



CONTROL # 3

Tiempo: 3 horas

- P1.** a) (3 pts.) Sean X e Y variables independientes con distribución normal estándar. Determine la función de densidad conjunta de $U = X$ y $V = X/Y$. Muestre que X/Y tiene distribución de Cauchy, es decir, su densidad es

$$\frac{1}{\pi(1+x^2)}.$$

- b) (3 pts.) La densidad conjunta de X e Y está dada por

$$f(x, y) = \begin{cases} \frac{e^{-x/y} e^{-y}}{y} & 0 < x < \infty, 0 < y < \infty \\ 0 & \text{en otro caso.} \end{cases}$$

Calcule la densidad condicional de X dado que $Y = y$. Calcule $P(X > 1 | Y = y)$.

- P2.** a) Se sabe que el valor esperado del puntaje que obtiene un alumno en el examen final de un ramo es de 75.

- 1) (1 pto.) Dé una cota superior de la probabilidad que el puntaje sea mayor que 85.
- 2) (1 pto.) Suponga de aquí en adelante que se sabe que la varianza es 25. ¿Qué puede decirse sobre la probabilidad de que el puntaje obtenido por el alumno esté entre 65 y 85?
- 3) (1 pto.) ¿Cuántos alumnos tienen que dar el examen para asegurar que, con probabilidad de al menos un 99 %, el promedio de notas esté entre 70 y 80? Obtenga un resultado sin utilizar el teorema del límite central, y otro utilizándolo.

- b) Sea X_1, \dots, X_n una m.a.s. con densidad común dada por

$$f(x) = \begin{cases} e^{-(x-\theta)} & x \geq \theta \\ 0 & \text{en otro caso.} \end{cases}$$

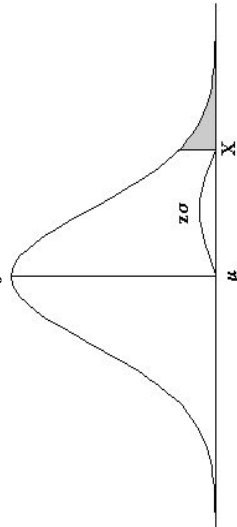
- 1) (1,5 pts.) Encuentre un estimador $\hat{\theta}_1$ mediante el método de los momentos.
- 2) (1,5 pts.) Encuentre un estimador $\hat{\theta}_2$ mediante el método de máxima verosimilitud.

- P3.** La duración de unas determinadas baterías es una variable aleatoria $\mathcal{N}(\mu, \sigma^2)$ con parámetros desconocidos. Se prueban 16 baterías, obteniendo una duración promedio de 7,0 y con s_{n-1}^2 igual a 0,9.

- a) (1,5 pts.) Encontrar un intervalo de confianza al 95 % para μ .
- b) (1,5 pts.) Encontrar un intervalo de confianza al 95 % para σ^2 .
- c) (1,5 pts.) Suponga que se sabe que la varianza real es $\sigma^2 = 1$. ¿Cuál es el intervalo de confianza para μ en este caso?
- d) (1,5 pts.) Si se desea reducir un 20 % el largo del intervalo anterior, manteniendo el nivel de confianza, ¿cuántas baterías adicionales se deberían probar?

TABLA 1: DISTRIBUCIÓN NORMAL

Áreas bajo la curva normal



Ejemplo:

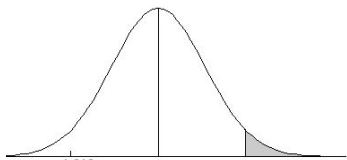
$$Z = \frac{X - \mu}{\sigma}$$

$$P[Z > 1] = 0.1587$$
$$P[Z > 1.96] = 0.0250$$

Desv. normal x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010

TABLA 2: DISTRIBUCIÓN t DE STUDENT

Puntos de porcentaje de la distribución t



Ejemplo

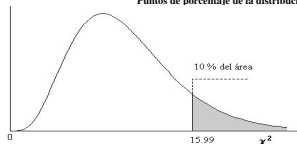
Para φ = 10 grados de libertad:

$$P[t > 1.812] = 0.05$$
$$P[t < -1.812] = 0.05$$

α	0.25	0.2	0.15	0.1	0.05	0.025	0.01	0.005	0.0005
1	1.000	1.376	1.963	3.078	6.314	12.706	31.821	63.656	636.578
2	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	31.600
3	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	12.924
4	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.689
28	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.660
30	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.551
60	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.460
120	0.677	0.845	1.041	1.289	1.658	1.980	2.358	2.617	3.373
∞	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.290

TABLA 3: DISTRIBUCIÓN χ²

Puntos de porcentaje de la distribución χ²



Ejemplo:

Para φ = 10 grados de libertad

$$P[\chi^2 > 15.99] = 0.10$$

α	0.995	0.99	0.975	0.95	0.9	0.75	0.5	0.25	0.1	0.05	0.025	0.01	0.005	α
1	3.326	1.754	0.824	0.336	0.158	0.072	0.452	1.325	2.71	3.84	5.02	6.83	7.88	1
2	1.00E-02	2.01E-02	5.06E-02	0.103	0.211	0.355	1.386	2.77	4.61	5.99	7.38	9.21	10.60	2
3	7.17E-02	0.115	0.216	0.352	0.584	1.231	2.37	4.11	6.25	7.87	9.35	11.34	12.84	3
4	0.291	0.229	0.184	0.134	0.094	0.064	0.343	0.74	1.05	1.24	1.46	1.75	1.93	4
5	0.412	0.554	0.631	1.145	1.160	2.67	4.35	6.83	9.24	11.07	12.83	15.09	16.75	5
6	0.676	0.872	1.237	1.635	2.20	3.45	5.35	7.84	10.64	12.59	14.45	16.81	18.55	6
7	0.99	1.099	2.71	2.83	3.25	6.35	9.04	12.02	14.07	16.01	18.48	20.3	7	
8	1.344	1.647	2.18	2.73	3.49	5.07	7.34	10.22	13.36	15.51	17.53	20.1	22.0	8
9	1.735	2.09	2.70	3.33	4.17	5.90	8.34	11.39	14.68	16.92	19.02	21.7	23.6	9
10	2.16	2.56	3.25	3.94	4.87	5.78	9.34	12.55	15.99	18.31	20.5	23.2	25.2	10
11	2.60	3.05	3.82	4.57	5.68	7.54	10.34	13.70	17.28	19.68	21.9	24.7	26.8	11
12	3.07	3.57	4.40	5.23	6.30	8.44	11.34	14.85	18.51	21.0	23.3	26.2	28.3	12
13	3.57	4.07	5.01	5.88	7.04	9.34	12.34	15.98	19.81	22.4	24.7	27.7	29.8	13
14	4.07	4.66	5.63	6.57	7.79	10.17	13.14	17.12	21.1	23.7	26.1	29.1	31.3	14
15	4.60	5.23	6.26	7.26	8.55	11.14	13.44	18.25	22.3	25.0	27.5	30.6	32.8	15
16	5.14	5.81	6.91	7.96	9.31	11.94	15.34	19.37	23.5	26.3	28.8	32.0	34.3	16
17	5.70	6.41	7.56	8.67	10.09	12.79	16.34	20.5	24.6	27.6	30.2	33.4	35.7	17
18	6.26	7.01	8.23	9.39	10.86	13.68	17.34	21.6	26.0	28.9	31.5	34.8	37.2	18
19	6.84	7.61	8.91	10.15	11.73	13.83	18.34	22.8	27.3	30.4	33.0	36.3	38.8	19
20	7.43	8.26	9.59	10.85	12.44	14.45	19.34	23.8	28.4	31.4	34.2	37.6	40.0	20
21	8.03	8.90	10.28	11.59	13.24	15.34	20.3	24.9	29.6	32.7	35.5	38.9	41.4	21
22	8.64	9.54	10.98	12.34	14.04	17.24	21.3	26.0	30.8	33.9	36.8	40.3	42.8	22
23	9.26	10.20	11.69	13.09	14.85	18.14	22.3	27.1	32.0	35.2	38.1	41.6	44.2	23
24	9.89	10.86	12.40	13.95	15.66	19.04	23.3	28.2	33.2	36.4	39.4	43.0	45.6	24
25	10.52	11.52	13.12	14.61	16.47	19.94	24.3	29.3	34.4	37.7	40.6	44.3	46.9	25
26	11.17	12.20	13.84	15.38	17.29	20.8	25.3	30.4	35.6	38.8	41.9	45.6	48.3	26
27	11.81	12.87	14.56	16.14	18.19	21.7	26.3	31.4	36.6	39.8	42.9	46.6	49.4	27
28	12.46	13.56	15.31	16.93	19.04	22.7	27.3	32.6	37.9	41.3	44.5	48.3	51.0	28
29	13.12	14.26	16.05	17.71	19.77	23.6	28.3	33.7	39.1	42.6	45.7	49.6	52.3	29
30	13.79	14.95	16.75	18.49	20.5	24.5	29.3	34.8	40.3	43.8	47.0	50.9	53.7	30
31	14.47	15.66	17.49	19.28	21.3	25.3	30.4	35.8	41.6	44.9	48.1	51.9	54.8	31
32	15.16	16.38	18.25	20.07	22.1	26.3	31.4	36.8	42.9	46.2	49.4	53.2	56.1	32
33	15.86	17.10	19.04	20.86	22.9	27.3	32.6	37.9	44.1	47.4	50.6	54.0	56.9	33
34	16.57	17.83	19.84	21.66	23.7	28.3	33.7	39.1	45.2	48.5	51.7	55.1	58.0	34
35	17.29	18.56	20.65	22.47	24.5	29.3	34.8	40.3	46.3	49.6	52.8	56.2	59.1	35
36	18.02	19.32	21.49	23.29	25.3	30.4	35.8	41.6	47.4	50.7	53.9	57.3	60.2	36
37	18.76	20.07	22.34	24.12	26.1	31.4	36.8	42.9	48.5	51.8	55.0	58.3	61.3	37
38	19.51	20.84	23.21	24.96	26.9	32.4	37.9	44.1	49.6	52.9	56.1	59.4	62.4	38
39	20.27	21.62	24.09	25.81	27.7	33.4	39.1	45.2	50.7	54.0	57.2	60.5	63.5	39
40	21.04	22.42	24.94	26.65	28.5	34.4	40.3	46.3	51.8	55.1	58.3	61.6	64.6	40
41	21.82	23.22	25.81	27.50	29.3	35.4	41.6	47.4	52.9	56.2	59.4	62.7	65.7	41
42	22.61	24.04	26.69	28.36	30.1	36.4	42.9	48.5	54.0	57.3	60.5	63.8	66.8	42
43	23.41	24.87	27.59	29.23	30.9	37.4	44.1	49.6	55.1	58.3	61.6	64.9	67.9	43
44	24.22	25.72	28.50	30.11	31.7	38.4	45.2	50.7	56.2	59.4	62.7	66.0	69.0	44
45	25.04	26.59	29.42	31.00	32.5	39.4	46.3	51.8	57.3	60.5	63.8	67.1	70.1	45
46	25.87	27.48	30.35	31.90	33.3	40.4	47.4	52.9	58.3	61.6	64.9	68.2	71.2	46
47	26.71	28.38	31.29	32.81	34.1	41.4	48.5	54.0	59.4	62.7	66.0	69.3	72.3	47
48	27.56	29.30	32.24	33.73	34.9	42.4	49.6	55.1	60.5	63.8	67.1	70.4	73.4	48
49	28.42	30.23	33.19	34.66	35.7	43.4	50.7	56.2	61.6	64.9	68.2	71.5	74.5	49
50	29.29	31.18	34.15	35.60	36.5	44.4	51.8	57.3	62.7	66.0	69.3	72.6	75.6	50
51	30.17	32.14	35.12	36.55	37.3	45.4	52.9	58.3	63.8	67.1	70.4	73.7	76.7	51
52	31.06	33.11	36.09	37.51	38.1	46.4	54.0	59.4	64.9	68.2	71.5	74.8	77.8	52
53	31.96	34.09	37.07	38.48	38.9	47.4	55.1	60.5	66.0	69.3	72.6	75.9	78.8	53
54	32.87	35.08	38.06	39.46	39.7	48.4	56.2	61.6	67.1	70.4	73.7	77.0	79.8	54
55	33.79	36.08	39.06	40.45	40.5	49.4	57.3	62.7	68.2	71.5	74.8	78.1	80.8	55
56	34.72	37.09	40.06	41.45	41.3	50.4	58.3	63.8	69.3	72.6	75.9	79.2	81.8	56
57	35.66	38.11	41.07	42.46	42.1	51.4	59.4	64.9	70.4	73.7	77.0	80.3	82.8	57
58	36.61	39.14	42.09	43.47	42.9	52.4	60.5	66.0	71.5	74.8	78.1	81.4	83.8	58
59	37.57	40.18	43.12	44.49	43.7	53.4	61.6	67.1	72.6	75.9	79.2	82.5	84.9	59
60	38.54	41.23	44.16	45.52	44.5	54.4	62.7	68.2	73.7	77.0	80.3	83.6	85.9	60
61	39.52	42.29	45.21	46.56	45.3	55.4	63.8	69.3	74.8	78.1	81.4	84.7	86.9	61
62	40.51	43.36	46.27	47.61	46.1	56.4	64.9	70.4	75.9	79.2	82.5	85.8	88.0	62
63	41.51	44.44	47.34	48.67	46.9	57.4	66.0	71.5	77.0	80.3	83.6	86.9	89.1	63
64	42.52	45.53	48.41	49.74	47.7	58.4	67.1	72.6	78.1	81.4	84.7	88.0	90.2	64
65	43.54	46.63	49.49	50.82	48.5	59.4	68.2	73.7	79.2	82.5	85.8	89.1	91.3	65
66	44.57	47.74	50.58	51.91	49.3	60.4	69.3	74.8	80.3	83.6	86.9	90.2	92.4	66
67	45.61	48.86	51.68	53.01	50.1	61.4	70.4	75.9	81.4	84.7	88.0	91.3	93.5	67
68	46.66	49.99	52.79	54.11	50.9	62.4	71.5	77.0	82.5	85.8	89.1	92.4	94.6	68
69	47.72	51.13	53.90	55.22	51.7	63.4	72.6	78.1	83.6	86.9	90.2	93.5	95.7	69
70	48.79	52.28	55.02	56.33	52.5	64.4	73.7	79.2	84.7	88.0	91.3	94.6	96.8	70
71	49.87	53.44	56.15	57.45	53.3	65.4	74.8	80.3	85.8	89.1	92.4	95.7	97.9	71
72	50.96	54.61	57.29	58.57	54.1	66.4	75.9	81.4	86.9	90.2	93.5	96.8	99.0	72
73	52.06	55.79	58.43	59.70	54.9	67.4	77.0	82.5	88.0	91.3	94.6	97.9	100.1	73
74	53.17	56.98	59.59	60.83	55.7	68.4	78.1	83.6	89.1	92.4	95.7	99.0	101.2	74
75	54.29	58.18	60.75	61.97	56.5	69.4	79.2	84.7	90.2	93.5	96.8	100.1	102.3	75
76	55.42	59.39	61.92	63.12	57.3	70.4	80.3	85.8	91.3	94.6	97.9	101.2	103.4	76
77	56.56	60.61	63.10	64.27	58.1	71.4	81.4	86.9	92.4	95.7	99.0	102.3	104.5	77
78	57.71	61.84	64.29	65.43	58.9	72.4	82.5	88.0	93.5	96.8	100.1	103.4	105.6	78
79	58.87	63.08	65.49	66.60	59.7	73.4	83.6	89.1	94.6	97.9	101.2	104.5	106.7	79
80	59.94	64.33	66.69	67.77	60.5	74.4	84.7	90.2	95.7	99.0	102.3	105.6	107.8	80
81	61.02	65.59	67.90	68.95	61.3	75.4	85.8	91.3	96.8	100.1	103.4	106.7	108.9	81
82	62.11	66.86	69.12	70.12	62.1	76.4	86.9	92.4	97.9	101.2	104.5	107.8	110.0	82
83	63.21	68.14	70.35	71.30	62.9	77.4	88.0	93.5	99.0	102.3	105.6	108.9	111.1	83
84	64.32	69.43	71.59	72.48	63.7	78.4	89.1	94.6	100.1	103.4	106.7	110.0	112.2	84
85	65.44	70.73	72.84	73.67	64.5	79.4	90.2	95.7	101.2	104.5	107.8	111.1	113.3	85
86	66.57	72.04	74.09	74.86	65.3	80.4	91.3	96.8	102.3	105.6	108.9	112.2	114.4	86
87	67.71	73.36	75.35	76.06	66.1	81.4	92.4	97.9	103.4	106.7	110.0	113.3	115.5	87
88	68.86	74.69	76.62	77.26	66.9	82.4	93.5	99.0						