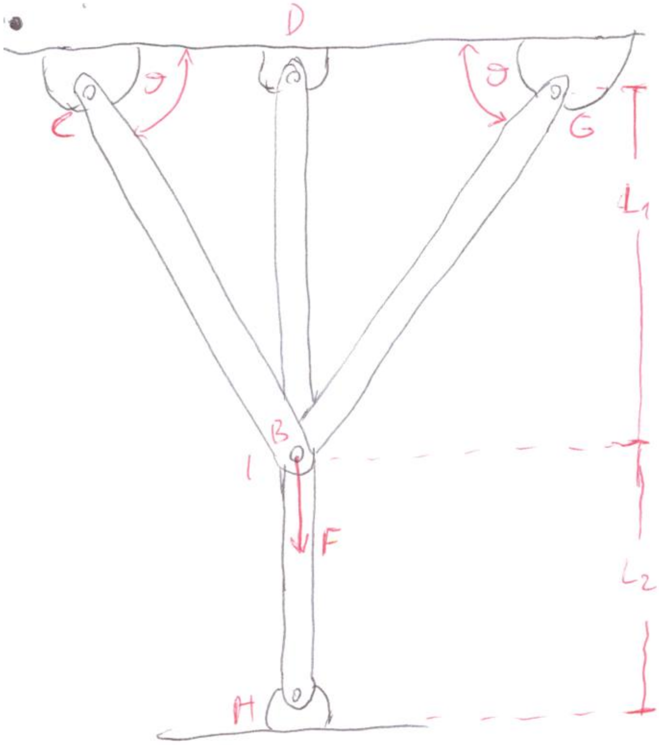
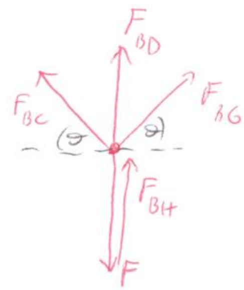


* DCL pasador B



Simetría

$F_{BG} = F_{BC}$

$$\sum F_y = 0 \Rightarrow F + F_{BH} - F_{BD} - 2F_{BG}\,\text{Sen}\,\theta = 0$$

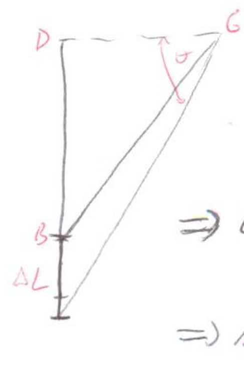
$$\boxed{\therefore F = F_{BH} + F_{BD} + 2F_{BG}\,\text{Sen}\,\theta} \quad (*)$$

* Aplicando Ley de Hooke sobre cada barra:

$$\sigma_{BH} = \frac{F_{BH}}{A} = E \cdot \varepsilon_{BH} = E\frac{\Delta L}{L_{BH}} \rightsquigarrow \boxed{F_{BH} = AE\frac{\Delta L}{L_{BH}}} \quad (i)$$

$$\sigma_{BD} = \frac{F_{BD}}{A} = E \cdot \varepsilon_{BD} = E\frac{\Delta L}{L_{BD}} \rightsquigarrow \boxed{F_{BD} = AE\frac{\Delta L}{L_{BD}}} \quad (ii)$$

* ¿Qué pasa con BG?



$\rightsquigarrow$ Suponemos pequeñas deformaciones

$\hookrightarrow \theta$ constante

$$\Rightarrow L_{BG}(\text{Final}) = \frac{L_{BD} + \Delta L}{\text{Sen}\,\theta} \wedge L_{BG} = \frac{L_{BD}}{\text{Sen}\,\theta}$$

$$\Rightarrow \Delta L_{BG} = L_{BG}(\text{Final}) - L_{BG} = \frac{L_{BD} + \Delta L}{\text{Sen}\,\theta} - \frac{L_{BD}}{\text{Sen}\,\theta} \Bigg\} \boxed{\Delta L_{BG} = \frac{\Delta L}{\text{Sen}\,\theta}}$$

$$\therefore \sigma_{BG} = \frac{F_{BG}}{A} = E \cdot \varepsilon_{BG} = E \cdot \frac{\Delta L}{L_{BG}\,\text{Sen}\,\theta} \rightsquigarrow \boxed{F_{BG} = A \cdot E \frac{\Delta L}{L_{BG}\,\text{Sen}\,\theta}}$$

$$\boxed{\therefore F_{BG} = A \cdot E \cdot \frac{\Delta L}{L_{BD}}}$$ (iii)

* Reemplazando en ☆:

$$F = AE\frac{\Delta L}{L_{BH}} + AE\frac{\Delta L}{L_{BD}} + 2AE\frac{\Delta L}{L_{BD}}\operatorname{sen}\theta \qquad / \quad L_{BH} = L_2, \quad L_{BD} = L_1$$

$$\boxed{\therefore \Delta L = \frac{F}{AE\left(\frac{1}{L_2} + \frac{1}{L_1} + \frac{2\operatorname{sen}\theta}{L_1}\right)} = 5{,}032 \cdot 10^{-5}\ m}$$

* Def. Factor de Seguridad $\rightsquigarrow$ $F.S. = \dfrac{\sigma_0}{\sigma_{adm}} \Rightarrow \sigma_{adm} = \dfrac{\sigma_0}{F.S} = \dfrac{200}{1{,}5}$

↳ $\boxed{\text{Hay fallo si } \sigma > \sigma_{adm}}$ $(F > F_{adm})$ ⟷ $\boxed{\sigma_{adm} = 133{,}33\ MPa}$

$\Rightarrow$ Barra BH $\rightsquigarrow$ $\sigma_{BH} = E \cdot \dfrac{\Delta L}{L_2} = \dfrac{\cancel{E}}{L_2} \cdot \dfrac{F}{A\cancel{E}\left(\frac{1}{L_2} + \frac{1}{L_1} + \frac{2\operatorname{sen}\theta}{L_1}\right)}$

$$\therefore F = \sigma_{BH} L_2 A\left(\frac{1}{L_2} + \frac{1}{L_1} + \frac{2 \cdot \operatorname{sen}\theta}{L_1}\right)$$

Para acotar, $\sigma_{BH} = \sigma_{adm}$

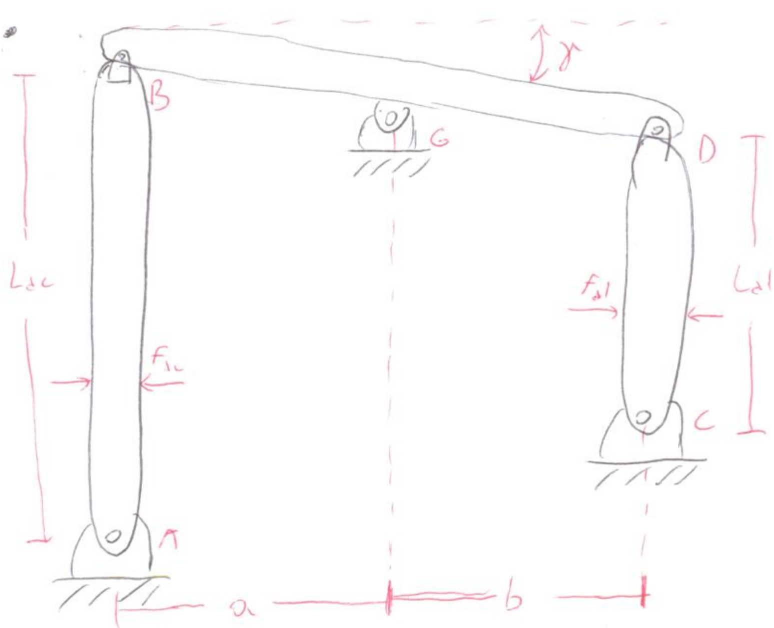
$$\Rightarrow \boxed{F = 0{,}2524\ MPa}$$

$\Rightarrow$ Barra BD $\sim$ $F = \sigma_{BD} L_1 A\left(\dfrac{1}{L_2} + \dfrac{1}{L_1} + \dfrac{2\operatorname{sen}\theta}{L_1}\right) = 0{,}5047\ MPa$

$\Rightarrow$ Barra BG $\sim$ $F = \sigma_{BG}\dfrac{L_1}{\operatorname{sen}\theta} A\left(\dfrac{1}{L_2} + \dfrac{1}{L_1} + \dfrac{2\operatorname{sen}\theta}{L_1}\right) = 0{,}5828\ MPa$

* El valor operacional del sistema debe ser para el F más bajo. Por lo tanto

$$\boxed{F_{max} = 0{,}2524\ MPa = 252{,}4\ kPa}$$



* Deformación térmica

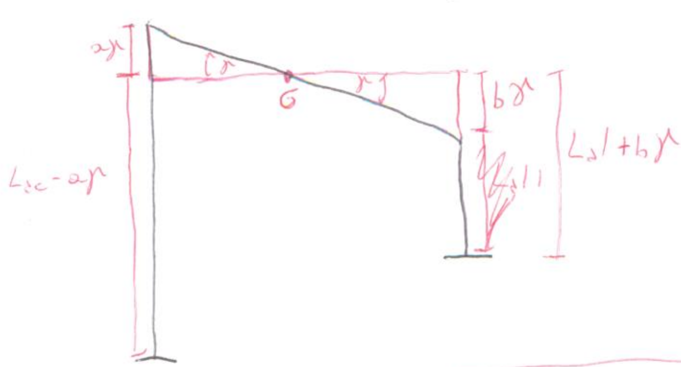
$$\varepsilon = \alpha \cdot \Delta T$$

$$\frac{l_f - l_0}{l_0}$$

* Para que BGD quede horizontal, el acero debe acortarse y el aluminio alargarse, sin embargo ambas se alargan producto de ΔT $\Rightarrow$ Utilizaremos ppio. de superposición
  - $\rightarrow$ Asumimos que ambas barras se alargan térmicamente
  - $\rightarrow$ Calculamos la def. elástica necesaria para que BGD quede horizontal

* Por def. térmica :
$$\left.\begin{aligned} L_{ac}^f &= L_{ac}(1+\alpha_{ac}\Delta T) \\ L_{al}^f &= L_{al}(1+\alpha_{al}\Delta T) \end{aligned}\right\} (*)$$

* Ahora supongamos que las barras se han alargado más de lo necesario, pero como BGD es rígido, deben acortarse un poco:



- Estado inicial luego de def. térmica $(L_{ac}^f), (L_{al}^f)$
- Estado final luego de def. elástica restitutiva $(L_{ac}-a\gamma), (L_{al}+b\gamma)$

$$\therefore \quad \varepsilon = \frac{l_f - l_i}{l_i} \Rightarrow \boxed{\varepsilon_{ac} = \frac{L_{ac}^f - (L_{ac}-a\gamma)}{L_{ac}^f} \wedge \varepsilon_{al} = \frac{L_{al}^f - (L_{al}+b\gamma)}{L_{al}^f}}$$

· Aplicando Ley de Hooke

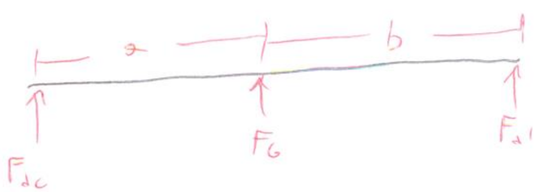
$$\frac{\sigma_{dc}}{E_{dc}} = \frac{F_{dc}}{A_{dc}E_{dc}} = \frac{L_{dc}^{f} - L_{dc} + a\gamma}{L_{dc}^{f}}$$

$$\frac{\sigma_{dl}}{E_{dl}} = \frac{F_{dl}}{A_{dl}\,E_{dl}} = \frac{L_{dl}^{f} - L_{dl} - b\gamma}{L_{dl}^{f}}$$

· Aplicando ✱ y despejando F_i:

$$\boxed{\begin{aligned} F_{dc} &= A_{dc}E_{dc}\left(\frac{L_{dc}(1+\alpha_{dc}\Delta T) - L_{dc} + a\gamma}{L_{dc}(1+\alpha_{dc}\Delta T)}\right) \\ F_{dl} &= A_{dl}\,E_{dl}\left(\frac{L_{dl}(1+\alpha_{dl}\Delta T) - L_{dl} - b\gamma}{L_{dl}(1+\alpha_{dl}\Delta T)}\right)\end{aligned}} \quad (☆)$$

· ¿Qué falta? => DCL en barra BGD



$$\sum_G M_z = 0 \Rightarrow \boxed{F_{dc}\cdot a = F_{dl}\cdot b} \quad (\perp)$$

· Aplicando (☆) en ($\perp$), obtendremos una expresión para ΔT:

$$A\Delta T^2 + B\Delta T + C = 0$$

$* \; A = \alpha_{dl}\alpha_{dc}L_{dc}L_{dl}\left(a A_{dc}E_{dc} - b A_{dl}E_{dl}\right)$

$* \; B = \left[a A_{dc}E_{dl}L_{dl}(L_{dc}\alpha_{dc} + a\gamma\alpha_{dl}) + b A_{dl}E_{dl}L_{dc}(L_{dl}\alpha_{dl} - b\gamma\alpha_{dc})\right]$

$* \; C = \gamma\left[a^2 A_{dc}E_{dc}L_{dl} + b^2 A_{dl}E_{dl}L_{dc}\right]$

· Resolviendo el sistema:

$$\Delta T = 91{,}2^\circ \checkmark \quad ó \quad 32882^\circ \times \quad \Rightarrow \boxed{\Delta T = 91{,}2^\circ}$$

b) Reemplazando el valor obtenido de ΔT en ☆

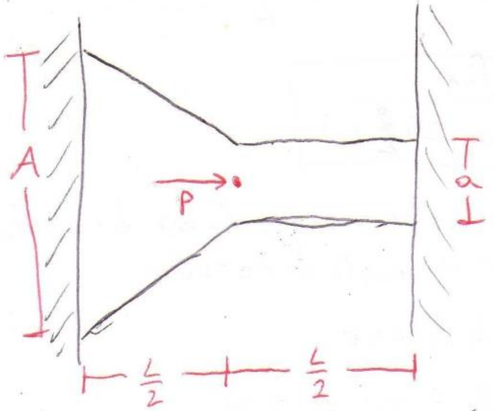
$$F_{al} = 3197,55 \text{ N}$$
$$F_{ac} = 6395,11 \text{ N}$$

* Calculando las cargas críticas:

* $P_{cr}^{ac} = \dfrac{\pi^2}{L_{ac}^2} E_{ac} \cdot \dfrac{t_{ac}^3 \cdot e_{ac}}{12} = 2229,37 \text{ N}$ ⇝ Falla por pandeo

* $P_{cr}^{al} = \dfrac{\pi^2}{L_{al}^2} E_{al} \cdot \dfrac{t_{al}^3 \cdot e_{al}}{12} = 973,266 \text{ N}$ ⇝ Falla por pandeo

P3)



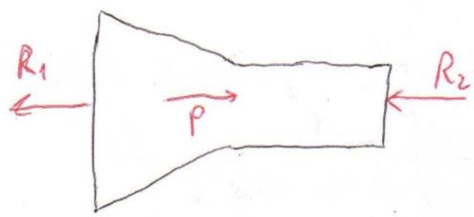
Espesor e

Módulo de Elasticidad E

\# Principal problema: Área en el primer tramo varía con la distancia

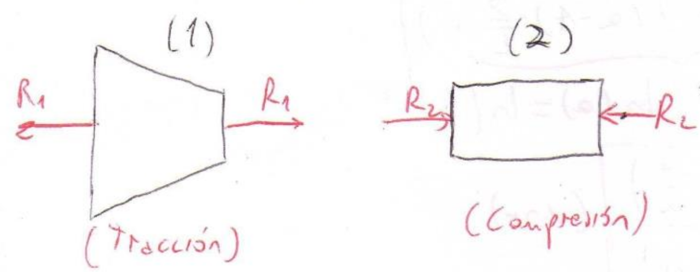
$\Rightarrow$ Esfuerzo varía con la distancia.

- DCL completo



$$\sum F_x = 0 \Rightarrow \boxed{P = R_1 + R_2} \quad (i)$$

- Cortes antes y después de P:



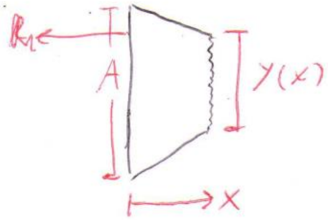
\# Como el largo total no cambia, lo que se alarga (1), se acorta (2)

$$\Rightarrow \boxed{\Delta L_1 = \Delta L_2} \quad (ii)$$

- Aplicación de Ley de Hooke:

$$(2): \sigma = \frac{R_2}{a \cdot e} \Rightarrow \frac{R_2}{a \cdot e} = E \cdot \frac{\Delta L_2}{L/2} \Rightarrow \boxed{\Delta L_2 = \frac{1}{2} \cdot \frac{R_2 L}{a \cdot e E}} \quad (iii)$$

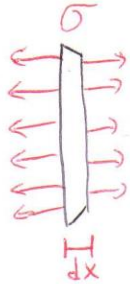
(1):



$$\sigma = \frac{R_1}{e \cdot Y(x)} \quad \wedge \quad Y(x) = A + (a-A)\cdot\frac{2}{L}x$$

$$\boxed{\therefore \sigma(x) = \frac{R_1}{e\left(A+(a-A)\frac{2}{L}x\right)}}$$

* Cada segmento diferencial del tramo (1) se deforma en magnitudes diferentes ya que el esfuerzo cambia con X. Al hacer un corte diferencial:



$$\varepsilon = \frac{\Delta(dx)}{dx}$$

Lo que se alarga cada trozo diferencial

Largo inicial del trozo diferencial

Ley de Hooke

$$\sigma = E\cdot\varepsilon \Rightarrow \varepsilon = \frac{\sigma}{E}$$

$$\frac{\Delta(dx)}{dx} = \frac{\sigma(x)}{E}$$

$$\boxed{\Delta(dx) = \frac{\sigma(x)}{E}dx}$$

* Integrando entre 0 y $\frac{L}{2}$, se obtiene el alargamiento total:

$$\int \Delta(dx) = \Delta L_1 = \int_0^{L/2} \frac{\sigma(x)}{E}dx = \frac{R_1}{eE}\int_0^{L/2}\frac{1}{A+(a-A)\frac{2}{L}x}dx$$

$$\Rightarrow \Delta L_1 = \frac{R_1}{eE}\cdot\frac{L}{2(a-A)}\cdot \underbrace{\ln\left(A+(a-A)\cdot\frac{2}{L}x\right)}_{\ln(A)-\ln(a)=\ln\left(\frac{A}{a}\right)}\Bigg|_0^{L/2}$$

$$\boxed{\therefore \Delta L_1 = \frac{R_1 L\cdot\ln\left(\frac{A}{a}\right)}{2eE(a-A)}} \quad (iv)$$

* Debido a (ii) $\Rightarrow$ (iii) = (iv):

$$\frac{1}{2}\frac{R_2 \cancel{L}}{a\cancel{e}\cancel{E}} = \frac{R_1\cancel{L}\cdot\ln\left(\frac{A}{a}\right)}{\cancel{2}\cancel{e}\cancel{E}(a-A)} \Rightarrow \boxed{R_2 = \frac{a}{a-A}\ln\left(\frac{A}{a}\right)R_1} \quad (v)$$

* Reemplazando (v) en (i), se tiene finalmente que:

$$R_1\left(1+\frac{a}{a-A}\ln\left(\frac{A}{a}\right)\right)=P$$

$$\boxed{\therefore\ R_1=\frac{P}{1+\frac{a}{a-A}\ln\left(\frac{A}{a}\right)}}$$

$$\boxed{R_2=\frac{P}{1+\frac{1}{\frac{a}{a-A}\ln\left(\frac{A}{a}\right)}}}$$