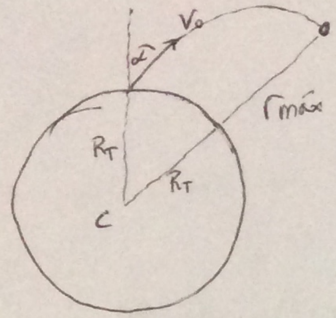
Control #3 I Física 2015-2. M. Casanova.

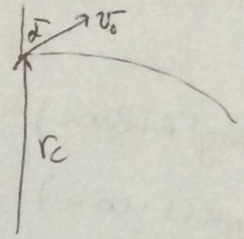
P1



$$V_0 = \sqrt{GM_T/R_T}$$

$$\left[V(r) = -\frac{GM_T m}{r} \right]$$

a) $\vec{L}_c = \vec{r}_c \times \vec{p}$ (1) origen en centro de la tierra.



$$|\vec{L}_c| = |r_c||p| \sin\alpha$$

$$|\vec{L}_c| = R_T m v_0 \sin\alpha.$$

$$\left[|L_c| = R_T m \sqrt{\frac{GM_T}{R_T}} \sin\alpha \right] \ (1)$$

b) $E_{mecanica} = K + U(r)$ (1) ~~cnste~~ en la sup. de la tierra.

$$\left[E_M = \frac{1}{2} m v_0^2 - \frac{GM_T m}{R_T} = \frac{1}{2} m \frac{GM_T}{R_T} - \frac{GM_T m}{R_T} = -\frac{1}{2} \frac{GM_T m}{R_T} \right] \ (1)$$

c) r_{max}

por conservación de la energía.

obs: $\vec{v}_0 = v_r \hat{r} + v_\theta \hat{\theta}$

pero en altura máx $v_r = 0$

y $v_\theta = v_0 \sin\theta$.

$v_\theta^2 = v_0^2 \sin^2\theta$

$$E_{R_T} = E_{r_{max}}$$

$$-\frac{1}{2} \frac{GM_T m}{R_T} = \frac{1}{2} m v_\theta^2 - \frac{GM_T m}{r_{max}} \ (1)$$

$$-\frac{1}{2} \frac{GM_T \cancel{m}}{R_T} = \frac{1}{2} \cancel{m} v_0^2 \sin^2\theta - \frac{GM_T \cancel{m}}{r_{max}}$$

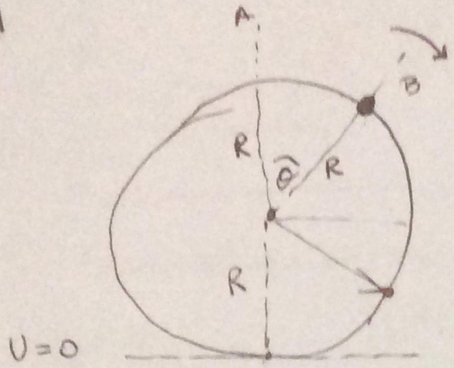
$$-\frac{1}{2} \frac{\cancel{GM_T}}{R_T} = \frac{1}{2} \frac{\cancel{GM_T}}{R_T} \sin^2\theta - \frac{\cancel{GM_T}}{r_{max}}$$

$$\frac{-1}{2R_T} = \frac{1}{2} \frac{\sin^2\theta}{R_T} - \frac{1}{r_{max}}$$

$$\frac{1}{r_{max}} = \frac{1}{2} \frac{\sin^2\theta}{R_T} + \frac{1}{2R_T} = \frac{1}{2R_T}\left(\sin^2\theta + 1\right) =$$

$$\left[r_{max} = \frac{2R_T}{1 + \sin^2\theta} \right] \ (1)$$

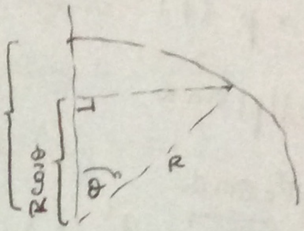
P2)



$V_{inicial} = 0.$

a) ¿$v(\theta)$?

conservación de energía. punto más alto y cuando ha bajado θ.



$$E_{M(A)} = mg\,2R \quad 0,5$$

$$E_{M(B)} = \frac{1}{2}mv^2 + mg(R + R\cos\theta) \quad 0,5$$

$$2\cancel{m}gR = \frac{1}{2}\cancel{m}v^2 + \cancel{m}gR(1+\cos\theta) \quad 0,5$$

$$2gR - gR - gR\cos\theta = \frac{1}{2}v^2$$

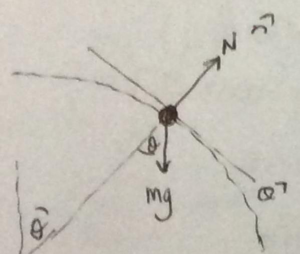
$$\boxed{2gR(1-\cos\theta) = v^2} \quad 0,5$$

b) $$\vec{a}_c = -\frac{v^2}{R}(\hat{r}) = -\frac{2g\cancel{R}(1-\cos\theta)}{\cancel{R}} = -2g(1-\cos\theta)$$

(1)

$$\boxed{\vec{a}_c = 2g(\cos\theta - 1)(\hat{r})} \quad (1)$$

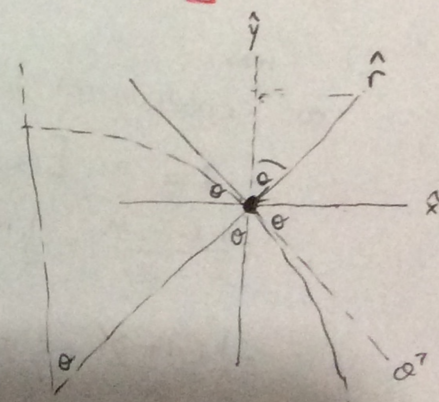
c)



$$\hat{r})\ N - mg\cos\theta = -m\frac{v^2}{R}$$

$$\hat{\theta})\ \cancel{m}g\sin\theta = \cancel{m}a_T \quad (0.5)$$

$$\boxed{\hat{\theta}\, g\sin\theta = \vec{a}_T} \quad (0.5)$$



la aceleración es $\vec{a} = a_r\hat{r} + a_T\hat{\theta}$

$$\vec{a} = 2g(\cos\theta - 1)(\sin\theta, \cos\theta) + g\sin\theta(\cos\theta, -\sin\theta)$$

$\hat{r} = (\sin\theta, \cos\theta)$

$\hat{\theta} = (\cos\theta, -\sin\theta)$

tomando solo $\hat{y}$

$$\vec{a}_y = 2\cancel{g}(\cos\theta - 1)\cos\theta + \cancel{g}\sin\theta(-\sin\theta) = 0. \quad (0.5)$$

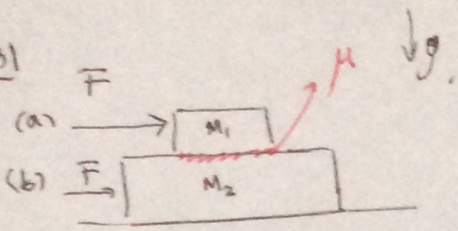
$$2\cos^2\theta - 2\cos\theta - \sin^2\theta = 0$$

$$2\cos^2\theta - 2\cos\theta - (1 - \cos^2\theta) = 0.$$

$$\underbrace{3}_{a}\cos^2\theta \underbrace{-2}_{b}\cos\theta \underbrace{-1}_{c} = 0.$$

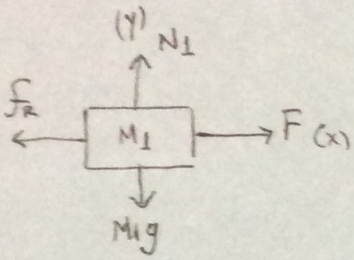
$$\cos\theta = \frac{2 \pm \sqrt{4 - 4\cdot 3\cdot -1}}{2\cdot 3} = \frac{2 \pm \sqrt{4+12}}{6} = \frac{2 \pm 4}{6} = \begin{cases} 1 \rightarrow \theta = \pi/2 \\ -1/3 \rightarrow \boxed{\theta \approx 109^\circ} \end{cases} \quad (0.5)$$

P3)



a) Fuerza aplicada en M_1.

del M_1



x) $F - f_R = M_1 a_x$

y) $N_1 - M_1 g = 0 \Rightarrow N_1 = M_1 g$

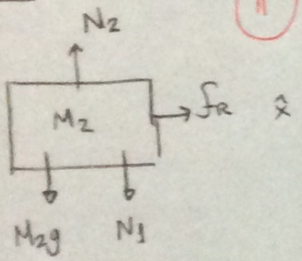
$f_R = \mu \cdot N_1 = \mu M_1 g$

x) $F - \mu M_1 g = M_1 a_x$

$\left[F = M_1 (\mu g + a_x) \right]$

(1)

del M_2



(1)

y) $N_2 = M_2 g + M_1 g = (M_1 + M_2) g$

x) $f_R = M_2 a_x$

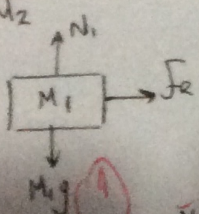
$\mu \cdot M_1 g = M_2 a_x$

$\left[\mu g \frac{M_1}{M_2} = a_x \right]$ reemplazamos en F.

(1)

$$\boxed{F = M_1 \left(\mu g + \mu g \frac{M_1}{M_2} \right) = M_1 \mu g \left(1 + \frac{M_1}{M_2} \right) = \mu M_1 g \left(\frac{M_1 + M_2}{M_2} \right) = \mu \frac{M_1}{M_2} g (M_1 + M_2)}$$ //

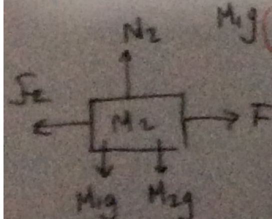
b) F en M_2



(1)

x) $f_R = M_1 a_x \Rightarrow \mu \cancel{M_1} g = \cancel{M_1} a_x$ $\boxed{\mu g = a_x}$

y) $N_1 = M_1 g$



y) $N_2 = (M_1 + M_2) g$

x) $F - f_R = M_2 a_x$

(1)

$$\boxed{F = \mu M_1 g + M_2 \mu g = \mu g (M_1 + M_2)}$$ (A)