w_1^2 - w_1^2(0) = \cot g^2\alpha \left(w_3^2 - w_3^2(0)\right) \quad (4)$$

- $$w_2 \dot{w}_2 = -w_3 \dot{w}_3 \cot g^2\alpha \quad // \int_0^t$$

$$\cancel{\tfrac{1}{2}}\left(w_2^2 - w_2^2(0)\right) = -\cot g^2\alpha \left(w_3^2 - w_3^2(0)\right)\cancel{\tfrac{1}{2}}$$

$$= \left(w_2^2 - w_2^2(0)\right) = -\cot g^2\alpha \left(w_3^2 - w_3^2(0)\right) \quad (5)$$

No nos olvidemos de nuestras condiciones iniciales

$$w_1(0) = \Omega\cos\alpha \;,\; w_2(0) = 0 \;,\; w_3(0) = \Omega \operatorname{sen}\alpha$$

Reemplazando

(4) queda: $$w_1^2 - \Omega^2\cos^2\alpha = \cot g^2\alpha \left(w_3^2 - \Omega^2 \operatorname{sen}^2\alpha\right)$$

$$= w_1^2 - \cancel{\Omega^2\cos^2\alpha} = \cot g^2\alpha\, w_3^2 - \cancel{\Omega^2\cos^2\alpha}$$

$$\boxed{w_1^2 = w_3^2 \cot g^2\alpha} \quad *$$

(5) queda: $$\boxed{w_2^2 = -\cot g^2\alpha \left(w_3^2 - \Omega^2 \operatorname{sen}^2\alpha\right)} \quad **$$

Pero de la ecuación (3) podemos hacer el truco:

$$\dot{\omega}_2 = \omega_1 \omega_3 \Rightarrow \dot{\omega}_2^{\,2} = \omega_1^2 \omega_3^2$$

$$\Rightarrow \omega_1^2 = \frac{\dot{\omega}_2^{\,2}}{\omega_3^2}$$

Reemplazando en * tenemos

$$\frac{\dot{\omega}_2^{\,2}}{\omega_3^2} = \omega_3^2 \cot g^2 \alpha \Rightarrow \dot{\omega}_2^{\,2} = \omega_3^4 \cot g^2 \alpha$$

$$\Rightarrow \boxed{\dot{\omega}_2 = \omega_3^2 \cot g\, \alpha} \ ***$$

- Despejando ω_3^2 de la ecuación (**)

$$\omega_3^2 = \Omega^2 \operatorname{sen}^2 \alpha - \omega_2^2 \operatorname{tg}^2 \alpha$$

$$\Rightarrow \dot{\omega}_2 = (\Omega^2 \operatorname{sen}^2 \alpha - \omega_2^2 \operatorname{tg}^2 \alpha) \cot g\, \alpha$$

$$\Rightarrow \frac{\dot{\omega}_2}{\omega_2^2 \operatorname{tg}^2 \alpha - \Omega^2 \operatorname{sen}^2 \alpha} = -\cot g\, \alpha \qquad \Bigg| \quad \dot{\omega}_2 = \frac{d\omega_2}{dt}$$

$$\Rightarrow \frac{d\omega_2}{\omega_2^2 \operatorname{tg}^2 \alpha - \Omega^2 \operatorname{sen}^2 \alpha} = -\cot g\, \alpha \cdot dt$$

$$\Rightarrow \int \frac{d\omega_2}{\omega_2^2 \operatorname{tg}^2 \alpha - \Omega^2 \operatorname{sen}^2 \alpha} = -\cot g\, \alpha \int dt$$

Esa primitiva es dada y queda:

$$+\frac{1}{tg\alpha \cdot \Omega sen\alpha} \tanh^{-1}\left[\frac{\omega_2 \, tg\alpha}{\Omega sen\alpha}\right] = + t \cot g \alpha$$

$$\Rightarrow \tanh^{-1}\left[\frac{\omega_2 \, tg\alpha}{\Omega sen\alpha}\right] = t \cancel{\cot g\alpha}\, \cancel{tg\alpha}\, \Omega sen\alpha$$

$$\Rightarrow \tanh^{-1}\left[\frac{\omega_2 \, tg\alpha}{\Omega sen\alpha}\right] = t \Omega sen\alpha$$

$$\Rightarrow \frac{\omega_2 \, tg\alpha}{\Omega sen\alpha} = \tanh(t \Omega sen\alpha)$$

$$\Rightarrow \boxed{\omega_2 = \Omega \cos\alpha \tanh(t \Omega sen\alpha)}$$

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