DECOHERENCE AND THE TRANSITION FROM QUANTUM TO CLASSICAL

The environment surrounding a quantum system can, in effect, monitor some of the system's observables. As a result, the eigenstates of those observables continuously decohere and can behave like classical states.

Wojciech H. Zurek

Quantum mechanics works exceedingly well in all practical applications. No example of conflict between its predictions and experiment is known. Without quantum physics we could not explain the behavior of solids, the structure and function of DNA, the color of the stars, the action of lasers or the properties of superfluids. Yet well over half a century after its inception, the debate about the relation of quantum mechanics to the familiar physical world continues. How can a theory that can account with precision for everything we can measure still be deemed lacking?

What is wrong with quantum theory?

The only "failure" of quantum theory is its inability to provide a natural framework that can accommodate our prejudices about the workings of the universe. States of quantum systems evolve according to the *deterministic*, *linear* Schrödinger equation,

$$i\hbar \frac{d}{dt}|\psi\rangle = H|\psi\rangle \tag{1}$$

That is, just as in classical mechanics, given the initial state of the system and its Hamiltonian H, one can compute the state at an arbitrary time. This deterministic evolution of $|\psi\rangle$ has been verified in carefully controlled experiments. Moreover, there is no indication of a border between quantum and classical behavior at which equation 1 fails (see figure 1).

There is, however, a very poorly controlled experiment with results so tangible and immediate that it has an enormous power to convince: Our perceptions are often difficult to reconcile with the predictions of equation 1.

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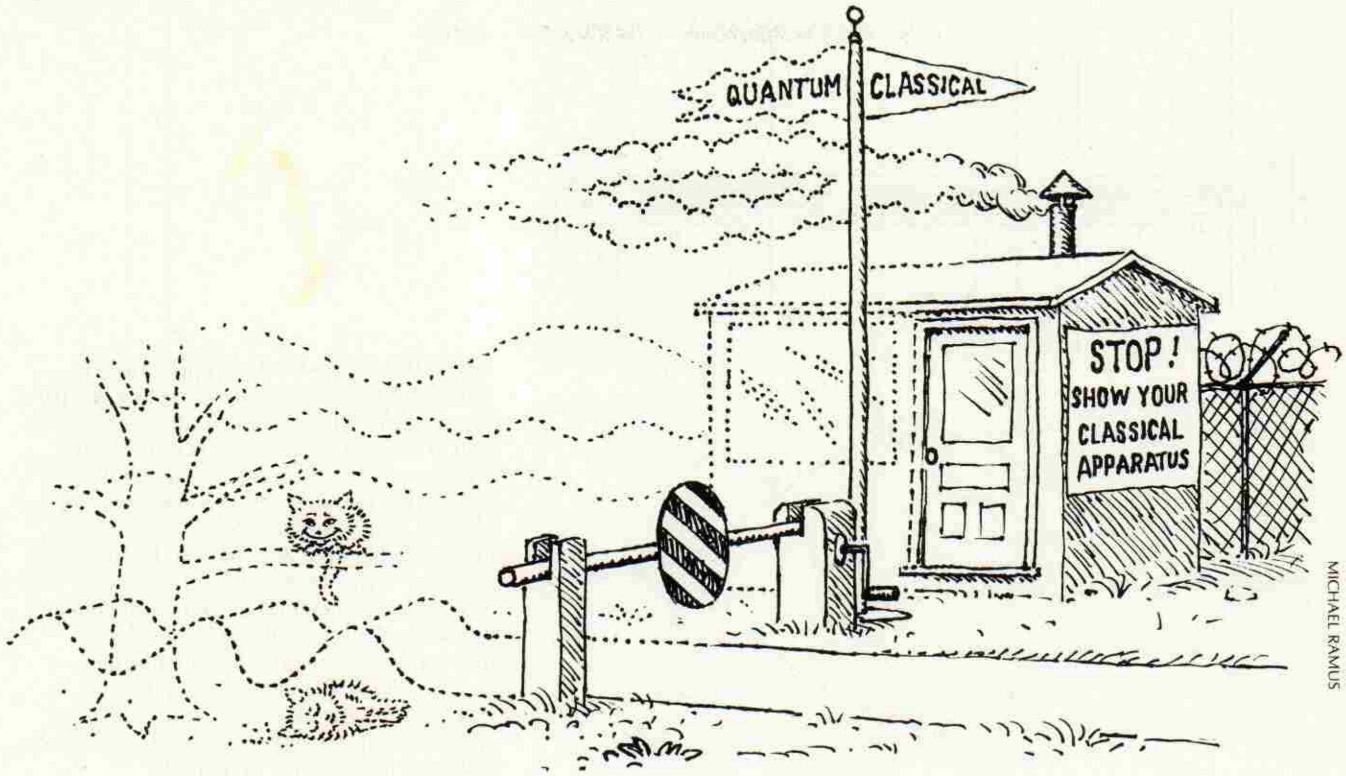
Why? Given almost any initial condition the universe described by $|\psi\rangle$ evolves into a state that simultaneously contains many alternatives never seen to coexist in our world. Moreover, while the ultimate evidence for the choice of one such option resides in our elusive "consciousness," there is every indication that the choice occurs long before consciousness ever gets involved. Thus at the root of our unease with quantum mechanics is the clash between the principle of superposition—the consequence of the linearity of equation 1—and the everyday classical reality in which this principle appears to be violated.

The problem of measurement has a long and fascinating history. The first widely accepted explanation of how a single outcome emerges from the many possibilities was the Copenhagen interpretation, proposed by Niels Bohr,[1,2] who insisted that a classical apparatus is necessary to carry out measurements. Thus quantum theory was not to be universal. The key feature of the Copenhagen interpretation is the dividing line between quantum and classical. Bohr emphasized that the border must be mobile, so that even the "ultimate apparatus"—the human nervous system—can be measured and analyzed as a quantum object, provided that a suitable classical device is available to carry out the task.

In the absence of a crisp criterion to distinguish between quantum and classical, an identification of the "classical" with the "macroscopic" has often been tentatively accepted. The inadequacy of this approach has become apparent as a result of relatively recent developments: A cryogenic version of the Weber bar—a gravity-wave detector—must be treated as a quantum harmonic oscillator even though it can weigh a ton.[3] Nonclassical squeezed states can describe oscillations of suitably prepared electromagnetic fields with macroscopic numbers of photons. (See the article by Malvin C. Teich and Bahaa E. A. Saleh in PHYSICS TODAY, June 1990, page 26.) Superconducting Josephson junctions have quantum states associated with currents involving macroscopic numbers of electrons, and yet they can tunnel between the



Delineating the border between the quantum realm ruled by the Schrödinger equation and the classical realm ruled by Newton's laws is one of the unresolved problems of physics. **Figure 1**

minima of the effective potential.[4]

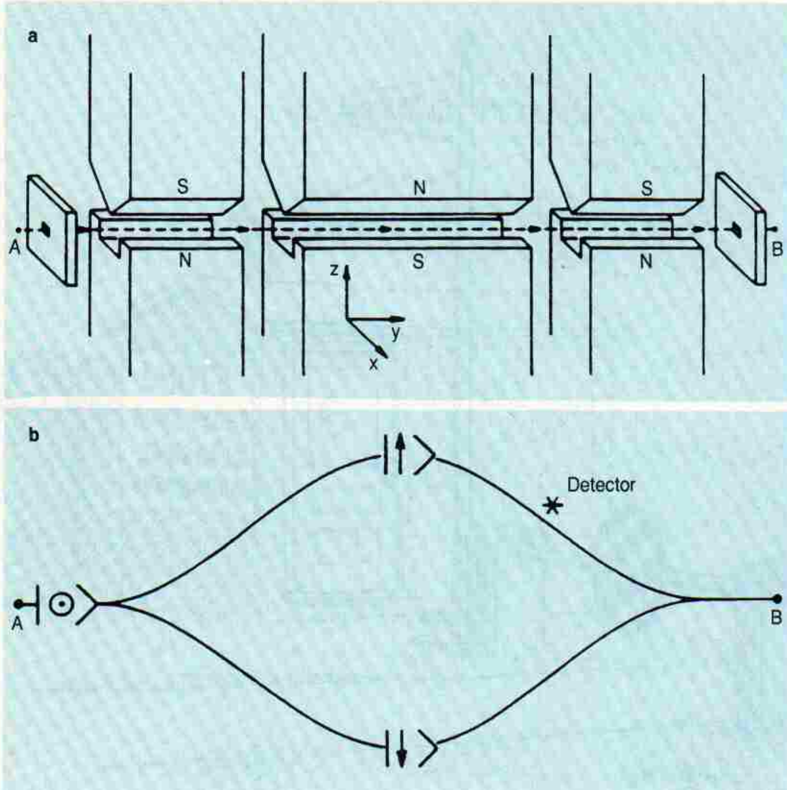
If macroscopic systems cannot always be safely placed on the classical side of the boundary, might there be no boundary at all? The many-worlds interpretation (or, more accurately, the many-universes interpretation) claims to do away with the boundary.[5] The many-worlds interpretation was developed in the 1950s by Hugh Everett III with the encouragement of John Archibald Wheeler. In this interpretation all of the universe is described by quantum theory. Superpositions evolve forever according to the Schrödinger equation. Each time a suitable interaction takes place between any two quantum systems, the wavefunction of the universe splits, so that it develops ever more "branches."

Everett's work was initially almost unnoticed. It was taken out of mothballs over a decade later by Bryce DeWitt, who managed—in part, through his PHYSICS TODAY article (September 1970, page 30)—to upgrade its status from virtually unknown to very controversial.[6] The many-worlds interpretation is a natural choice for quantum cosmology, which describes the whole universe by means of a state vector. There is nothing more macroscopic than the universe. It can have no *a priori* classical subsystems. There can be no observer "on the outside." In this context, classicality has to be an *emergent* property of the selected observables or systems.

At a first glance, the two interpretations—many-worlds and Copenhagen—have little in common. The Copenhagen interpretation demands an *a priori* "classical domain" with a border that enforces a classical "embargo" by letting through just one potential outcome. The many-worlds interpretation aims to abolish the need for the border altogether: Every potential outcome is accommodated by the ever proliferating branches of the wavefunction of the universe. The similarity of the difficulties faced by these two viewpoints nevertheless becomes apparent when we ask the obvious question "Why do I, the observer, perceive only one of the outcomes?" Quantum theory, with its freedom to rotate bases in Hilbert space, does not even clearly define which states of the universe correspond to branches. Yet our perception of a reality with alternatives and not a coherent superposition of alternatives demands an explanation of when, where and how it is decided what the observer actually perceives. Considered in this context, the many-worlds interpretation in its original version does not abolish the border but pushes it all the way to the boundary between the physical universe and consciousness. Needless to say, this is a very uncomfortable place to do physics.

In spite of the profound difficulties and the lack of a breakthrough for some time, recent years have seen a growing consensus that progress is being made in dealing with the measurement problem. The key (and uncontroversial) fact has been known almost since the inception of quantum theory, but its significance for the transition from quantum to classical is being recognized only now: Macroscopic quantum systems are never isolated from their environments. Therefore, as H. Dieter Zeh emphasized,[7] they should not be expected to follow Schrödinger's equation, which is applicable only to a closed system. As a result systems usually regarded as classical suffer (or benefit) from the natural loss of quantum coherence, which "leaks out" into the environment.[8] The resulting "decoherence" cannot be ignored when one addresses the problem of the reduction of wavepackets: It imposes, in effect, the required embargo on the potential outcomes by allowing the observer to maintain records of alternatives and to be aware of only one branch.

This article aims to explain the physics and thinking behind this approach. The reader should be warned that I



Reversible Stern–Gerlach apparatus (a) splits a beam of atoms into two branches **(b)** that are correlated with the component of the spin of the atoms and then recombines the branches before the atoms leave the device. Eugene Wigner used this *gedanken* experiment to show that a correlation between the spin and the location of an atom can be reversibly undone.[12] The introduction of a one-bit—that is, two-state—quantum detector that changes its state when the atom passes nearby prevents this: The detector inherits the correlation between the spin and the trajectory, so the reversible Stern–Gerlach apparatus can no longer undo the correlation. (Adapted from ref. 8.) **Figure 2**

am not a disinterested witness to this development[7–10] but rather one of its proponents. I shall, nevertheless, attempt to paint a fairly honest picture and point out difficulties as well as accomplishments.

Correlations and measurements

A convenient starting point for the discussion of the measurement problem, and more generally of the emergence of classical behavior from quantum dynamics, is the analysis of quantum measurements due to John von Neumann.[11] In contrast to Bohr, who assumed at the outset that apparatus must be classical, von Neumann analyzed the case of quantum apparatus. I shall reproduce his analysis for the simplest case: a measurement on a two-state system $\mathcal{S}$ (which can be thought of as spin $\frac{1}{2}$) with the result recorded by a quantum two-state (one bit) detector.

The Hilbert space $\mathcal{H}_{\mathcal{S}}$ of the system is spanned by the orthonormal states $|\uparrow\rangle$ and $|\downarrow\rangle$, while the states $|d_\uparrow\rangle$ and $|d_\downarrow\rangle$ span the space $\mathcal{H}_{\mathcal{D}}$ of the detector $\mathcal{D}$. A two-dimensional $\mathcal{H}_{\mathcal{D}}$ is the absolute minimum needed to record the possible outcomes. One can devise a quantum detector (see figure 2) that begins in the $|d_\downarrow\rangle$ state and "clicks," $|\uparrow\rangle|d_\downarrow\rangle \to |\uparrow\rangle|d_\uparrow\rangle$, when the spin is in the state $|\uparrow\rangle$ but remains unperturbed otherwise.[7,12]

I shall assume that before the interaction the system was in a pure state $|\psi_{\mathcal{S}}\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$, with the complex coefficients satisfying $|\alpha|^2 + |\beta|^2 = 1$. The composite system starts as $|\Phi^{\rm i}\rangle = |\psi_{\mathcal{S}}\rangle|d_\downarrow\rangle$. Interaction results in the evolution of $|\Phi^{\rm i}\rangle$ into a correlated state $|\Phi^{\rm c}\rangle$:

$$|\Phi^{\rm i}\rangle = (\alpha|\uparrow\rangle + \beta|\downarrow\rangle)\,|d_\downarrow\rangle \\ \to \alpha|\uparrow\rangle|d_\uparrow\rangle + \beta|\downarrow\rangle|d_\downarrow\rangle = |\Phi^{\rm c}\rangle \qquad (2)$$

This essential and uncontroversial first stage of the measurement process can be accomplished with a Schrödinger equation with an appropriate interaction. It might be tempting to halt the discussion of measurements with equation 2. After all, the correlated state vector $|\Phi^{\rm c}\rangle$ implies that if the detector is seen in the state $|d_\uparrow\rangle$, the system is guaranteed to be found in the state $|\uparrow\rangle$. Why ask for anything more?

The reason for dissatisfaction with $|\Phi^{\rm c}\rangle$ as a description of a completed measurement is simple and fundamental: In the real world, even when we do not know the outcome, we do know what the alternatives are, and we can safely act as if only one of them has already occurred. As we shall see in the next section, this is not true for a system described by $|\Phi^{\rm c}\rangle$. But how can an observer who has not yet consulted the detector express his ignorance about the outcome without giving up his certainty about the "menu" of the possibilities? Quantum theory provides the right formal tool for the occasion: A *density matrix* can be used to describe the probability distribution for the alternative outcomes.

Von Neumann was well aware of these difficulties. Indeed, he postulated[11] that in addition to the unitary evolution given by equation 1 there is an *ad hoc* "process 1"—a nonunitary reduction of the state vector—that takes a pure, correlated state $|\Phi^{\rm c}\rangle$ into an appropriate mixture. This process makes the outcomes independent of one another by taking the pure-state density matrix

$$\rho^{\rm c} = |\Phi^{\rm c}\rangle\langle\Phi^{\rm c}| \\ = |\alpha|^2\,|\uparrow\rangle\langle\uparrow|\,|d_\uparrow\rangle\langle d_\uparrow| + \alpha\beta^*\,|\uparrow\rangle\langle\downarrow|\,|d_\uparrow\rangle\langle d_\downarrow| \\ + \alpha^*\beta\,|\downarrow\rangle\langle\uparrow|\,|d_\downarrow\rangle\langle d_\uparrow| + |\beta|^2\,|\downarrow\rangle\langle\downarrow|\,|d_\downarrow\rangle\langle d_\downarrow| \qquad (3)$$

and canceling the off-diagonal terms, which express quantum correlations, leaving a reduced density matrix

$$\rho^{\rm r} = |\alpha|^2\,|\uparrow\rangle\langle\uparrow|\,|d_\uparrow\rangle\langle d_\uparrow| + |\beta|^2\,|\downarrow\rangle\langle\downarrow|\,|d_\downarrow\rangle\langle d_\downarrow| \qquad (4)$$

Why is the reduced $\rho^{\rm r}$ easier to interpret as a description of a completed measurement than $\rho^{\rm c}$? After all, both $\rho^{\rm r}$ and $\rho^{\rm c}$ contain identical diagonal elements, and both outcomes are still present. What, if anything, was gained at the substantial price of introducing the nonunitary "process 1"?

The preferred basis: What was measured?

The key advantage of ρ^r over ρ^c is that its coefficients may be interpreted as classical probabilities. The density matrix ρ^r can be used to describe the alternative states of a composite spin–detector system that has the *classical* correlations: When the off-diagonal terms are absent one can safely maintain that the apparatus and the system are each separately in a definite but unknown state, and that the correlation between them exists in the preferred basis defined by the states appearing on the diagonal. By the same token, two halves of a split coin are classically correlated: Holding an unopened envelope containing one of them we can be sure that its state is either heads or tails (and not some superposition of the two), and that the second envelope contains the other, matching alternative.[13] (See the box on this page.)

By contrast, it is impossible to interpret ρ^c as denoting such "classical ignorance." In particular, the detector has not even decided on the set of alternatives! This can be illustrated by choosing $\alpha = -\beta = 1/\sqrt{2}$, so that

$$|\Phi^c\rangle = (|\uparrow\rangle|d_\uparrow\rangle - |\downarrow\rangle|d_\downarrow\rangle)/\sqrt{2} \tag{5}$$

This state is invariant under rotations of the basis. For instance, instead of using the eigenstates $|\uparrow\rangle$ and $|\downarrow\rangle$ of $\hat{\sigma}_z$ one can rewrite $|\Phi^c\rangle$ in terms of the eigenstates of $\hat{\sigma}_x$, $|\odot\rangle = (|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$, and $|\otimes\rangle = (|\uparrow\rangle - |\downarrow\rangle)/\sqrt{2}$. This immediately yields

$$|\Phi^c\rangle = (|\otimes\rangle|d_\otimes\rangle - |\odot\rangle|d_\odot\rangle)/\sqrt{2} \tag{6}$$

where the states $|d_\otimes\rangle = (|d_\uparrow\rangle + |d_\downarrow\rangle)/\sqrt{2}$ and $|d_\odot\rangle = (|d_\downarrow\rangle - |d_\uparrow\rangle)/\sqrt{2}$ are perfectly "legal" states of the quantum detector. Therefore, the density matrix $\rho^c = |\Phi^c\rangle\langle\Phi^c|$ could have many different states of the subsystems on the diagonal.

This should not come as a surprise. Except for notation the state vector $|\Phi^c\rangle$ is the same as the wavefunction of a pair of correlated spin-$\frac{1}{2}$ systems in David Bohm's version of the Einstein–Podolsky–Rosen paradox.[14] Related experiments[15] show that nonseparable quantum correlations violate Bell's inequalities.[16] The key point is that before the measurement neither of the two spins in a system described by $|\Phi^c\rangle$ *has* a definite state—their states are not merely unknown. We conclude that when a detector is quantum, *a superposition of the records exists and is a record of a superposition of outcomes*—a very nonclassical state of affairs.

Missing information and decoherence

Unitary evolution condemns every closed quantum system to "purity." Yet if the outcomes of a measurement are to become independent, with consequences that can be explored separately, a way must be found to dispose of the excess information. This disposal can be caused by interaction with the degrees of freedom external to the system, which we shall summarily refer to as "the environment."

Reduction of the state from ρ^c to ρ^r decreases the information available to the observer about the composite system $\mathcal{SD}$. Thus its entropy $S = -\mathrm{Tr}\rho\ln\rho$ increases as it must, $\Delta S = S(\rho^r) - S(\rho^c) = -(|\alpha|^2\ln|\alpha|^2 + |\beta|^2\ln|\beta|^2)$. The initial state described by ρ^c was pure, and the reduced state is mixed. Information gain—the objective of measurement—is accomplished only when the observer interacts and becomes correlated with the detector in the already precollapsed state ρ^r. This must be preceded by an increase in entropy if the outcomes are to become classical, so that they can be used as initial conditions to predict the future.

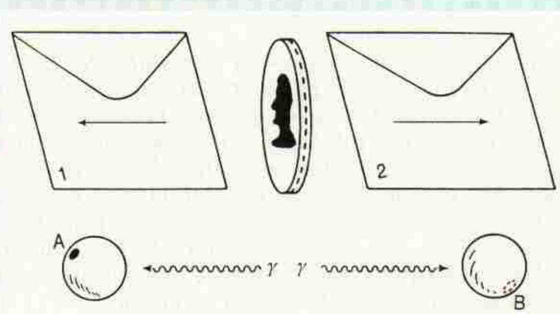
Classical and Quantum Correlations

When a coin is split into two halves that are put into two envelopes, which are then shuffled, numbered and sent to two observers, the state of the system can be described by a statistical operator—the density matrix,

$$\rho_{\mathrm{coin}} = \tfrac{1}{2}(|H_1\rangle\langle H_1|\,|T_2\rangle\langle T_2| + |T_1\rangle\langle T_1|\,|H_2\rangle\langle H_2|)$$

This correctly represents the certainty about the two alternatives—that is, whether the heads (H) or tails (T) half is in the selected envelope; the correlation between the contents of the two envelopes; and the (classical) ignorance about which of the two alternatives is actually the case. A density matrix representing a pointer of an apparatus correlated with a measured system ought to have a similar form.

By contrast, in David Bohm's version of the Einstein–Podolsky–Rosen experiment the two photons have correlated polarizations, so that their state must be described by $|\psi\rangle = (|A_1\rangle|B_2\rangle - |B_1\rangle|A_2\rangle)/\sqrt{2}$, where polarizations A and B correspond to the opposite poles of the Stokes sphere. Given $|\psi\rangle$, we have all the information quantum theory allows one to have about their combined state. Such complete information comes at a price: Neither of the two photons has a state "of its own." Hence even the alternatives are undefined. A quantum apparatus correlated with a quantum system is described by an analogous state vector and would suffer from an analogous ambiguity about the alternative outcomes. (Figure adapted from ref. 13.)



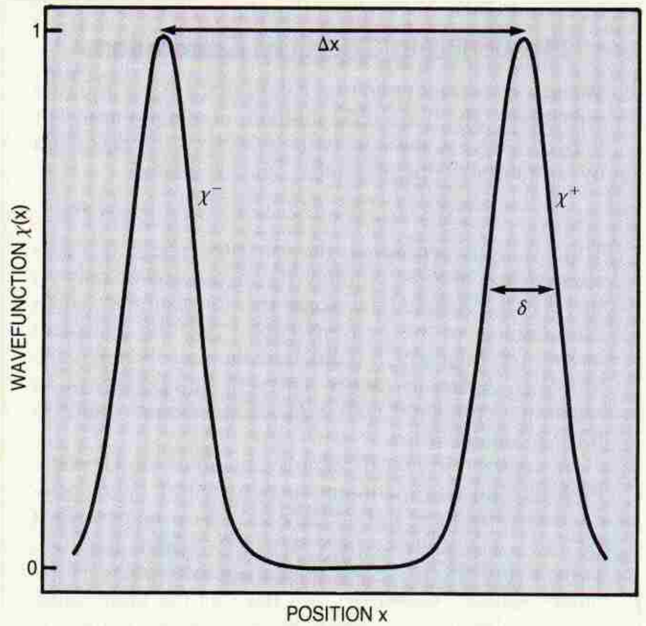
As an illustration of the process of environment-induced decoherence consider a system $\mathcal{S}$, a detector $\mathcal{D}$ and an environment $\mathcal{E}$. The environment is also a quantum system. Following the first step of the measurement process—establishment of the correlation as shown in equation 2—the environment similarly interacts and becomes correlated with the apparatus:

$$|\Phi^c\rangle|\mathcal{E}_0\rangle = (\alpha|\uparrow\rangle|d_\uparrow\rangle + \beta|\downarrow\rangle|d_\downarrow\rangle)|\mathcal{E}_0\rangle$$
$$\to \alpha|\uparrow\rangle|d_\uparrow\rangle|\mathcal{E}_\uparrow\rangle + \beta|\downarrow\rangle|d_\downarrow\rangle|\mathcal{E}_\downarrow\rangle = |\Psi\rangle \tag{7}$$

The final state of such a combined "von Neumann chain" of correlated systems $\mathcal{SDE}$ extends the correlation beyond the $\mathcal{SD}$ pair. When the states of the environment $|\mathcal{E}_i\rangle$ corresponding to states $|d_\uparrow\rangle$ and $|d_\downarrow\rangle$ of the detector are orthogonal, $\langle\mathcal{E}_i|\mathcal{E}_j\rangle = \delta_{ij}$, the density matrix that describes the detector–system combination obtained by ignoring (tracing over) the uncontrolled (and unmeasured) degrees of freedom is

$$\rho_{\mathcal{SD}} \equiv \mathrm{Tr}_{\mathcal{E}}|\Psi\rangle\langle\Psi| = \Sigma_i\langle\mathcal{E}_i|\Psi\rangle\langle\Psi|\mathcal{E}_i\rangle = \rho^r \tag{8}$$

The result ρ^r is precisely the reduced density matrix of equation 4 that von Neumann called for. Now, *a*



Coherent superposition of two Gaussian wavepackets. Such a wavefunction may describe a particle inside a Stern–Gerlach apparatus (figure 2) or may develop in the course of a double-slit experiment. The phase between the two components has been chosen to be zero. **Figure 3**

superposition of the records—of the states of $\mathcal{D}$—is no longer a record of a superposition. Any coherent superposition of the states $|d_\uparrow\rangle$ and $|d_\downarrow\rangle$ is continuously reduced to a mixture. A preferred basis of the detector, sometimes called a "pointer basis," has been singled out. An effective superselection rule has emerged—decoherence prevents superpositions of the preferred basis states from persisting. Moreover, we have obtained all this—or so it appears—without having to appeal to anything beyond the ordinary, unitary Schrödinger evolution.

The preferred basis of the detector—or for that matter, of any open quantum system—is selected by the dynamics. Not all aspects of this process are completely clear, but the detector–environment interaction Hamiltonian certainly plays a decisive role. In particular, when the interaction with the environment dominates, the reduced density matrix ends up being diagonal in the eigenspaces of an observable Λ that commutes with the interaction Hamiltonian,[8] $[\Lambda, H_{\rm int}] = 0$. This commutation relation has a simple physical interpretation: It guarantees that the pointer observable Λ will be a constant of motion of the interaction Hamiltonian. Thus when a system is in the eigenstate of Λ, interaction with the environment will leave it unperturbed.

In the real world, the spreading of quantum correlations is practically inevitable. For example, if in the course of the experiment depicted in figure 2 a photon had scattered from the spin-carrying atom while it was traveling along one of its two alternative routes, this would have resulted in a correlation with the environment and would have necessarily led to decoherence. The density matrix of the $\mathcal{SD}$ pair would have lost its off-diagonal terms. Moreover, given that it is impossible to catch up with the photon, such a loss of coherence would have been irreversible. Irreversibility can also arise from more familiar, statistical causes: Environments are notorious for having large numbers of interacting degrees of freedom, making extraction of lost information as difficult as reversing trajectories in a Boltzmann gas.

Decoherence: How long does it take?

A tractable model of the environment is afforded by a collection of harmonic oscillators[17–19] or, equivalently, by a quantum field.[20] If a particle is present, excitations of the field will scatter off it. The resulting "ripples" will constitute a record of its shape, orientation and so on, and most importantly its instantaneous location and hence its trajectory.

A boat traveling on a quiet lake or a stone falling into water leaves such an imprint on the surface. Our eyesight relies on the perturbation left in the preexisting state of the electromagnetic field. It is hardly surprising that an imprint is left whenever two quantum systems interact, even when "nobody is looking," and even when the lake is stormy and full of waves, and the field is full of excitations. "Messy" initial states of the environment make it difficult to decipher the record, but do not interfere with its existence.

A specific model—a particle with position x and a scalar field $\varphi(q,t)$ propagating in direction q, interacting through $H_{\rm int} = \epsilon x\, {\rm d}\varphi/{\rm d}t$—is one particularly attractive implementation of the above ideas.[17–20] Computations can be carried out furthest in the case where x and q differ (that is, when the field propagates in a direction orthogonal to x). Conclusions are especially easy to formulate in the so-called high-temperature limit, in which only effects of the thermal excitations of the field φ are taken into account.

In this case the density matrix $\rho(x,x')$ of the particle in the position representation evolves according to the *master equation*

$$\frac{{\rm d}\rho}{{\rm d}t} = -\frac{\rm i}{\hbar}[H,\rho] - \gamma(x-x')\left(\frac{\partial\rho}{\partial x} - \frac{\partial\rho}{\partial x'}\right) - \frac{2m\gamma k_{\rm B}T}{\hbar^2}(x-x')^2\rho \qquad (9)$$

where H is the particle's Hamiltonian (although with the potential $V(x)$ adjusted because of $H_{\rm int}$), γ is the relaxation rate, $k_{\rm B}$ is the Boltzmann constant, and T is the temperature of the field. Such master equations can be derived in a number of different ways: Amir Caldeira and Anthony Leggett[19] used a modification of the path integral formalism due to Richard Feynman and Frank Vernon[17] to obtain the high-temperature limit presented above, while William Unruh and I have studied the coupled field–harmonic oscillator equations of motion to arrive at a version that also takes into account zero-point fluctuations.[20]

I will not analyze equation 9 in detail; for our purposes it suffices to note that it naturally separates into three distinct terms, each of them responsible for a different aspect of classical behavior. The first term, the von Neumann equation, can be derived from the Schrödinger equation. Classically, it generates Newton's equations of motion for the averages of observables (Ehrenfest's theorem). The second term causes dissipation: the loss of energy and a decrease of the average momentum. The relaxation rate is $\gamma = \eta/2m$, where the viscosity $\eta = \epsilon^2/2$ is caused by the interaction. The last term is responsible for the fluctuations or random kicks that lead to Brownian motion.

For us, however, the effect of the last term on quantum superpositions will be of much greater interest. I shall show that it *destroys quantum coherence*, eliminating off-diagonal terms responsible for quantum correlations between *spatially separated* pieces of the wavepacket. It is therefore responsible for the classical structure of phase space, as it converts superpositions into mixtures of localized wavepackets that, in the classical limit, turn into the familiar points in phase space. This effect is best illustrated by an example. Consider a coherent superposition of two Gaussians $\chi(x) \sim \chi^+(x) + \chi^-(x)$ with widths δ and separated by a distance Δx, as shown in figure 3. For the case of wide separation ($\Delta x \gg \delta$) the corresponding density matrix $\rho(x,x') = \chi(x)\,\chi^*(x')$ has four peaks: two on and two off the diagonal (see figure 4). Quantum coherence is due to the off-diagonal peaks, for which x and x' are very different. With their disappearance, position emerges as an approximate preferred basis.

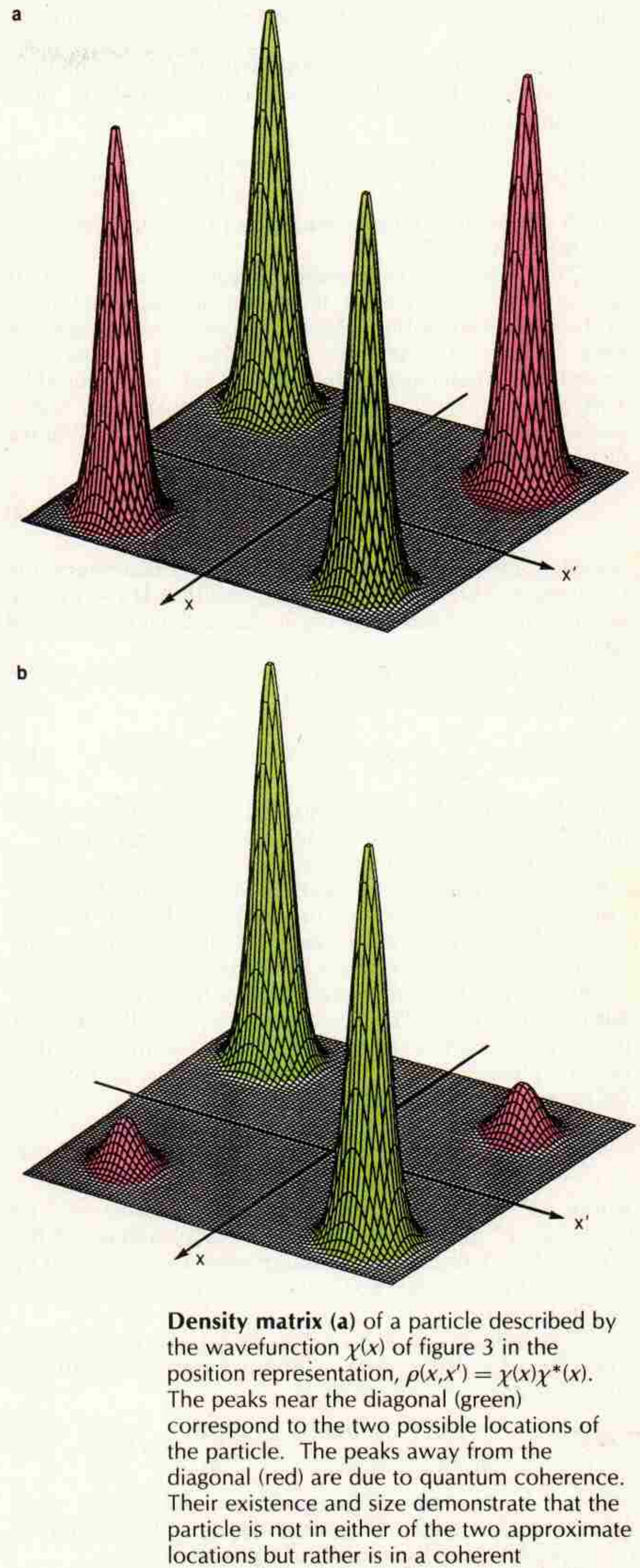
The effect of the last term of equation 9 on the diagonal peaks is small: Near the diagonal $x \simeq x'$, so that the last term, which is proportional to $(x-x')^2$, is negligible. By contrast, for the off-diagonal peaks $(x-x')^2 \simeq (\Delta x)^2$, the square of the separation. Therefore the off-diagonal terms decay at the rate $\tau_D^{-1} \sim 2\gamma m k_B T(\Delta x)^2/\hbar^2$. It follows that the quantum coherence will disappear exponentially on a *decoherence time scale*[20]

$$\tau_D \simeq \tau_R \frac{\hbar^2}{2mk_B T(\Delta x)^2} = \gamma^{-1}\left(\frac{\lambda_T}{\Delta x}\right)^2 \qquad (10)$$

where $\lambda_T = \hbar/\sqrt{2mk_B T}$ is the thermal de Broglie wavelength. For macroscopic objects, the decoherence time scale τ_D predicted by equation 10 is typically orders of magnitude smaller than the relaxation time $\tau_R = \gamma^{-1}$. For instance, for a system at room temperature ($T = 300$ K) with mass $m = 1$ g and separation $\Delta x = 1$ cm, the ratio $\tau_D/\tau_R = 10^{-40}$! Thus, even if the relaxation time was of the order of the age of the universe, $\tau_R \sim 10^{17}$ sec, quantum coherence would be destroyed in $\tau_D \sim 10^{-23}$ sec. Such enormous ratios obtain only for macroscopic objects, and can be inferred only when all of the assumptions made in the derivation of equation 10 are valid. Nevertheless it is now easy to understand why the decoherence between macroscopically distinguishable positions can be nearly instantaneous even for rather well-isolated systems. Of course, equation 10 does not imply that everything will become effectively classical: For a massive Weber bar,[3] tiny Δx ($\sim 10^{-17}$ cm) and cryogenic temperatures ($T \sim 10^{-3}$–1 K) suppress decoherence. For an electron $m = 10^{-27}$ g, and hence τ_D can be much more than τ_R on atomic and larger scales.

Classical limit of quantum dynamics

The Schrödinger equation was deduced from classical mechanics in the Hamilton–Jacobi form. Thus it is no surprise that it yields classical equations of motion when $\hbar$ can be regarded as small. This fact, Ehrenfest's theorem, Bohr's correspondence principle and the kinship of quantum commutators with classical Poisson brackets are all a part of the standard lore found in the textbooks. However, establishing the quantum–classical correspondence involves the states as well as the equations of motion. Quantum mechanics is formulated in Hilbert space, which can accommodate localized wavepackets with sensible classical limits as well as the most bizarre



Density matrix (a) of a particle described by the wavefunction $\chi(x)$ of figure 3 in the position representation, $\rho(x,x') = \chi(x)\chi^*(x)$. The peaks near the diagonal (green) correspond to the two possible locations of the particle. The peaks away from the diagonal (red) are due to quantum coherence. Their existence and size demonstrate that the particle is not in either of the two approximate locations but rather is in a coherent superposition of them. Environment-induced decoherence causes decay of the off-diagonal terms of $\rho(x,x')$. **b:** Partially decohered $\rho(x,x')$. Further decoherence would result in a density matrix with diagonal peaks only. It can then be regarded as a classical probability distribution with an equal probability of finding the particle in either of the locations corresponding to the Gaussian wavepackets. **Figure 4**

superpositions. By contrast, classical dynamics happens in phase space.

To facilitate the study of this aspect of the problem, it is convenient to employ the Wigner transform of a wavefunction $\psi(x)$,

$$W(x,p)=\frac{1}{2\pi\hbar}\int_{-\infty}^{\infty}e^{ipy/\hbar}\,\psi^*\left(x+\frac{y}{2}\right)\psi\left(x-\frac{y}{2}\right)dy \quad (11)$$

which expresses quantum states as functions of position and momentum.

The Wigner distribution $W(x,p)$ is real, but it can be negative. Hence it cannot be regarded as a probability distribution. Nevertheless, when integrated over either of the two variables, it yields the probability distribution for the other (for example, $\int W(x,p)dp=|\psi(x)|^2$). For a minimum-uncertainty wavepacket $\psi(x)\sim\exp\{-(x-x_0)^2/2\delta^2+ip_0x/\hbar\}$, the Wigner distribution is a Gaussian in x and p:

$$W(x,p)=\frac{1}{\pi\hbar}\exp\left\{-\frac{(x-x_0)^2}{\delta^2}-\frac{(p-p_0)^2\delta^2}{\hbar^2}\right\} \quad (12)$$

A system described by this type of Wigner distribution is localized in both x and p. Nothing else that Hilbert space has to offer is closer to being a classical point in phase space.

The Wigner distribution is easily generalized to the case of a general density matrix:

$$W(x,p)=\frac{1}{2\pi\hbar}\int_{-\infty}^{-\infty}e^{ipy/\hbar}\rho\left(x-\frac{y}{2},x+\frac{y}{2}\right)dy \quad (13)$$

We will be using the evolving (initially pure but eventually mixed) density matrix of the particle, discussed above.

The Wigner transform suggests a strategy for exhibiting classical behavior: Whenever $W(x,p)$ is a mixture of localized wavepackets (as in equation 12), it can be regarded as a classical probability distribution in the phase space. However, when the underlying state is truly quantum, its Wigner distribution function will have alternating sign. This property alone will make it impossible to regard the function as a probability distribution in phase space. For the *superposition* of the two Gaussians discussed above,

$$W\sim\frac{W^++W^-}{2}+\frac{1}{\pi\hbar}\exp\left(-\frac{p^2\delta^2}{\hbar^2}-\frac{x^2}{\delta^2}\right)\cos\left(\frac{\Delta x}{\hbar}p\right) \quad (14)$$

where W^+ and W^- are Wigner transforms of the Gaussians χ^+ and χ^-. The resulting $W(x,p)$ is shown in figure 5a. A *mixture* of two Gaussian wavepackets would be described by the same two Gaussians, but without the last, oscillating term.

The equation of motion for $W(x,p)$ can be obtained from equation 9 for $\rho(x,x')$. For a harmonic oscillator ($V\sim x^2$) it does not depend on $\hbar$:

$$\frac{dW}{dt}=\left(-\frac{p}{m}\frac{\partial W}{\partial x}+\frac{\partial V}{\partial x}\frac{\partial W}{\partial p}\right)+2\gamma\frac{\partial(pW)}{\partial p}+D\frac{\partial^2W}{\partial p^2} \quad (15)$$

where V is the renormalized potential and $D=2m\gamma k_BT=\eta k_BT$. The three terms of this equation correspond to the three terms of equation 9.

The first term is easily identified as a classical Poisson bracket $\{H,W\}$. Thus classical dynamics in its Liouville form follows from quantum dynamics, at least for the harmonic oscillator case (for more general $V(x)$ the Poisson bracket would have to be supplemented by quantum corrections of order $\hbar$). The second term is friction. The last term results in the diffusion of $W(x,p)$ in momentum at a rate D.

This last term has precisely the right effect on nonclassical $W(x,p)$ to produce the correct structure of the classical phase space by barring all but the probability distributions of well-localized wavepackets. This can be seen as follows: A superposition of two spatially separated wavepackets results in a sinusoidal modulation of the Wigner distribution in the momentum coordinate (see figure 5a). The oscillating term $\cos(p\Delta x/\hbar)$ is an eigenfunction of the diffusion operator $\partial^2/\partial p^2$ in the last term of equation 15. As a result, the modulation of W in p will be washed out by diffusion at a rate $\tau_D^{-1}=2m\gamma k_BT(\Delta x)^2/\hbar^2$. Negative valleys of $W(x,p)$ will fill in on a time scale of order τ_D, the same rate given by equation 10. In the example described here $W(x,p)$ will retain just two peaks (see figure 5b), which now correspond to the two classical alternatives. Superpositions of momenta will also decohere once the resulting difference in velocities has spread out the wavefunction in x.

The ratio of the decoherence and relaxation time scales depends on $\hbar^2/m$ (see equation 10). Therefore when m is large and $\hbar$ small, decoherence can be nearly instantaneous ($\tau_D\approx0$), while at the same time the motion of compact wavepackets (points in classical phase space) in the smooth potential becomes nearly reversible. I suggest that this idealization is responsible for our confidence in classical mechanics and, more generally, our belief in classical reality. Consequently, the discussion above demonstrates that decoherence and the transition from quantum to classical (usually regarded as esoteric) is an inevitable counterpart of the ubiquitous phenomenon of friction.

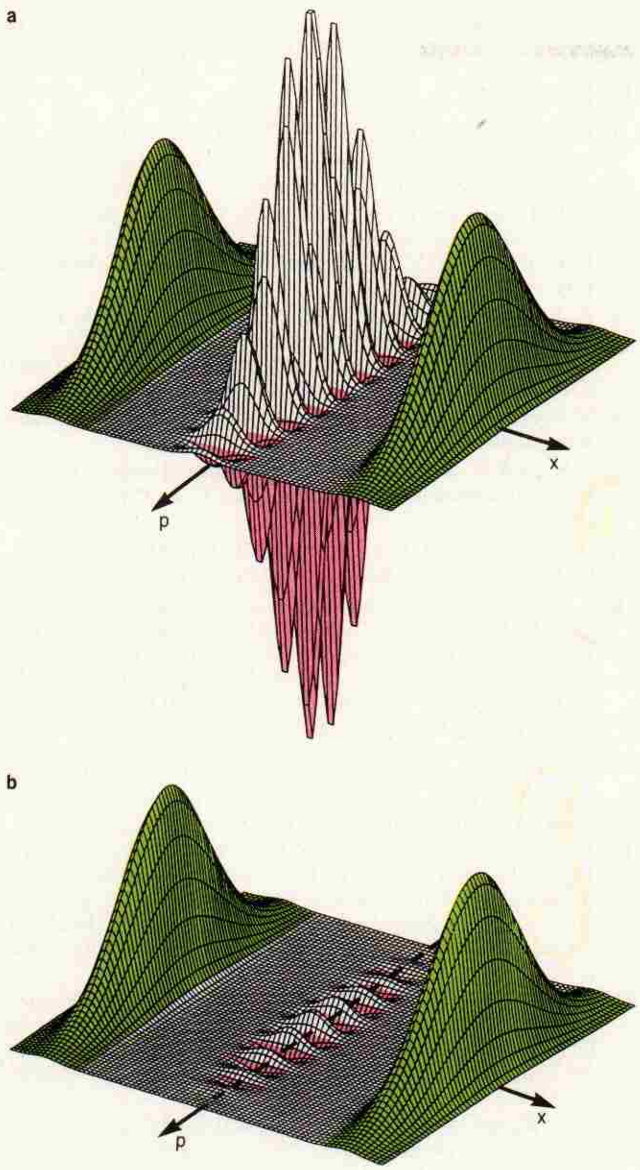
Decoherence, histories, and the universe

The universe is, of course, a closed system. As far as quantum phase information is concerned, it is practically the only system that is effectively closed. Of course, an observer inhabiting a quantum universe can monitor only very few observables, and decoherence can arise when the unobserved degrees of freedom are "traced over."[21] A more fundamental issue, however, is that of the emergence of the effective classicality as a feature of the universe that is more or less independent of special observers, or of coarse grainings such as a fixed separation of the universe into observed and unobserved systems.

The quantum mechanics of the universe must allow for possible large quantum fluctuations of space–time geometry at certain epochs and scales. In particular, it may include important effects of quantum gravity early in the expansion of the universe. Nontrivial issues such as the emergence of the usual notion of time in quantum mechanics must then be addressed. Here we shall neglect such considerations and simply treat the universe as a closed system with a simple initial condition.

Significant progress in the study of decoherence in this context has been reported by Murray Gell-Mann and James B. Hartle,[10] who are pursuing a program suitable for quantum cosmology that may be called the many-histories interpretation. The many-histories interpretation builds on the foundation of Everett's many-worlds interpretation, but with the addition of three crucial ingredients: the notion of sets of alternative coarse-grained *histories* of a quantum system, the decoherence of the histories in a set, and their approximate determinism near the effectively classical limit.

A set of coarse-grained alternatives for a quantum system at a given time can be represented by a set of mutually exclusive projection operators, each corresponding to a different range of values for some properties of the system at that time. (A completely fine-grained set of alternatives would be a complete set of commuting operators.) An exhaustive set of mutually exclusive coarse-grained alternative histories can be obtained, each



Wigner distributions. a: Distribution corresponding to the pure state of figure 3. Note the two separate positive peaks (green) as well as the oscillating interference term in between them. $W(x,p)$ cannot be regarded as a classical distribution in the phase space as long as it has negative (red) contributions. **b:** Decoherence results in diffusion of $W(x,p)$ in the direction of momentum. As a result, the negative and positive ripples of the interference term diffuse into each other and cancel out. This process is almost instantaneous for open, macroscopic systems. In the appropriate limit both the classical structure of the phase space and classical dynamics emerge naturally. **Figure 5**

one represented by a time-ordered sequence of such projection operators.

The definition of consistent histories for a closed quantum system was first proposed by Robert Griffiths.[22] He demonstrated that when the sequences of projection operators satisfy a certain condition (the vanishing of the real part of every interference term between sequences), the histories characterized by these sequences can be assigned classical probabilities—in other words, the probabilities of alternative histories can be added. Griffiths's idea was further extended by Roland Omnés,[23] who developed the "logical interpretation" of quantum mechanics by demonstrating how the rules of ordinary logic can be recovered when making statements about properties that satisfy the Griffiths criterion.

Recently Gell-Mann and Hartle pointed out that in practice somewhat stronger conditions than Griffiths's tend to hold whenever histories decohere. The strongest condition is connected with the idea of records and the crucial fact that noncommuting projection operators in a historical sequence can be registered through commuting operators designating records. They defined a decoherence functional in terms of which the Griffiths criterion and the stronger versions of decoherence are easily stated.

Given the initial state of the universe (perhaps a mixed state) and the time evolution dictated by the quantum field theory of all the elementary particles and their interactions, one can in principle predict probabilities for any set of alternative decohering coarse-grained histories of the universe. Gell-Mann and Hartle raise the question of which sets exhibit the classicality of familiar experience. Decoherence is a precondition for such classicality; the remaining criterion, approximate determinism, is not yet defined with precision and generality.

Within the many-histories program, one is studying[10] the stringent requirements put on the coarseness of histories by their classicality. Besides the familiar and comparatively trivial indeterminacy imposed by the uncertainty principle, there is the further coarse graining required for decoherence of histories. Still further coarseness—for example, that encountered in following hydrodynamic variables averaged over macroscopic scales—can supply the high inertia that resists the noise associated with the mechanics of decoherence and so permits decohering histories to exhibit approximate predictability. Thus the effectively classical domain through which quantum mechanics can be perceived necessarily involves a much greater indeterminacy than is generally attributed to the quantum character of natural phenomena.

Quantum theory of classical reality

We have seen how classical reality emerges from the substrate of quantum physics: Open quantum systems are forced into states described by localized wavepackets. These essentially classical states obey classical equations of motion, although with damping and fluctuations of possibly quantum origin. What else is there to explain?

The origin of the question about the interpretation of quantum physics can be traced to the clash between predictions of the Schrödinger equation and our perceptions. It is therefore useful to conclude this paper by revisiting the source of the problem—our awareness of definite outcomes. If the mental processes that produce this awareness were essentially unphysical, there would be no hope of addressing the ultimate question—why do we perceive just one of the quantum alternatives?—within the context of physics. Indeed, one might be tempted to follow Eugene Wigner in giving consciousness the last word in collapsing the state vector.[24] I shall assume the opposite. That is, I shall examine the idea that the higher mental processes all correspond to well-defined but, at present, poorly understood information processing functions that are carried out by physical systems, our brains.

Described in this manner, awareness becomes susceptible to physical analysis. In particular, the process of decoherence is bound to affect the states of the brain: Relevant observables of individual neurons, including chemical concentrations and electrical potentials, are macroscopic. They obey classical, dissipative equations of motion. Thus any quantum superposition of the states of neurons will be destroyed far too quickly for us to become conscious of quantum goings-on: Decoherence applies to our own "state of mind."

One might still ask why the preferred basis of neurons becomes correlated with the classical observables in the familiar universe. The selection of available interaction Hamiltonians is limited and must constrain the choices of the detectable observables. There is, however, another process that must have played a decisive role: Our senses did not evolve for the purpose of verifying quantum mechanics. Rather, they developed through a process in which survival of the fittest played a central role. And when nothing can be gained from prediction, there is no evolutionary reason for perception. Moreover, only classical states are robust in spite of decoherence and therefore have predictable consequences. Hence one might argue that we had to evolve to perceive classical reality.

There is little doubt that the process of decoherence sketched in this paper is an important fragment central to the understanding of the big picture—the transition from quantum to classical: Decoherence destroys superpositions. The environment induces, in effect, a superselection rule that prevents certain superpositions from being observed. Only states that survive this process can become classical.

There is even less doubt that the rough outline of this big picture will be further extended. Much work needs to be done both on technical issues (such as studying more realistic models that could lead to experiments) and on issues that require new conceptual input (such as defining what constitutes a "system" or answering the question of how an observer fits into the big picture).

Decoherence is of use within the framework of either of the two major interpretations: It can supply a definition of the branches in Everett's many-worlds interpretation, but it can also delineate the border that is so central to Bohr's point of view. And if there is one lesson to be learned from what we already know about such matters, it is undoubtedly the key role played by information and its transfer in the quantum universe.

The natural sciences were built on a tacit assumption: Information about a physical system can be acquired without influencing the system's state. Until recently, information was regarded as unphysical, a mere record of the tangible, material universe, existing beyond and essentially decoupled from the domain governed by the laws of physics. This view is no longer tenable (see, for example, Rolf Landauer's article in PHYSICS TODAY, May 1991, page 23). Quantum theory has helped to put an end to such Laplacean dreams of a mechanical universe. The dividing line between what is and what is known to be has been blurred forever. Conscious observers have lost their monopoly on acquiring and storing information. The environment can also monitor a system, and the only difference from a man-made apparatus is that the records maintained by the environment are nearly impossible to decipher. Nevertheless, such monitoring causes decoherence, which allows the familiar approximation known as classical objective reality—a perception of a selected subset of all conceivable quantum states evolving in a largely predictable manner—to emerge from the quantum substrate.

* * *

I would like to thank John Archibald Wheeler for many inspiring and enjoyable discussions on quantum measurements, Murray Gell-Mann and Jim Hartle for useful and frequent exchanges of ideas about decoherence and for detailed comments on the manuscript, and Mike Warren for help with computer graphics.

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