Pauta Auxiliar N° 9 ME3204

2014

* Notación indicial

- Convenio de suma de Einstein: $a_i b_i = a_1 b_1 + a_2 b_2 + a_3 b_3$

$$A_{ij} b_j = A_{11} b_1 + A_{12} b_2 + A_{13} b_3 \quad \left(\begin{array}{l} \text{3 componentes} \\ \hookrightarrow \text{Vector} \end{array} \right)$$

- Operadores (En coord. cartesianas)

↳ Producto punto: $\underset{\sim}{a} \cdot \underset{\sim}{b} = a_i b_j (\underset{\sim}{e_i} \cdot \underset{\sim}{e_j}) = a_i b_j \delta_{ij} = a_i b_i$

↳ Producto cruz: $\underset{\sim}{a} \times \underset{\sim}{b} = \varepsilon_{ijk} a_i b_j \underset{\sim}{e_k}$

↳ Producto tensorial: $\underset{\sim}{a} \otimes \underset{\sim}{b} = a_i b_j (\underset{\sim}{e_i} \otimes \underset{\sim}{e_j})$

$$* \ \delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \ , \quad \varepsilon_{ijk} = \begin{cases} 1 & 1,2,3 \ ; \ 3,1,2 \ ; \ 2,3,1 \\ -1 & 3,2,1 \ ; \ 1,3,2 \ ; \ 2,1,3 \\ 0 & i = j \lor j = k \lor k = i \end{cases}$$

- Operadores diferenciales (en Coord. cartesianas)

↳ Divergencia: $\operatorname{div} \underset{\sim}{A} = \underset{\sim}{\nabla} \cdot \underset{\sim}{A} = \dfrac{\partial}{\partial x_k} (A_{ij} \underset{\sim}{e_i} \otimes \underset{\sim}{e_j}) \cdot \underset{\sim}{e_k}$

$$= \frac{\partial A_{ij}}{\partial x_k} \underset{\sim}{e_i} (\underset{\sim}{e_j} \cdot \underset{\sim}{e_k}) = \frac{\partial A_{ij}}{\partial x_k} e_i \delta_{jk} = \frac{\partial A_{ik}}{\partial x_k} \underset{\sim}{e_i}$$

(1° orden)

↳ Curl: $\operatorname{curl} \underset{\sim}{A} = \nabla \times \underset{\sim}{v} = \varepsilon_{kij} \dfrac{\partial v_i}{\partial x_k} \underset{\sim}{e_j}$

↳ Gradiente: • $\operatorname{Grad} \underset{\sim}{A} = \underset{\sim}{\nabla} \otimes \underset{\sim}{A} = \dfrac{\partial}{\partial x_k} (A_{ij} \underset{\sim}{e_i} \otimes \underset{\sim}{e_j}) \otimes \underset{\sim}{e_k}$

$$= \frac{\partial A_{ij}}{\partial x_k} \underset{\sim}{e_i} \otimes \underset{\sim}{e_j} \otimes \underset{\sim}{e_k} \quad \text{(3° orden!)}$$

• $\operatorname{Grad} \underset{\sim}{v} = \dfrac{\partial}{\partial x_j} (v_i \underset{\sim}{e_i}) \otimes \underset{\sim}{e_j} = \dfrac{\partial v_i}{\partial x_j} \underset{\sim}{e_i} \otimes \underset{\sim}{e_j}$ (2° orden!)

$\boxed{\text{ETC!!}}$

P1)

a) $\cdot\ \underline{A}(\underline{u}\otimes\underline{v}) = A_{ij}\, \underline{e}_i \otimes \underline{e}_j (u_k \underline{e}_k \otimes v_l \underline{e}_l)$

$$= A_{ij} u_k v_l (\underline{e}_i \otimes \underline{e}_j)(\underline{e}_k \otimes \underline{e}_l)$$

$$= A_{ij} u_k v_l (\underline{e}_i \otimes \underline{e}_l)\delta_{jk}$$

$$= A_{ij} u_j v_l (\underline{e}_i \otimes \underline{e}_l)$$

$$= (A_{ij} u_j \underline{e}_i) \otimes v_l \underline{e}_l = (\underline{A}\underline{u}) \otimes \underline{v}$$

$\cdot\ (\underline{a}\otimes\underline{b})^T = [(a_i \underline{e}_i)\otimes(b_j \underline{e}_j)]^T = (a_i b_j\, \underline{e}_i \otimes \underline{e}_j)^T$

$$= (a_i b_j\, \underline{e}_j \otimes \underline{e}_i) = [(b_j \underline{e}_j)\otimes(a_i \underline{e}_i)] = (\underline{b}\otimes\underline{a})$$

$\cdot\ tr(\underline{a}\otimes\underline{b}) = tr(a_i b_j\, \underline{e}_i \otimes \underline{e}_j) = a_i b_j\, tr(\underline{e}_i \otimes \underline{e}_j)$

$$= a_i b_j \delta_{ij} = a_i b_i = \underline{a}\cdot\underline{b}$$

b) $\cdot\ \varepsilon_{ijk}\varepsilon_{lmn}$ ~~$= \varepsilon_{ijk}\varepsilon_{lmn}$ → Separemos por casos:~~

↳ La def. del símbolo de permutación en función del delta de Kronecker es:

$$\varepsilon_{ijk}\varepsilon_{lmn} = \begin{vmatrix} \delta_{il} & \delta_{im} & \delta_{in} \\ \delta_{jl} & \delta_{jm} & \delta_{jn} \\ \delta_{kl} & \delta_{km} & \delta_{kn} \end{vmatrix}$$

⇝ En nuestro caso:

$$\varepsilon_{ijk}\varepsilon_{imn} = \begin{vmatrix} \delta_{ii} & \delta_{im} & \delta_{in} \\ \delta_{ji} & \delta_{jm} & \delta_{jn} \\ \delta_{ki} & \delta_{km} & \delta_{kn} \end{vmatrix} = \begin{vmatrix} 1 & \delta_{im} & \delta_{in} \\ \delta_{ji} & \delta_{jm} & \delta_{jn} \\ \delta_{ki} & \delta_{km} & \delta_{kn} \end{vmatrix}$$

$$=(\delta_{jm}\delta_{kn}-\delta_{jn}\delta_{km})-\delta_{im}(\delta_{jn}\delta_{ki}-\delta_{ji}\delta_{kn})$$
$$+\delta_{in}(\delta_{ji}\delta_{km}-\delta_{jm}\delta_{ki})$$

* <u>Del 2° término</u>

$$\delta_{im}(\delta_{jn}\delta_{ki}-\delta_{ji}\delta_{kn})=\delta_{im}\delta_{ki}\delta_{jn}-\delta_{im}\delta_{ji}\delta_{kn})$$
$$=\delta_{mk}\delta_{jn}-\delta_{mj}\delta_{kn}$$

* <u>Del 3° término</u>

$$\delta_{in}(\delta_{ji}\delta_{km}-\delta_{jm}\delta_{ki})=\delta_{in}\delta_{ji}\delta_{km}-\delta_{in}\delta_{ki}\delta_{jm}$$
$$=\delta_{jn}\delta_{mk}-\delta_{mj}\delta_{kn}$$

$$\Rightarrow \varepsilon_{ijk}\varepsilon_{imn}=(\delta_{jm}\delta_{kn}-\delta_{jn}\delta_{km})-\cancel{\delta_{mk}\delta_{jn}}+\cancel{\delta_{mj}\delta_{kn}}$$
$$+\cancel{\delta_{jn}\delta_{mk}}-\cancel{\delta_{mj}\delta_{kn}}$$

$$\boxed{\therefore\ \varepsilon_{ijk}\varepsilon_{imn}=(\delta_{jm}\delta_{kn}-\delta_{jn}\delta_{km})}$$

- $\varepsilon_{jmn}\varepsilon_{imn}=2\delta_{ij}$. A partir de la ec. anterior:

$$\boxed{\begin{aligned}\varepsilon_{jmn}\varepsilon_{imn}&=\varepsilon_{mnj}\varepsilon_{mni}\\&=(\delta_{nn}\delta_{ji}-\delta_{ni}\delta_{jn})=(3\delta_{ji}-\delta_{ij})=2\delta_{ij}\end{aligned}}$$

- $\varepsilon_{ijk}\varepsilon_{ijk}=6$. A partir de la ec. anterior

$$\varepsilon_{ijk}\varepsilon_{ijk}=2\cdot\delta_{ii}=2\cdot 3\rightsquigarrow\boxed{\varepsilon_{ijk}\varepsilon_{ijk}=6}$$

c) $\det(\underset{\sim}{A} - \lambda \underset{\sim}{I}) = -\lambda^3 + \lambda^2 tr(\underset{\sim}{A}) - \frac{1}{2}\lambda\left[(tr(\underset{\sim}{A}))^2 - tr(\underset{\sim}{A}^2)\right] + \det(\underset{\sim}{A}$

$$\boxed{\det(\underset{\sim}{A} - \lambda \underset{\sim}{I}) = -\lambda^3 + I_1\lambda^2 - I_2 \cdot \frac{\lambda}{2} + I_3}$$

- El determinante en notación indicial se escribe como:

$$\det(\underset{\sim}{S}) = \frac{1}{6}\varepsilon_{ijk}\varepsilon_{pqr} S_{ip} S_{jq} S_{kr}$$

- Aplicándolo a $\det(\underset{\sim}{A} - \lambda\underset{\sim}{I})$: $(\underset{\sim}{I} = \delta_{ij})$

$$\det(\underset{\sim}{A} - \lambda\underset{\sim}{I}) = \frac{1}{6}\varepsilon_{ijk}\varepsilon_{pqr}(A_{ip} - \lambda\delta_{ip})(A_{jq} - \lambda\delta_{jq})(A_{kr} - \lambda\delta_{kr})$$

- Expandiendo y dejando factorizado en potencias de λ, se tiene:

$$\det(\underset{\sim}{A} - \lambda\underset{\sim}{I}) = \frac{1}{6}\varepsilon_{ijk}\varepsilon_{pqr}\Big[\overbrace{A_{ip}A_{jq}A_{kr}}^{(1)} - \underbrace{\lambda(A_{ip}A_{jq}\delta_{kr} + A_{ip}\delta_{jq}A_{kr} + \delta_{ip}A_{jq}A_{kr})}_{(2)} + \underbrace{\lambda^2(A_{ip}\delta_{jq}\delta_{kr} + \delta_{ip}A_{jq}\delta_{kr} + \delta_{ip}\delta_{jq}A_{kr})}_{(3)} - \underbrace{\lambda^3\delta_{ip}\delta_{jq}\delta_{kr}}_{(4)}\Big]$$

- De (1) $\rightsquigarrow \frac{1}{6}\varepsilon_{ijk}\varepsilon_{pqr}A_{ip}A_{jq}A_{kr} = \det(\underset{\sim}{A})$ (Por definición)

- De (2) $\rightsquigarrow -\frac{1}{6}\varepsilon_{ijk}\varepsilon_{pqr}\lambda(A_{ip}A_{jq}\delta_{kr} + A_{ip}\delta_{jq}A_{kr} + \delta_{ip}A_{jq}A_{kr})$

  * Tomemos el 1º término

  $\rightsquigarrow A_{ip}A_{jq}\delta_{kr}\varepsilon_{ijk}\varepsilon_{pqr} = A_{ip}A_{jq}\varepsilon_{ijk}\varepsilon_{pqk} = A_{ip}A_{jq}(\varepsilon_{kij}\varepsilon_{kpq})$

  (Por (b)) $= A_{ip}A_{jq}(\delta_{ip}\delta_{jq} - \delta_{iq}\delta_{jp})$

  $= A_{ip}A_{jq}\delta_{ip}\delta_{jq} - A_{ip}A_{jq}\delta_{iq}\delta_{jp}$

  $= A_{ii}A_{jj} - A_{ij}A_{ji} = tr(A)\cdot tr(A) - tr(A^2)$

  $= (tr(A))^2 - tr(A^2)$ //

* Esto sucede análogamente con los otros 2 términos, luego

$-\cancel{\frac{1}{6}\lambda \varepsilon_{ijk}\varepsilon_{pqr}}$ $\therefore (2) = \frac{1}{6}\lambda\left(3(trA)^2 + 3\,tr(A^2)\right) = \frac{1}{2}\lambda\left((trA)^2 + tr(A^2)\right)$ //

· De (3) $\sim \frac{\lambda^2}{6}\left(A_{ip}\delta_{jq}\delta_{kr} + \delta_{ip}A_{jq}\delta_{kr} + \delta_{ip}\delta_{jq}A_{kr}\right)\varepsilon_{ijk}\varepsilon_{pqr}$

* Tomemos el 1º término:

$\sim \varepsilon_{ijk}\varepsilon_{pqr}A_{ip}\delta_{jq}\delta_{kr} = \varepsilon_{ijk}\varepsilon_{pjk}A_{ip} = \varepsilon_{jki}\varepsilon_{jkp}A_{ip}$

(Por (b)) $= 2\delta_{ip}A_{ip} = 2A_{ii} = 2\,tr(\underline{A})$ //

* Esto sucede análogamente con los otros 2 términos, luego:

$\therefore (3) = \frac{\lambda^2}{6}\left(6\,tr(\underline{A})\right) = \lambda^2\,tr(\underline{A})$

· De (4) $\sim -\frac{\lambda^3}{6}\delta_{ip}\delta_{jq}\delta_{kr}\varepsilon_{ijk}\varepsilon_{pqr} = -\frac{\lambda^3}{6}\varepsilon_{ijk}\varepsilon_{ijk}$

$= -\frac{\lambda^3}{\cancel{6}}\cdot\cancel{6}$ (Por (b))

$\therefore (4) = -\lambda^3$

$$\boxed{\begin{aligned} &\therefore \det(\underline{A} - \lambda\underline{I}) = -\lambda^3 + I_1\lambda^2 - I_2\lambda + I_3 \\ &\qquad I_1 = tr(\underline{A}) \\ &\qquad I_2 = \tfrac{1}{2}\left[(tr\underline{A})^2 - tr(\underline{A}^2)\right] \\ &\qquad I_3 = \det(\underline{A}) \end{aligned}}$$ //

[2]

$$* \quad \frac{\partial x_i}{\partial x_j} = \delta_{ij} \quad ; \quad \boxed{\frac{\partial T_{ij}}{\partial T_{kl}} = \delta_{ik}\,\delta_{jl}}$$

$$\bullet \quad \frac{\partial I_1}{\partial \underset{\sim}{T}} = \frac{\partial\, tr(\underset{\sim}{T})}{\partial \underset{\sim}{T}} = \frac{\partial T_{ii}}{\partial T_{jk}} = \delta_{ij}\,\delta_{ik} = \delta_{jk} = \underset{\approx}{I}$$

$$\cdot \quad \frac{\partial I_2}{\partial \underset{\sim}{T}} = \frac{1}{2}\,\frac{\partial}{\partial \underset{\sim}{T}}\left((tr\,\underset{\sim}{T})^2 - tr(\underset{\sim}{T})^2\right) = \frac{1}{2}\left(\frac{\partial(tr\,\underset{\sim}{T})^2}{\partial \underset{\sim}{T}} - \frac{\partial(tr(T^2))}{\partial \underset{\sim}{T}}\right)$$

$$= \frac{1}{2}\left(\frac{\partial(T_{ii}T_{jj})}{\partial T_{kl}} - \frac{\partial(T_{ij}T_{ji})}{\partial T_{kl}}\right)$$

$$= \frac{1}{2}\left(\delta_{ik}\delta_{il}T_{jj} + T_{ii}\delta_{jk}\delta_{kl} - \delta_{ik}\delta_{jl}T_{ji} - T_{ij}\delta_{jk}\delta_{il}\right)$$

$$= \frac{1}{2}\left(\delta_{kl}T_{jj} + \delta_{kl}T_{ii} - 2\,T_{lk}\right) \quad \text{(original } T_{kl}!\text{)}$$

$$= \frac{1}{2}\left(\underset{\sim}{I}\cdot tr(\underset{\sim}{T}) + \underset{\sim}{I}\cdot Tr(\underset{\sim}{T}) - 2\,T^T\right)$$

$$\boxed{\therefore \quad \frac{\partial I_2}{\partial \underset{\sim}{T}} = tr(\underset{\sim}{T})\cdot\underset{\sim}{I} - \underset{\sim}{T}^T}$$

b) $\cdot\ \nabla(A_{mn}x_n x_m) = \frac{\partial}{\partial x_p}(A_{mn}x_n x_m)\,\underset{\sim}{e_p}$

$$= \left(A_{mn}\frac{\partial x_n}{\partial x_p}x_m + A_{mn}\frac{\partial x_m}{\partial x_p}x_n\right)\underset{\sim}{e_p}$$

$$= \left(A_{mn}\delta_{np}x_m + A_{mn}\delta_{mp}x_n\right)\underset{\sim}{e_p}$$

$$= \left(A_{mp}x_m + A_{pn}x_n\right)\underset{\sim}{e_p} \quad \left(\text{m, n índices mudos}\right)$$

$$= \left(A_{np}x_n + A_{pn}x_n\right)\underset{\sim}{e_p}$$

$$= (A_{np} + A_{pn})\,\underset{\sim}{x_n}\,\underset{\sim}{e_p} \,//$$

$\cdot\ \operatorname{div}(\phi\underset{\sim}{u}) = \frac{\partial}{\partial x_j}(\phi u_i \underset{\sim}{e_i}) = u_i \underset{\sim}{e_i}\cdot\frac{\partial\phi}{\partial x_j}\cdot\underset{\sim}{e_j} + \phi\frac{\partial u_i}{\partial x_j}(e_i\cdot e_j)$

$$= u_i\frac{\partial\phi}{\partial x_j}(\underset{\sim}{e_i}\cdot\underset{\sim}{e_j}) + \phi\frac{\partial u_i}{\partial x_j}(\underset{\sim}{e_i}\cdot\underset{\sim}{e_j})$$

$$= u_i\frac{\partial\phi}{\partial x_j}\delta_{ij} + \phi\frac{\partial u_i}{\partial x_j}\delta_{ij}$$

$$= u_i\frac{\partial\phi}{\partial x_i} + \phi\frac{\partial u_i}{\partial x_i}$$

$$= \underset{\sim}{u}\cdot\operatorname{grad}\phi + \phi\operatorname{div}\underset{\sim}{u} \,//$$

$\cdot\ \operatorname{grad}(\phi\underset{\sim}{u}) = \underset{\sim}{\nabla}\otimes(\phi\underset{\sim}{u}) = \frac{\partial}{\partial x_j}(\phi\, u_i \underset{\sim}{e_i})\otimes\underset{\sim}{e_j}$

$$= u_i\frac{\partial\phi}{\partial x_j}(\underset{\sim}{e_i}\otimes\underset{\sim}{e_j}) + \phi\frac{\partial u_i}{\partial x_j}(\underset{\sim}{e_i}\otimes\underset{\sim}{e_j})$$

$$= \underset{\sim}{u}\otimes\operatorname{grad}\phi + \phi\operatorname{grad}\underset{\sim}{u} \,//$$

c) Analicemos $\underset{\sim}{u} \cdot \underset{\sim}{A}\underset{\sim}{n}$

$$\underset{\sim}{u} \cdot \underset{\sim}{A}\underset{\sim}{n} = u_i \underset{\sim}{e_i} \cdot (A_{jk} \underset{\sim}{e_j} \otimes \underset{\sim}{e_k} \, n_l \underset{\sim}{e_l})$$
$$= u_i \underset{\sim}{e_i} \cdot (A_{jk} n_l (\underset{\sim}{e_j} \otimes \underset{\sim}{e_k}) \underset{\sim}{e_l})$$
$$= u_i \underset{\sim}{e_i} \cdot (A_{jk} n_l \underset{\sim}{e_j} \delta_{kl})$$
$$= u_i \underset{\sim}{e_i} \cdot (A_{jk} n_k \underset{\sim}{e_j})$$
$$= u_i A_{jk} n_k \, e_i \cdot e_j = u_i A_{jk} n_k \delta_{ij}$$
$$= u_i A_{ik} n_k = A_{ik} u_i n_k = \underset{\sim}{A}^T \underset{\sim}{u} \underset{\sim}{n}$$

$$\Rightarrow \int_{\partial B} \underset{\sim}{u} \cdot \underset{\sim}{A}\underset{\sim}{n} \, da = \int_{\partial B} (A^T \underset{\sim}{u}) \underset{\sim}{n} \, da = \int_{B} \operatorname{div}(\underset{\sim}{A}^T \underset{\sim}{u}) \, dV \;//$$

Teorema de la divergencia

P2 $$[\sigma_{ij}] = \begin{pmatrix} 0 & 0 & -Cx_2 \\ 0 & 0 & Cx_1 \\ -Cx_2 & Cx_1 & 0 \end{pmatrix} \quad \text{en } \underset{\sim}{x} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$$

a) $$\Rightarrow [\sigma_{ij}] = \begin{pmatrix} 0 & 0 & -2C \\ 0 & 0 & C \\ -2C & C & 0 \end{pmatrix}$$

* Calculando los invariantes

1-) $I_1 = tr(\underset{\sim}{\sigma}) = 0$

2-) $$\underset{\sim}{\sigma}^2 = \underset{\sim}{\sigma}\underset{\sim}{\sigma} = \begin{pmatrix} 4C^2 & -2C^2 & 0 \\ -2C^2 & C^2 & 0 \\ 0 & 0 & 5C^2 \end{pmatrix} \rightsquigarrow tr(\underset{\sim}{\sigma}^2) = 10C^2$$

$$\Rightarrow I_2 = \frac{1}{2}\left((tr(\underline{\sigma}))^2 - tr(\underline{\sigma}^2)\right) = -5c^2$$

$$3 \to I_3 = \det(\underline{\sigma}) = 0$$

* La ec. característica queda como:

$$-\lambda^3 + \lambda^2 \cancel{I_1} - \lambda I_2 + \cancel{I_3} = 0$$

$$-\lambda^3 - \lambda \cdot (-5c^2) = 0 \rightsquigarrow \boxed{\lambda(\lambda^2 - 5c^2) = 0}$$

$$\Rightarrow \lambda_1 = 0 \quad \wedge \quad \lambda_{2,3} = \frac{0 \pm \sqrt{20c^2}}{2} = \frac{\pm 2c\sqrt{5}}{2}$$

$$\boxed{\therefore \lambda_1 = 0 \,;\, \lambda_2 = c\sqrt{5} \,;\, \lambda_3 = -c\sqrt{5}}$$

(Esfuerzos ppales!!)

b) La ec. característica viene de $(\underline{\sigma} - \lambda_i \underline{I})\underline{n}_i = 0$. Luego:

$\boxed{\lambda_1 = 0}$

$$\begin{pmatrix} 0 & 0 & -2c \\ 0 & 0 & c \\ -2c & c & 0 \end{pmatrix}\begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \boxed{\begin{aligned} -2cn_3 &= 0 \quad (1) \\ cn_3 &= 0 \quad (2) \\ -2cn_1 + cn_2 &= 0 \quad (3) \end{aligned}}$$

* De (1) y (2) se sabe que $n_3 = 0$. De (3), se tiene que

$$\boxed{n_2 = 2n_1} \quad (4)$$

* Como los vectores deben ser unitarios:

$$n_1^2 + n_2^2 + n_3^2 = 1 \Rightarrow n_1^2 + (2n_1)^2 = 1$$

$$\therefore\ n_1 = 1/\sqrt{3} \implies n_2 = 2/\sqrt{3}$$

$$\boxed{\therefore\ \overset{1}{\underset{\sim}{n}} = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\2\\0\end{pmatrix}}$$

$\lambda_2 = c\sqrt{5}$

$$\begin{pmatrix}-c\sqrt{5} & 0 & -2c\\ 0 & -c\sqrt{5} & c\\ -2c & c & -c\sqrt{5}\end{pmatrix}\begin{pmatrix}n_1\\n_2\\n_3\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

$$\implies \boxed{\begin{aligned}&-c\sqrt{5}\,n_1 - 2c\,n_3 = 0 && (1)\\ &-c\sqrt{5}\,n_2 + c\,n_3 = 0 && (2)\\ &-2c\,n_1 + c\,n_2 - c\sqrt{5}\,n_3 = 0 && (3)\end{aligned}}$$

- De (1) $\implies \boxed{n_3 = -\frac{\sqrt{5}}{2}n_1 \quad (4)}$
- De (2) $\implies -c\sqrt{5}\,n_2 + c\left(-\frac{\sqrt{5}}{2}\right)n_1 = 0$

$$-c\sqrt{5}\,n_2 - c\frac{\sqrt{5}}{2}n_1 = 0$$

$$\boxed{n_2 = -\frac{1}{2}n_1 \quad (5)}$$

- Ec (3) es L.D. de los otros. Luego, asegurando que $\overset{2}{n}$ sea unitario

$$n_1^2 + n_2^2 + n_3^2 = 1$$

$$n_1^2 + \left(-\frac{1}{2}n_1\right)^2 + \left(-\frac{\sqrt{5}}{2}n_1\right)^2 = 1$$

$$n_1^2\left(1 + \frac{1}{4} + \frac{5}{4}\right) = 1$$

$$n_1 = \sqrt{\frac{2}{5}}$$

$$\implies n_2 = -\frac{1}{\sqrt{10}} \wedge n_3 = -\frac{\sqrt{2}}{2}$$

$$\therefore \ \overset{3}{\underset{\sim}{n}} = \begin{pmatrix} \sqrt{4/5} \\ -\sqrt{1/10} \\ -\sqrt{1/2} \end{pmatrix}$$

$\boxed{\lambda_3 = -C\sqrt{5}}$

$$\begin{pmatrix} C\sqrt{5} & 0 & -2C \\ 0 & C\sqrt{5} & C \\ -2C & C & C\sqrt{5} \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \boxed{\begin{array}{ll} C\sqrt{5}\,n_1 - 2C n_3 = 0 & (1) \\ C\sqrt{5}\,n_2 + C n_3 = 0 & (2) \\ -2C n_1 + C n_2 + C\sqrt{5}\,n_3 = 0 & (3) \end{array}}$$

- (3) es L.D. • De (1) $\longrightarrow \boxed{n_3 = \frac{\sqrt{5}}{2} n_1}$

- De (2) $\rightsquigarrow C\sqrt{5}\,n_2 + C\frac{\sqrt{5}}{2} n_1 = 0$

$$\boxed{n_2 = -\frac{1}{2} n_1}$$

- Verificando valor unitario:

$$n_1^2 + \left(\frac{\sqrt{5}}{2} n_1\right)^2 + \left(\frac{1}{2} n_1\right)^2 = 1$$

$$\therefore n_1 = \sqrt{\frac{2}{5}}$$

$$n_2 = \sqrt{\frac{1}{10}} \quad \wedge \quad n_3 = \frac{\sqrt{2}}{2}$$

$$\boxed{\therefore \ \overset{3}{\underset{\sim}{n}} = \begin{pmatrix} \sqrt{\frac{2}{5}} \\ -\sqrt{\frac{1}{10}} \\ \frac{\sqrt{2}}{2} \end{pmatrix}}$$

c) El est. de corte máximo está dado por:

$$\tau_{max} = \frac{\lambda_{max} - \lambda_{min}}{2} \quad / \quad \begin{aligned} \lambda_{max} &= C\sqrt{5} \\ \lambda_{min} &= 0 \end{aligned}$$

$$\boxed{\therefore \tau_{max} = C\frac{\sqrt{5}}{2}}$$