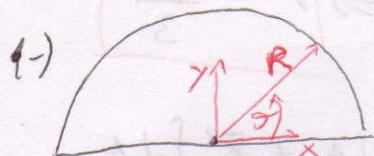


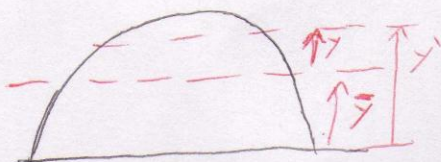
↳ Cálculo de ejes neutros y momentos de inercia de secciones particulares



En coordenadas polares $\rightarrow dA = r d\theta dr$
 $y = r \sin \theta$

$$\Rightarrow \bar{y} = \frac{\int y dA}{A} = \frac{\int_0^{\pi} \int_0^R (r \sin \theta) r d\theta dr}{\frac{1}{2} \pi R^2} = \frac{\int_0^{\pi} r^2 \sin \theta d\theta}{\frac{1}{2} \pi R^2}$$

$$= \underbrace{-\cos \theta \Big|_0^{\pi}}_2 \cdot \frac{R^3}{3} \cdot \frac{1}{\frac{1}{2} \pi R^2} \Rightarrow \boxed{\bar{y} = \frac{4R}{3\pi}}$$



$$\bar{I}_z = \int y'^2 dA = \int (y' - \bar{y})^2 dA$$

$$= \int y'^2 dA - 2\bar{y} \int y' dA + \bar{y}^2 \int dA$$

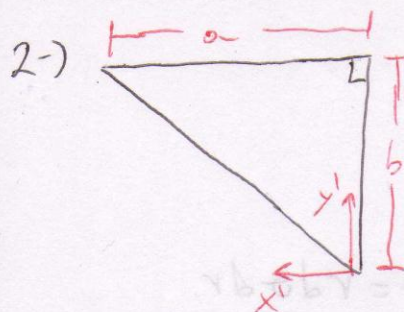
$$\rightarrow \bar{I}_z = \int_0^{\pi} \int_0^R R^2 \sin^2 \theta R d\theta dr - 2 \cdot \frac{4R}{3\pi} \left(\frac{4R}{3\pi} \right) + \frac{16R^2}{9\pi^2} \int_0^{\pi} \int_0^R r d\theta dr$$

$$2 \cdot \frac{4R}{3\pi} \int_0^{\pi} \int_0^R r \sin \theta r d\theta dr$$

$$= \frac{R^4}{4} \int_0^{\pi} \sin^2 \theta d\theta - \frac{8R^4}{9\pi} \int_0^{\pi} \sin \theta d\theta + \frac{8R^4}{9\pi^2} \cdot \pi$$

$$\underbrace{\frac{1}{2}(\theta - \sin \theta \cos \theta) \Big|_0^{\pi}}_{\frac{\pi}{2}}$$

$$= \frac{\pi R^4}{8} + \frac{16R^4}{9\pi} + \frac{8R^4}{9\pi} \Rightarrow \boxed{\bar{I}_z = \left(\frac{\pi}{8} - \frac{8}{9\pi} \right) R^4}$$



$$x' = \frac{a}{b} y' \Rightarrow \bar{y} = \frac{\int y' x' dy'}{\frac{ab}{2}}$$

$$\Rightarrow \bar{y} = \frac{\int_0^b y'^2 \frac{a}{b} dy'}{\frac{ab}{2}} = \frac{\frac{b^3}{3} \cdot \frac{a}{b}}{\frac{ab}{2}} \Rightarrow \boxed{\bar{y} = \frac{2b}{3}}$$

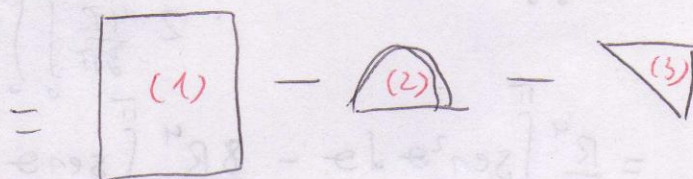
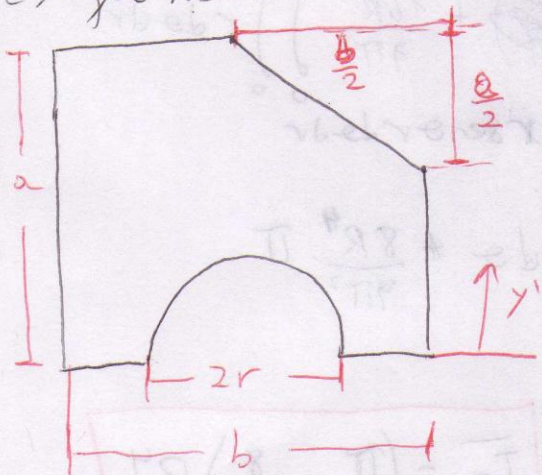
$$\bar{I}_2 = \int y'^2 dA = \int (y' - \bar{y})^2 dA = \int y'^2 dA - 2\bar{y} \int y' dA + \bar{y}^2 \int dA$$

$$= \int_0^b y'^3 \frac{a}{b} dy' - \frac{4b}{3} \int_0^b y'^2 \frac{a}{b} dy' + \frac{4b^2}{3} \int_0^b y' \frac{a}{b} dy'$$

$$= \frac{a}{b} \left(\frac{b^4}{4} - \frac{4b}{3} \cdot \frac{b^3}{3} + \frac{4b^2}{3} \cdot \frac{b^2}{2} \right) = \frac{a}{b} \left(\frac{b^4}{4} - \frac{4}{9} b^4 + \frac{2b^4}{3} \right)$$

$$\therefore \boxed{\bar{I}_2 = \frac{ab^3}{36}}$$

3-) ¿Y qué hacemos con esta figura?



$$\therefore \bar{y}_T = \frac{\bar{y}_1 \cdot A_1 - \bar{y}_2 \cdot A_2 - \bar{y}_3 \cdot A_3}{A_1 - A_2 - A_3}$$

$$* \bar{Y}_1 = \frac{a}{2} \quad \bar{I}_1 = \frac{b \cdot a^3}{12} \quad \wedge \quad A_1 = ab$$

$$\Rightarrow \bar{Y}_T = \frac{a}{2} ab - \left[\left(\frac{b}{2} \cdot \left(\frac{a}{2} \right)^2 \cdot \frac{1}{36} \right) + \frac{a}{2} \right] \cdot \frac{ab}{8} - \frac{4r}{3\pi}$$

$$\bar{Y}_T = \frac{a}{2} ab - \left[\frac{2b}{2} \cdot \frac{1}{3} + \frac{a}{2} \right] \frac{ab}{8} = \frac{4r}{3\pi} \cdot \frac{1}{2} \pi r^2$$

$$ab - \frac{ab}{8} - \frac{1}{2} \pi r^2$$

$$= \frac{a^2 b}{2} - \left(\frac{b}{3} + \frac{a}{2} \right) \frac{ab}{8} - \frac{4r}{3\pi} \cdot \frac{1}{2} \pi r^2$$

$$\frac{7ab}{8} - \frac{1}{2} \pi r^2$$

$$= \frac{a^2 b}{2} - \frac{ab^2}{24} - \frac{a^2 b}{16} - \frac{2}{3} r^3$$

$$\frac{7ab}{8} - \frac{1}{2} \pi r^2$$

$$\therefore \bar{Y}_T = \frac{ab}{8} \left(\frac{7a}{12} - \frac{b}{24} \right) - \frac{2}{3} r^3$$

$$\frac{7ab}{8} - \frac{1}{2} \pi r^2$$

* Los momentos de inercia se pueden sumar o restar siempre y cuando estén todos con respecto al eje neutro de la sección completa. Entonces, se calculan las inercias con respecto a $\bar{Y}_T$ usando el teorema de los ejes paralelos y luego se resta (No haremos eso aquí, pero RECUÉRDENLO!)