

$$g = F(x, y)$$

$$x = u^2 + v^2$$

$$y = u/v$$

$$\rightarrow g = F(u^2 + v^2, u/v)$$

$$\frac{\partial^2 g}{\partial u \partial v} ?$$

$$\frac{\partial x}{\partial u} = 2u$$

$$\frac{\partial y}{\partial u} = 1/v$$

$$\frac{\partial x}{\partial v} = 2v$$

$$\frac{\partial y}{\partial v} = \left(-\frac{u}{v^2}\right)$$

$$\begin{aligned} \textcircled{1} \quad \frac{\partial g}{\partial v} &= \frac{\partial F}{\partial v}(x, y) = \frac{\partial F}{\partial x} \cdot \frac{\partial x}{\partial v} + \frac{\partial F}{\partial y} \cdot \frac{\partial y}{\partial v} \\ &= \frac{\partial F}{\partial x} \cdot 2v + \frac{\partial F}{\partial y} \left(-\frac{u}{v^2}\right) \end{aligned}$$

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si malo esto -0,9

$$\textcircled{2} \quad \frac{\partial^2 g}{\partial u \partial v} = \frac{\partial}{\partial u} \left(\frac{\partial F}{\partial x} \cdot 2v + \frac{\partial F}{\partial y} \left(-\frac{u}{v^2}\right) \right)$$

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$$= \frac{\partial}{\partial u} \left(\frac{\partial F}{\partial x} \cdot 2v \right) + \frac{\partial}{\partial u} \left(\frac{\partial F}{\partial y} \left(-\frac{u}{v^2}\right) \right)$$

por si saben la formula

(*)

$$= \frac{\partial}{\partial u} \left(\frac{\partial F}{\partial x} \right) 2v + \frac{\partial F}{\partial x} \frac{\partial (2v)}{\partial u} + \frac{\partial}{\partial u} \left(\frac{\partial F}{\partial y} \right) \left(-\frac{u}{v^2}\right) + \frac{\partial F}{\partial y} \frac{\partial \left(-\frac{u}{v^2}\right)}{\partial u}$$

$$= \frac{\partial}{\partial u} \left(\frac{\partial F}{\partial x} \right) 2v + \frac{\partial}{\partial u} \left(\frac{\partial F}{\partial y} \right) \left(-\frac{u}{v^2}\right) - \frac{\partial F}{\partial y} \cdot \frac{1}{v^2}$$

si saben usarlo (1/2)

$$\textcircled{3} \quad \frac{\partial a}{\partial u} = \frac{\partial a}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial a}{\partial y} \frac{\partial y}{\partial u} = \frac{\partial^2 F}{\partial x^2} 2u + \frac{\partial^2 F}{\partial y \partial x} \frac{1}{v}$$

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$$\cdot \frac{\partial b}{\partial u} = \frac{\partial b}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial b}{\partial y} \frac{\partial y}{\partial u} = \frac{\partial^2 F}{\partial x \partial y} 2u + \frac{\partial^2 F}{\partial y^2} \frac{1}{v}$$

$$\therefore \frac{\partial^2 g}{\partial u \partial v} = 4uv \frac{\partial^2 F}{\partial x^2} + 2 \frac{\partial^2 F}{\partial y \partial x} - \frac{2u^2}{v^2} \frac{\partial^2 F}{\partial x \partial y} - \frac{u}{v^3} \frac{\partial^2 F}{\partial y^2} - \frac{1}{v^2} \frac{\partial F}{\partial y}$$