

# Simple Energy balance model: Budyko-Sellers computational lab

GF3004 - 2014

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In the present computer lab we will discuss a simple energy balance model for earth's climate. Contrary to a radiative convective-equilibrium model, the dimension of interest in this model is latitude, and there will be no vertical dependence of the model variables. The main objective of this session is to furnish ourselves with a simple model, so we can study the consequences of the ice-albedo feedback and the effect of the meridional heat transport in earth's climate.

## 1 Short description of the model

We will consider a zonally (east-west) symmetric climate, in such a way that all the dependence of the balance terms is only a function of latitude.

The energy balance in each of the latitude bands in the model, can be written as,

$$S_i(1 - \alpha(T_i)) = I \uparrow (T_i) + F(T_i), \quad (1)$$

where  $S_i$  is the incoming solar radiation as a function of latitude,  $I \uparrow$  is the outgoing longwave radiation and  $F$  is the heat transport between a latitude band and the rest of the planet.

Each one of the processes must be parameterized. In the case of the incoming solar radiation, an appropriate formula that represents the annual mean latitude distribution is the following,

$$S_i = Q(1 - 0.241(3x_i^2 - 1)), \quad (2)$$

where  $Q = S_o/4 = y$   $x_i = \sin \phi_i$ ,  $\phi_i$  is the mean latitude of each latitude band.

On the other hand, the albedo  $\alpha$  depends on temperature. One can parameterize this dependence simply as whether the surface is covered by ice or not. Being  $T_c$  a critical temperature

$$\alpha_i(T_i) = 0.6 \quad T_i \leq T_c \quad (3)$$

$$\alpha_i(T_i) = 0.3 \quad T_i \geq T_c \quad (4)$$

$T_c$  has usually a value between 0 and -10 C (by default we will use  $T_c = -10C$ ). Outgoing infrared radiation  $I \uparrow$  is parameterized as a linear function of temperature

$$I \uparrow (T_i) = A + BT_i \quad (5)$$

where the coefficients  $A$  y  $B$  are  $A = 204.0Wm^{-2}$  y  $B = 2.17Wm^{-2}(^{\circ}C)^{-1}$ . This is an empirical relation and therefore it takes into account the effect of water vapor, clouds, and other greenhouse gases in the present climate.

Finally the term  $F$  also requires to be parameterized. Probably the most simple way to do this is to parameterize the term  $F$  in each latitude band as proportional to the difference between the temperature of the latitude band  $T_i$  and the mean earth's temperature,  $\bar{T}$ ,

$$F_i = F(T_i) = k_t(T_i - \bar{T}), \quad (6)$$

where  $k_t$  is a heat transport coefficient that must be adjusted empirically. A suggested value for  $k_t$  is  $k_t = 3.81 \text{ W m}^{-2} (\text{°C})^{-1}$ .

For calculation purposes, one must note that the mean temperature is not simple the average arithmetic mean of the temperature in each band but rather  $\bar{T} = \frac{\sum T_i \cos \phi_i}{\sum \cos \phi_i}$ .

When parameterizations are incorporated into the balance equations (Eq. 1), one can solve for the temperature in the following way,

$$T_i = \frac{S_i(1 - \alpha_i) + k_t \bar{T} - A}{B + k_t} \quad (7)$$

## 2 Activities

Here, you are asked to write a MATLAB program so that equation 7 can be solved iteratively starting from some initial conditions. For instance, you may start defining your latitude regions every 10 degrees of latitude between 0 and 90. Write a value for the initial temperature in each band. With these initial values calculate the terms of the right hand side of the equation 7. By default, the value of  $T_c = -10\text{C}$ .

- Study the convergence of the model with respect to initial conditions. Choose initial conditions under the critical temperature and above the critical temperature. Describe the differences.
- For the parameters of the model used by default, find the value of the solar constant required so that the planet is completely covered by ice.
- Study the sensitivity of the previous value to the parameters of the model. In particular the sensitivity to the value of the transport coefficient. Choose small and large values for this coefficient. What differences you see with respect to your control simulation?. In particular, observe the latitudinal distribution of temperature and how does it change for different values of  $k_t$ . Attempt an explanation as to why the threshold for a "snowball earth" is sensitive to the values of the transport coefficient.
- Study the value of the solar constant required to have temperatures above the freezing at the equator, when the model is initialized with temperatures below the critical temperature. Make a plot of the mean temperature as a function of the solar constant that includes both type of initial condition, when the model is initialized with temperatures below and above the critical.
- Discuss reasons why would you think climate is not as sensitive to solar insolation as the model shows.
- Describe how or attempt to modify the simple model so that varying concentrations of greenhouse gases and cloud feedbacks can be accounted for in the simple model.
- Write a short report summarizing your results.