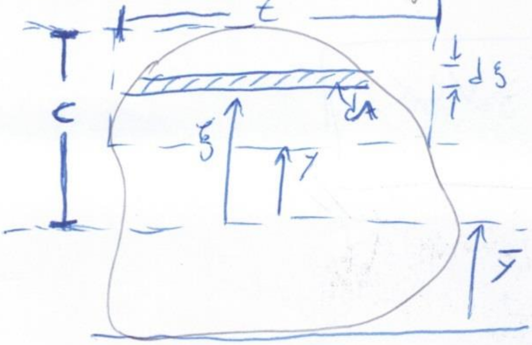
Pauta Aux 4 ME3204

2014

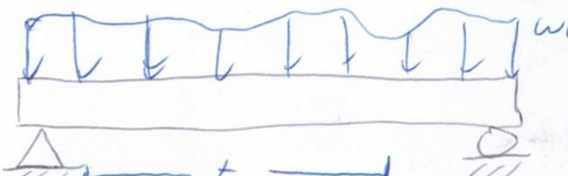
- Esfuerzo de corte en vigas

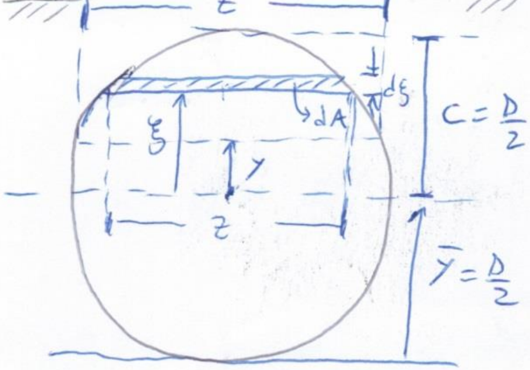


$$\boxed{\tau_{xy} = \frac{V(x)}{I_z \cdot t}\int_y^c \xi \, dA}$$

$$\boxed{\tau_{xy} = \frac{V(x)}{2 I_z}\left(\frac{h^2}{4} - y^2\right)}$$ (Sec. rectangular)

P1)





- Por la ec. del círculo

$$\frac{D^2}{4} = \left(\frac{t}{2}\right)^2 + y^2$$

$$\boxed{\therefore \; t = 2\sqrt{\frac{D^2}{4} - y^2}}$$

- El diferencial de área es $dA = z \cdot d\xi$

↳ Por ec. del círculo: $\frac{D^2}{4} = \left(\frac{z}{2}\right)^2 + \xi^2 \Rightarrow z = 2\sqrt{\frac{D^2}{4} - \xi^2}$

$$\boxed{\therefore \; dA = 2\sqrt{\frac{D^2}{4} - \xi^2}\, d\xi}$$

$$\Rightarrow \int_y^c \xi \, dA = \int_y^{D/2} 2\sqrt{\frac{D^2}{4} - \xi^2}\, d\xi = -\int_y^{D/2} \sqrt{v}\, dv \quad \left(\begin{array}{l} v = \frac{D^2}{4} - \xi^2 \\ dv = -2\xi\, d\xi \end{array}\right)$$

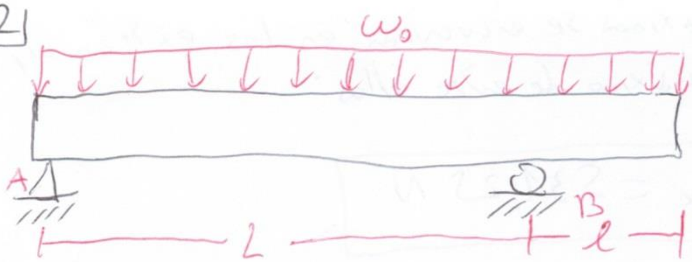
$$= -\frac{2}{3} v^{3/2} = -\frac{2}{3}\left(\frac{D^2}{4} - \xi^2\right)^{3/2} \Bigg|_y^{D/2}$$

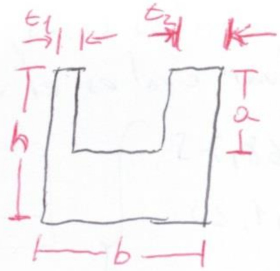
$$\boxed{\therefore \int_y^c \xi \, dA = \frac{2}{3}\left(\frac{D^2}{4} - y^2\right)^{3/2}}$$

$$\Rightarrow \tau_{xy} = \frac{V(x)}{I_z \cdot t}\int_c^y \xi \, dA = \frac{V(x)}{\frac{\pi \cdot D^4}{64} \cdot 2\sqrt{\frac{D^2}{4} - y^2}} \cdot \frac{2}{3}\left(\frac{D^2}{4} - y^2\right)^{3/2}$$

$$\boxed{\therefore \tau_{xy} = \frac{64}{3\pi} \, \frac{D^2/4 - y^2}{D^4} \cdot V(x)}$$ //

P2



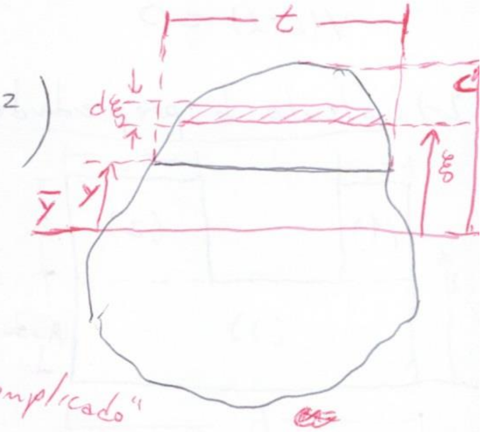


* Corte en vigas

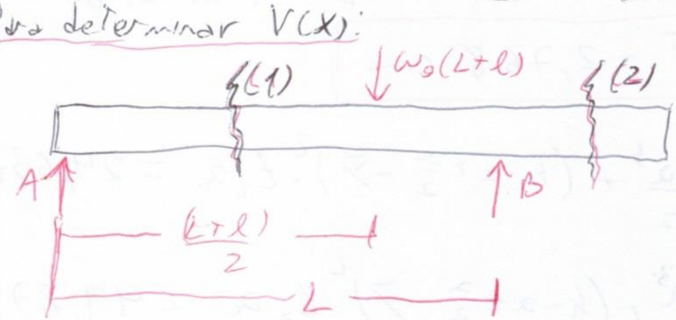
↳ Sección Rectangular: $\tau_{xy} = \dfrac{V(x)}{2I_z}\left(\dfrac{h^2}{4} - y^2\right)$

↳ Sección aleatoria: $\tau_{xy} = \dfrac{V(x)}{I_z t}\displaystyle\int_y^c \xi \, dA$

↳ Lo "complicado"



1-) DCL (Para determinar $V(x)$):



• $\sum F_y = 0$

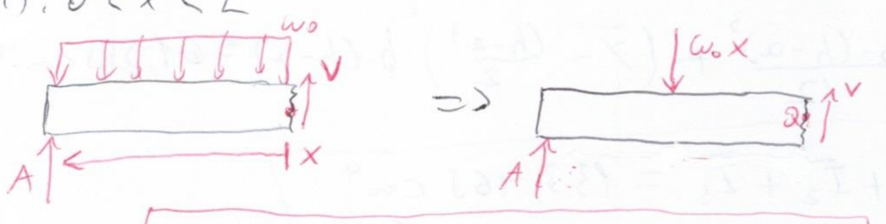
$\rightarrow A + B = w_0(L+l)$

• $\sum_A M_z = 0$

$\rightarrow \boxed{B = \dfrac{w_0(L+l)^2}{2L} = 781{,}25\ N}$

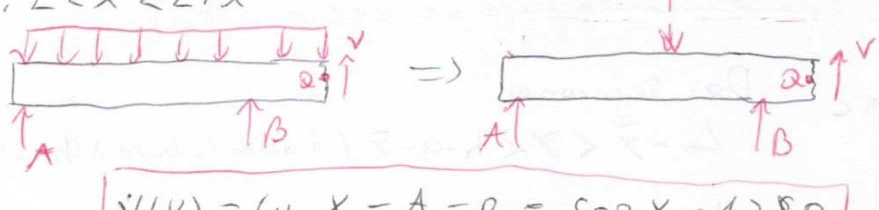
$\boxed{A = 468{,}75\ N}$

(1): $0 < x < L$



$\boxed{V(x) = w_0 x - A = 500x - 468{,}75}$

(2): $L < x < L + l$

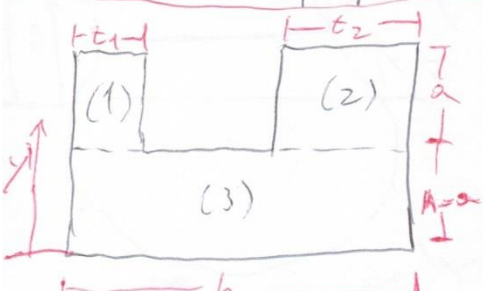


$\boxed{V(x) = w_0 x - A - B = 500x - 1250}$

* Dado que en ambos casos, los máximos se encuentran en los extremos, se debe evaluar cual es el valor máximo de entre ellos:

$$\Rightarrow \left.\begin{aligned} V(0) &= -468{,}75 \\ V(L^-) &= 531{,}25 \\ V(L^+) &= -250 \\ V(L+l) &= 0 \end{aligned}\right\} \quad \boxed{\therefore V_{max} = 531{,}25\ N}$$

2-) Cálculo de propiedades de área ($\bar{y}$, I_z)



$$\bar{y} = \frac{\sum_i y_i A_i}{\sum_i A_i}$$

$$\bar{y} = \frac{(h-a+\frac{a}{2})\cdot t_1 a + (h-a+\frac{a}{2})\, t_2 a + \frac{(h-a)}{2}\cdot b(h-a)}{t_1\cdot a + t_2\cdot a + (h-a)\cdot b}$$

$$\boxed{\bar{y} = 2{,}768\ cm}$$

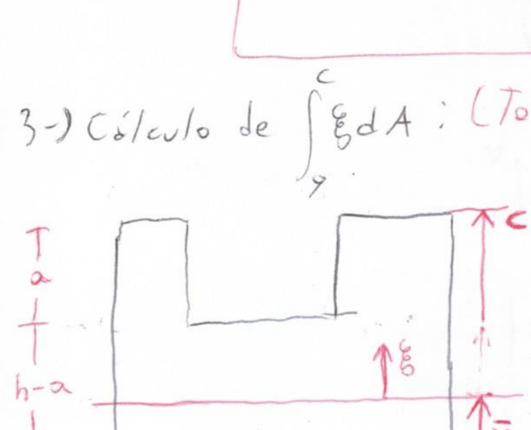
$$\Rightarrow \bar{I}_1 = I_1 + \delta^2 A_1 = \frac{t_1 a^3}{12} + \left(h-a+\frac{a}{2}-\bar{y}\right)^2 \cdot t_1 a = 24{,}637\ cm^4$$

$$\bar{I}_2 = I_2 + \delta^2 A_2 = \frac{t_2 a^3}{12} + \left(h-a+\frac{a}{2}-\bar{y}\right)^2 \cdot t_2 a = 49{,}273\ cm^4$$

$$\bar{I}_3 = I_3 + \delta^2 A_3 = \frac{b\cdot(h-a)^3}{12} + \left(\bar{y} - \frac{(h-a)}{2}\right)^2 b\cdot(h-a) = 61{,}555\ cm^4$$

$$\boxed{\begin{aligned} \therefore \bar{I}_T = \bar{I}_1 + \bar{I}_2 + \bar{I}_3 &= 135{,}465\ cm^4 \\ &= 1{,}355\cdot 10^{-6}\ m^4 \end{aligned}}$$

3-) Cálculo de $\int_y^c \xi\, dA$: (Todo se mide respecto del eje neutro)



Dos secciones:

↳ $-\bar{y} < y < h-a-\bar{y}$ (ó desde la base a $(h-a)$)

↳ $h-a-\bar{y} < y < c$ (ó desde $(h-a)$ hasta h)

* ξ es la coordenada que va desde y hasta $\underline{c}$

$\Rightarrow h-a-\bar{y} < y < c$

$$dA = (t_1 + t_2)\, d\xi$$

$$\rightsquigarrow \int_y^c \xi\, dA = \int_y^{h-\bar{y}} \xi (t_1+t_2)\, d\xi = (t_1+t_2)\left(\frac{(h-\bar{y})^2}{2} - \frac{y^2}{2}\right)$$

$$\boxed{\int_y^c \xi\, dA = (26{,}861 - 1{,}5 y^2)\cdot 10^{-6}\, m^4}$$

$\Rightarrow -\bar{y} < y < h-a-\bar{y}$

$$\rightsquigarrow \int_y^c \xi\, dA = \int_y^{h-a-\bar{y}} \xi\, b\, d\xi + \int_{h-a-\bar{y}}^{h-\bar{y}} \xi (t_1+t_2)\, d\xi$$

$$= \frac{b}{2}\left((h-a-\bar{y})^2 - y^2\right) + \frac{(t_1+t_2)}{2}\left((h-\bar{y})^2 - (h-a-\bar{y})^2\right)$$

$$\boxed{\int_y^c \xi\, dA = (30{,}654 - 4y^2)\cdot 10^{-6}\, m^4}$$

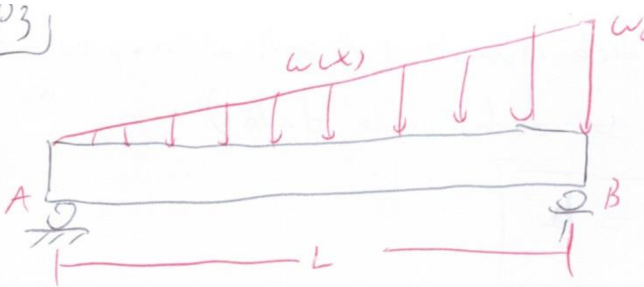
$$\boxed{\therefore \tau_{xy} = \frac{V(x)}{I_T}\cdot \begin{cases} (26{,}861 - 1{,}5y^2)\cdot 10^{-6} & h-a-\bar{y} < y < c \\ (30{,}654 - 4y^2)\cdot 10^{-6} & -\bar{y} < y < h-a-\bar{y} \end{cases}}$$

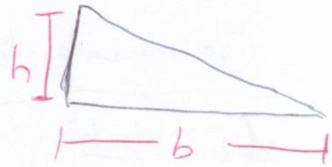
*Para determinar el esfuerzo máximo, necesitamos conocer V_{max} (listo) y el y para el cual se maximice el esfuerzo

↳ Para el 1º intervalo ⇝ $y = 0$ (justo en el eje neutro)
$\tau_{xy} = 150{,}196$ kPa

↳ Para el 2º intervalo ⟶ $y = 1{,}2317$* (el menor valor posible) $\Rightarrow \tau_{xy} = 321{,}39$ kPa

P3)

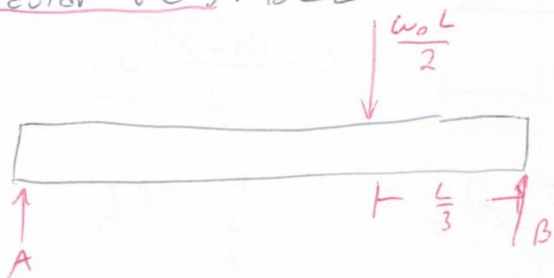




$$\tau_{xy} = \frac{V(x)}{I_z \cdot t}\int_y^c \xi \, dA$$

1-) Calcular $V(x)$
2-) Calcular I_z
3-) Calcular $\int_y^c \xi \, dA$

1-) Calcular $V(x)$: - DCL



$\cdot \sum_B M_z = 0$

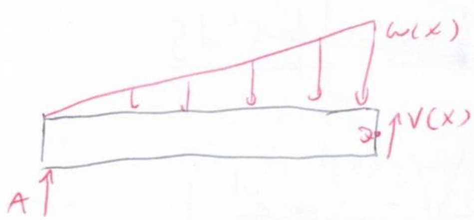
$\rightarrow \frac{L}{3} \cdot \frac{w_0 L}{2} - AL = 0$

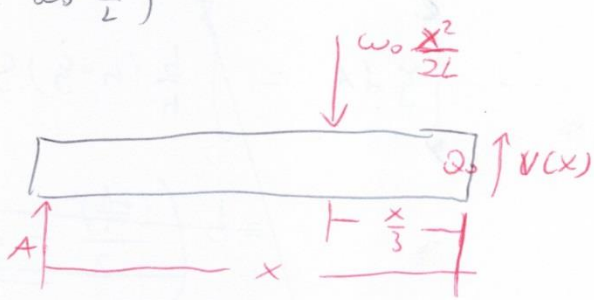
$$\boxed{\therefore A = w_0 \frac{L}{6}}$$

$\cdot \sum F_y = 0 \Rightarrow B = w_0 \frac{L}{2} - A = w_0 \frac{L}{2} - w_0 \frac{L}{6} = w_0 \frac{L}{3}$

$$\boxed{B = w_0 \frac{L}{3}}$$

- Corte en $0 < x < L$ $\left(w(x) = w_0 \frac{x}{L}\right)$





$\Rightarrow A - w_0 \frac{x^2}{2L} + V(x) = 0$

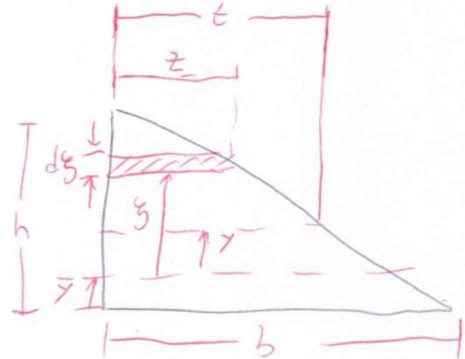
$$\boxed{\therefore V(x) = \frac{w_0}{2L} x^2 - \frac{w_0 L}{6}}$$

2-) Calcular I_z ⇝ Resuelto en Apéndice Aux 3

$$\boxed{I_z = \frac{bh^3}{36} \;;\; \bar{y} = \frac{h}{3}}$$

3-) Calcular $\int_y^c \xi \, dA$

↳ a) Calcular t



$$\rightsquigarrow t = \frac{(h-y')}{h} b \;, \text{ pero } y' = \bar{y} + y \Rightarrow t = \frac{(h - \frac{h}{3} - y)}{h} b$$

$$\boxed{t = \frac{2b}{3} - \frac{b}{h} y}$$

↳ b) Calcular dA: $dA = z \cdot d\xi = \left(\frac{2b}{3} - \frac{b}{h}\xi\right) d\xi$

$$\therefore \int_y^c \xi \, dA = \int_y^{2h/3} \left(\frac{2b}{3} - \frac{b}{h}\xi\right) \xi \, d\xi = \frac{2b}{3}\int_y^{2h/3} \xi \, d\xi - \frac{b}{h}\int_y^{2h/3} \xi^2 \, d\xi$$

$$= \frac{b}{3}\left(\left(\frac{2h}{3}\right)^2 - y^2\right) - \frac{b}{3h}\left(\left(\frac{2h}{3}\right)^3 - y^3\right)$$

$$\boxed{\therefore \int_y^c \xi \, dA = \frac{b}{3}\left(\frac{4}{27}h^2 + \frac{y^3}{h} - y^2\right)}$$

$$\boxed{\therefore \tau_{xy} = \frac{w_0\left(\frac{x^2}{2L} - \frac{L}{6}\right)}{\left(\frac{bh^3}{36}\right)\left(\frac{2b}{3} - \frac{b}{h}y\right)} \cdot \frac{b}{3}\left(\frac{4}{27}h^2 + \frac{y^3}{h} - y^2\right)}$$ //