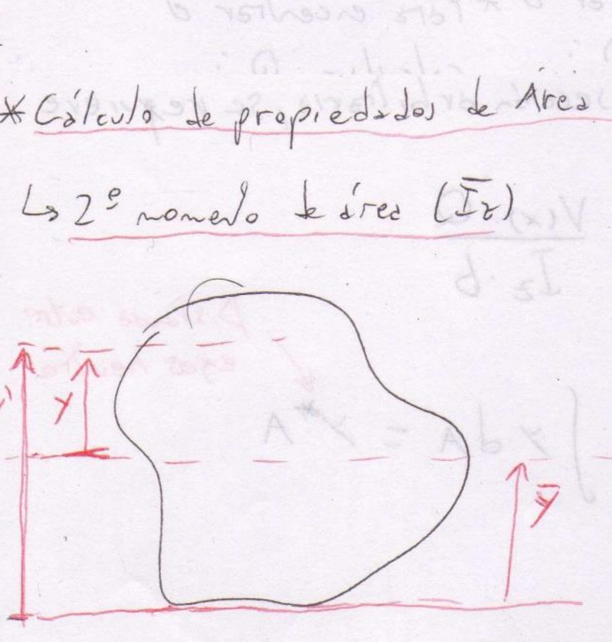
* Cálculo de propiedades de Área (I_z, Q)

↳ 2º momento de área ($\bar{I}_z$)



$$\bar{y}\ (\text{Eje neutro}) = \frac{\int y' \, dA}{A}$$

$$I_z = \int y^2 \, dA = \int (y' - \bar{y}) \, dA$$

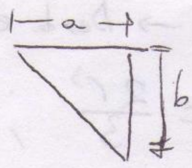
• Si hay varias fig. geométricas

$$\bar{Y}_T = \frac{\sum_i \bar{Y}_i \cdot A_i}{\sum A_i}$$

• Traslado de eje de rotación (Teorema de los ejes paralelos)

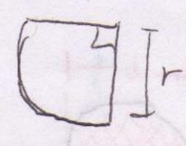
$$\bar{I}_z = I_z + \delta^2 A$$

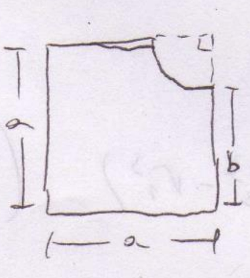
* Encontrar eje neutro y ~~...~~ I_z de:



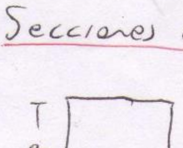
,

,

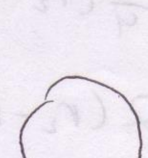




* Secciones conocidas:



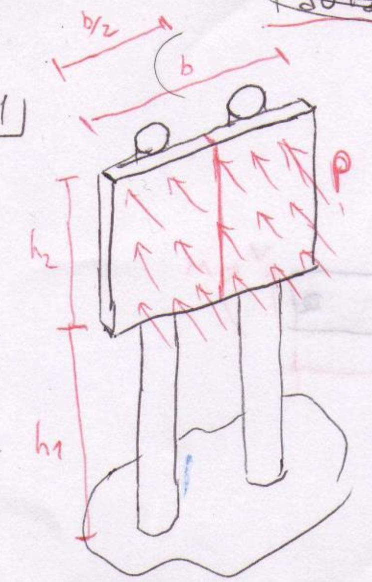
$I_z = \frac{ba^3}{12}$,

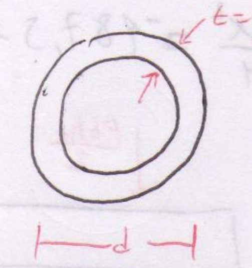


$I_z = \frac{\pi d^4}{64}$

~~Pauta Auxiliar N°4 ME3202~~ ~~2019~~

P1





$p = 75$ lb/ft

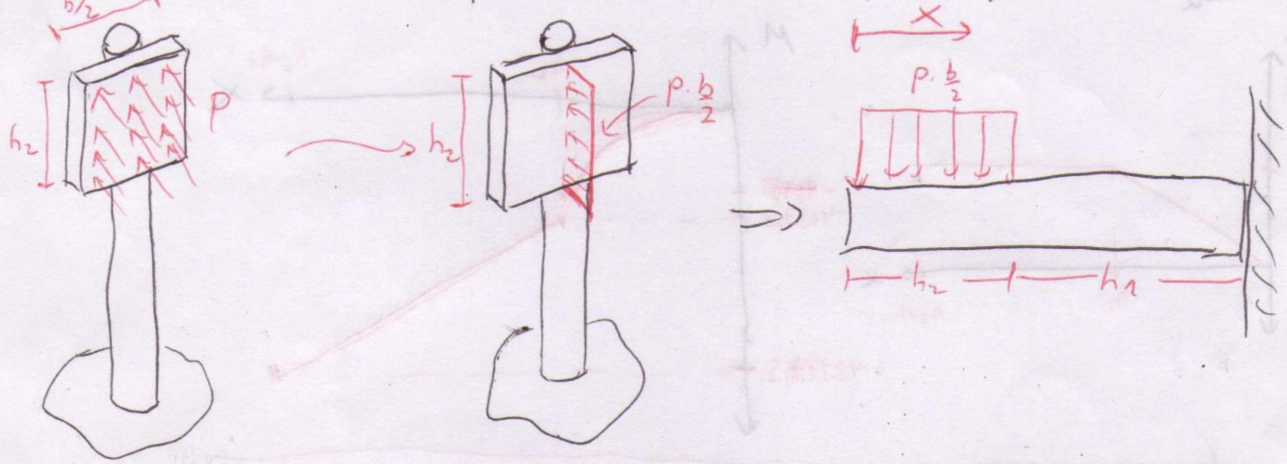
$t = \frac{d}{10}$

$h_1 = 20$ ft

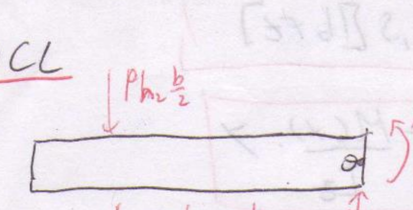
$h_2 = 5$ ft

$b = 10$ ft

• Veamos como afecta la presión del viento a cada poste



• DCL



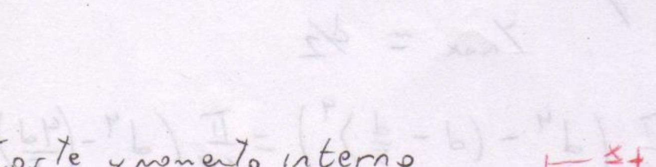
$\sum F_y = 0 \Rightarrow \boxed{R = \frac{P h_2 b}{2} = 1875\ lb}$

$\sum_{a} M_z = 0 \Rightarrow -M_R + P h_2 \frac{b}{2}\left(h_1 + \frac{h_2}{2}\right) = 0$

$\boxed{\therefore M_R = \cancel{3837{,}5}\ [lb \cdot ft]}$ $\boxed{42187{,}5\ [lb \cdot ft]}$

• Corte y momento interno

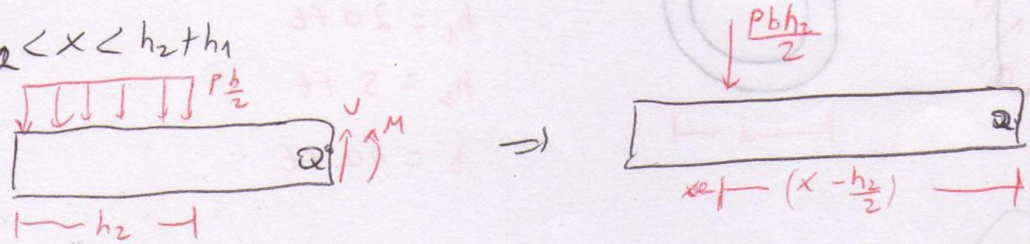
* $0 < x < h_1$



$\cdot \sum F_y = 0 \Rightarrow V(x) = \frac{Pb}{2}x = 375x$

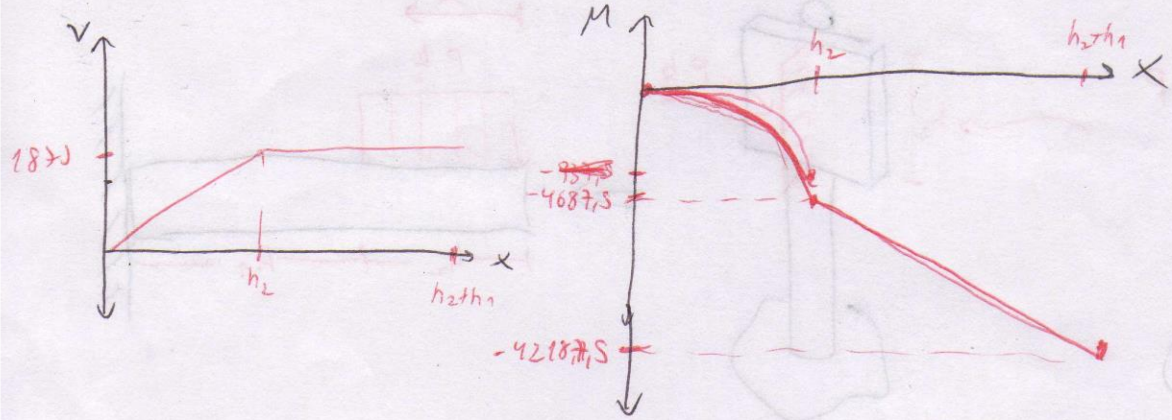
$\cdot \sum_a M_z = 0 \Rightarrow M(x) = -\frac{Pbx^2}{4} = -187{,}5x^2$

$\cdot\ h_2 < x < h_2 + h_1$



$\cdot \sum F_y = 0 \Rightarrow V(x) = \frac{Pbh_2}{2} = 1875$

$\cdot \sum_a M_z = 0 \Rightarrow M(x) = -\frac{Pbh_2}{2}\left(x - \frac{h_2}{2}\right) = -1875x + 4687{,}5$



$\boxed{\therefore V_{max} = 1875 \text{ lb} \ \text{ y } \ |M_{max}| = 42187{,}5 \text{ [lb ft]}}$

a) Esfuerzo normal debido a Flexión: $\boxed{\sigma_x = -\frac{M(x)}{I_z} \cdot y}$

$\rightarrow \sigma_{max} = -\frac{M_{max} \cdot y_{max}}{I_z}$ / $M_{max} = -42187{,}5 \text{ [lb·ft]}$, $y_{max} = d/2$

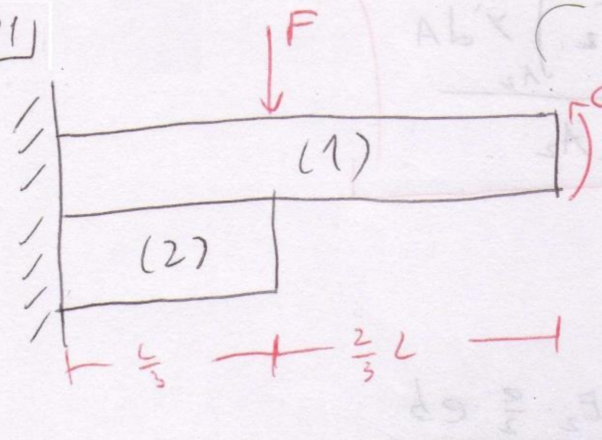
$* \ I_z = \frac{\pi}{64}\left(d^4 - (d-2t)^4\right) = \frac{\pi}{64}\left(d^4 - \left(d - \frac{d}{5}\right)^4\right) = \frac{\pi}{64}\left(d^4 - \left(\frac{4d}{5}\right)^4\right)$

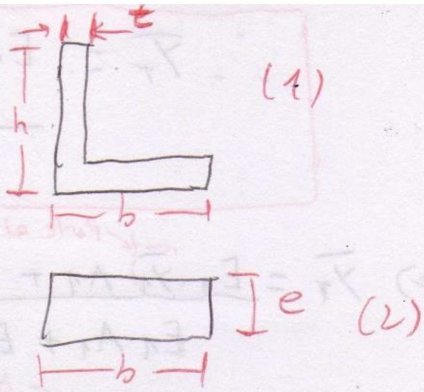
$= \frac{\pi}{64} \cdot \frac{369}{625} d^4 \Rightarrow \boxed{I_z = 0{,}02898\, d^4}$

$$\therefore \sigma_{max} = \frac{42187{,}5\,[lb\cdot ft]\cdot \frac{d}{2}}{0{,}02898\, d^4} = 7500\ \left[\frac{lb}{ft^2}\right]$$

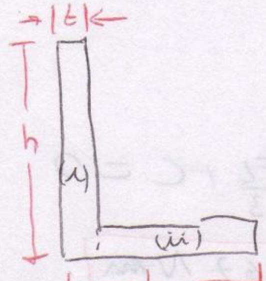
$$\Rightarrow \frac{727872{,}671}{d^3} = 7500 \quad \boxed{\therefore d = 4{,}595\ [ft]}$$

P1)





a) Viga (1)



$$\bar{y}_1 = \frac{\sum_i y_i \cdot A_i}{\sum_i A_i} = \frac{\bar{y}_{(i)} A_{(i)} + \bar{y}_{(ii)} \cdot A_{(ii)}}{A_{(i)} + A_{(ii)}}$$

$$= \frac{\frac{h}{2}\cdot h\cdot t + \frac{t}{2}\cdot t(b-t)}{ht + t(b-t)}$$

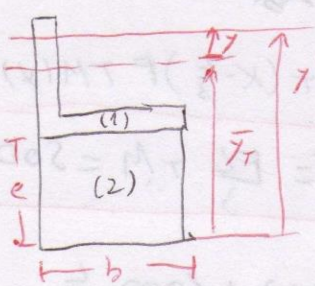
$$\Rightarrow \boxed{\bar{y}_1 = 3,038\ cm}$$

$$\to \bar{I}_{(i)} = I_{(i)} + \delta^2 A_{(i)} = \frac{th^3}{12} + \left(\bar{y} - \frac{h}{2}\right)\cdot t\cdot h = 249,249\ cm^4$$

$$I_{(ii)} = I_{(ii)} + \delta^2 A_{(ii)} = \frac{(b-t)\cdot t^3}{12} + \left(\bar{y} - \frac{t}{2}\right)^2 (b-t)\cdot t = 91,38\ cm^4$$

$$\boxed{\therefore I_1 = 3.406\cdot 10^{-6}\ m^4}$$

b)



- Para calcular el eje neutro de una viga compuesta se usa:

$$E_1 \int_{A_1} y\, dA + E_2 \int_{A_2} y\, dA = 0$$

- Haciendo $y = y' - \bar{y}_T$

$$\Rightarrow E_1 \int_{A_1} (y' - \bar{y}_T)\, dA + E_2 \int_{A_2} (y' - \bar{y}_T)\, dA = 0$$

$$\Rightarrow \left[E_1 \int_{A_1} y'\, dA + E_2 \int_{A_1} y'\, dA\right] + \left[E_2 \bar{y}_T A_1 + E_2 \bar{y}_T A_c\right] = 0$$

$$\boxed{\therefore \bar{Y}_T = \frac{E_1\int_{A_1} y'\,dA + E_2\int_{A_2} y'\,dA}{E_1 \cdot A_1 + E_2 A_2}}$$

Parte el $\qquad \frac{e}{2}$

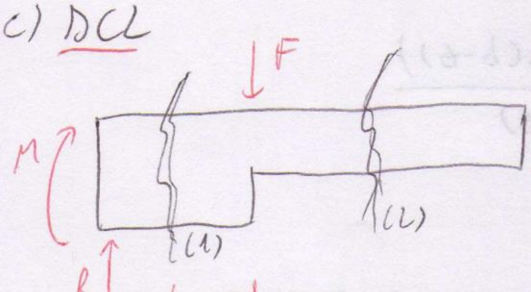
$$\Rightarrow \bar{Y}_T = \frac{E_1 \bar{y}_1 A_1 + E_2 \bar{y}_2 A_2}{E_1 A_1 + E_2 A_2}$$

Desde la base!

$$= \frac{E_1(\bar{Y}_1 + e)(ht + t(b-t)) + E_2 \cdot \frac{e}{2} \cdot eb}{E_1(ht + t(b-t)) + E_2 eb}$$

$$\boxed{\therefore \bar{Y}_T = 4{,}6967 \text{ cm}}$$

c) DCL



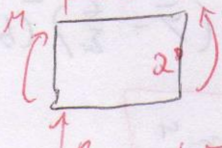
- $\sum M_z = 0 \rightsquigarrow M - \frac{FL}{3} + C = 0$

$\boxed{\therefore M = 166{,}667 \text{ N·mm}}$

- $\sum F_y = 0 \rightsquigarrow \boxed{R = F = 1000 \text{ N}}$

• Hacemos cortes para determinar la distribución de momento interno

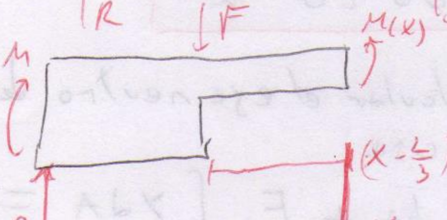
(1) $0 < x < \frac{L}{3} \Rightarrow$



- $\sum M_z = 0 \rightsquigarrow -M - Rx + M(x) = 0$

$\boxed{\therefore M(x) = 166{,}667 + 1000x}$

(2) $\frac{L}{3} < x < L \Rightarrow$



- $\sum M_z = 0 \rightsquigarrow$

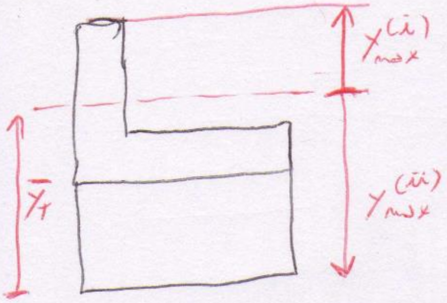
$\Rightarrow M - Rx + (x - \frac{L}{3})F + M(x) = 0$

$\boxed{\therefore M(x) = \frac{FL}{3} + M = 500 \text{ N·m}}$

• El momento máximo se da para cada intervalo (1): $M_{max} = M(\frac{L}{2}) = 166{,}667 + 1000 \cdot \frac{L}{3}$

$\boxed{M_{max} = 500 \text{ N·m}}$

• Para determinar Y_{max}, se hace para cada material



$$\Rightarrow Y_{max}^{(i)} = h + e - \bar{Y}_T = 0{,}143 \text{ cm (superior)}$$

$$Y_{max}^{(ii)} = -\bar{Y}_T = 4{,}6967 \text{ cm (inferior)}$$

• Usando las fórmulas para Esf. máximo en vigas compuestas:

$$\boxed{\sigma_{(i),max} = \frac{-E_1}{E_1 I_1 + E_2 I_2} \cdot M_{max} \cdot Y_{max}^{(i)} = -4{,}152 [MPa] \text{ (Compresión)}}$$

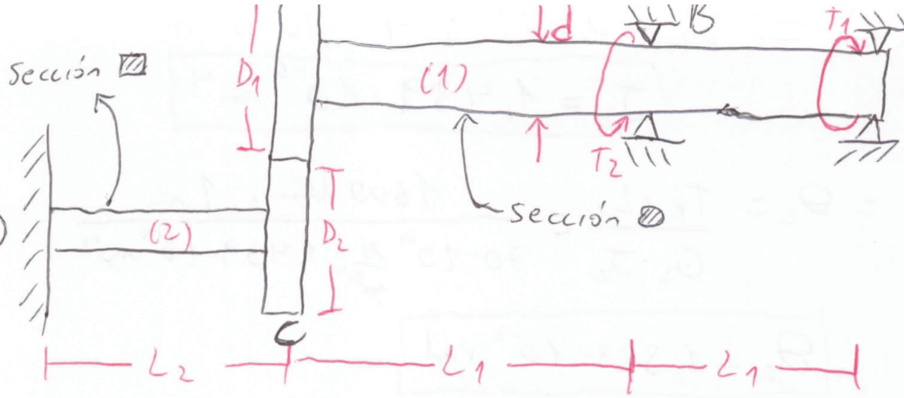
$$\boxed{\sigma_{(ii),max} = \frac{-E_2}{E_1 I_1 + E_2 I_2} M_{max} \cdot Y_{max}^{(ii)} = 1{,}507 [MPa] \text{ (Tracción)}}$$

• (2): $M_{max} = 500$ Nm (cte)

$$Y_{max} = 12 - \bar{Y}_1 = 8{,}962 [cm] \text{ (Parte superior)}$$

$$\Rightarrow \boxed{\sigma_{max} = \frac{-M_{max} \cdot Y_{max}}{I_1} = 13{,}454 [MPa]}$$

P3



- Eje 1: $d = 8\,cm$; $G_1 = 69\,GPa$; $T_1 = 200\,N\cdot m$; $T_2 = 80\,Nm$; $L_1 = 1{,}5\,m$
  $D_1 = 15\,cm$
- Eje 2: $a_1 = 4\,cm$; $a_1/a_2 = 0{,}5$; $G_2 = 70\,GPa$; $D_2 = 20\,cm$; $L_2 = 1\,m$
- Para Sección Cuadrada: $J = K_2\, a b^3$; $\tau = \dfrac{T}{k_1 a b^2}$ $(b \leq a)$

a/b	1	1,5	2	4	10	∞
K_1	0,208	0,231	0,246	0,282	0,312	1/3
K_2	0,141	0,196	0,281	0,281	0,312	1/3

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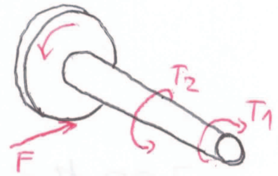
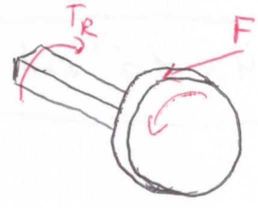
a) DCLs

(1)



$$-T_1 + T_2 + F\cdot\frac{D_1}{2} = 0 \Rightarrow \boxed{F = \frac{2}{D_1}(T_1 - T_2) = 16000\,N}$$

(2)



$$\boxed{T_R = F\cdot\frac{D_2}{2} = 1600\,Nm}$$

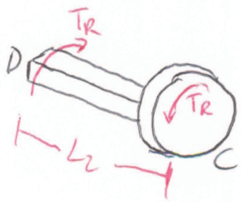
- Para encontrar el ángulo total en A, calculamos los ángulos de torsión desde D:

$$\frac{a_2}{a_1} = 2 \Rightarrow \boxed{K_1 = 0{,}246} \quad \boxed{K_2 = 0{,}281}$$

$\boxed{a_2 = 8\,cm}$

$\Rightarrow J_2 = K_2\, a\, b^3 \quad (b \le a) \Rightarrow J_2 = 0,281 \cdot 8 \cdot 4^3 \cdot (10^{-2})^4 \ m^4$

$$\boxed{J_2 = 1,439 \cdot 10^{-6} \ m^4}$$

* 

$$\theta_{DC} = \theta_2 = \frac{T_R \cdot L_2}{G_2 \cdot J_2} = \frac{1600 \ N\cancel{m} \cdot 1 \ \cancel{m}}{70 \cdot 10^9 \ \frac{N}{\cancel{m^2}} \cdot 1,439 \cdot 10^{-6} \ \cancel{m^4}}$$

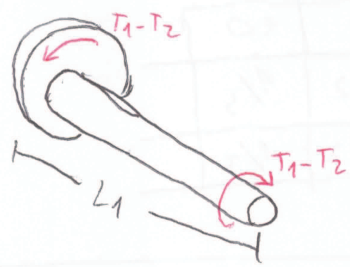
$$\boxed{\theta_2 = 1,588 \cdot 10^{-2} \ rad}$$

*Ángulo en el engranaje

Arcos iguales $\Big\}$ $\theta_1 \cdot \frac{D_1}{2} = \theta_2 \cdot \frac{D_2}{2} \Rightarrow \theta_1 = \frac{D_2}{D_1} \theta_2$

$$\boxed{\theta_1 = 2,117 \cdot 10^{-2} \ rad}$$

*Tramo CB

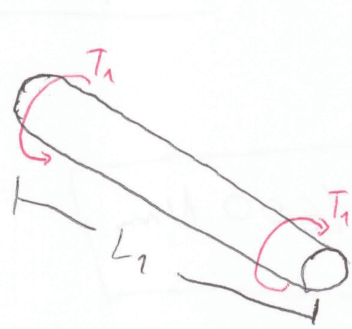


$$\theta_{CB} = \frac{(T_1 - T_2) L_1}{G_1 J_1} \quad \wedge \quad \boxed{J_1 = \frac{\pi d^4}{32} = 4,021 \cdot 10^{-6} \ m^4}$$

$$= \frac{120 \ N\cancel{m} \cdot 1,5 \ \cancel{m}}{60 \cdot 10^9 \ \frac{N}{\cancel{m^2}} \cdot 4,021 \cdot 10^{-6} \ \cancel{m^4}}$$

$$\boxed{\theta_{CB} = 7,461 \cdot 10^{-4} \ rad}$$

*Tramo BA



$$\theta_{BA} = \frac{T_1 \cdot L_1}{G_1 \cdot J_1} = \frac{200 \ N\cancel{m} \cdot 1,5 \ \cancel{m}}{60 \cdot 10^9 \ \frac{N}{\cancel{m^2}} \cdot 4,021 \cdot 10^{-6} \ \cancel{m^4}}$$

$$\boxed{\theta_{BA} = 1,243 \cdot 10^{-3} \ rad}$$

$\therefore$ El ángulo total de torsión en A es:

$$\theta_A = \theta_1 + \theta_{CB} + \theta_{BA}$$

$$\boxed{\theta_A = 3,77961 \cdot 10^{-2} \ rad}$$

b) Esfuerzos máximos:

- Eje 1: Tramo CB ⇝ $\boxed{\tau_{max} = \frac{(T_1 - T_2)\, d}{2 J_2} = 1{,}193\ [MPa]}$

  Tramo BA ⇝ $\boxed{\tau_{max} = \frac{T_1 \cdot d}{2 J_2} = 1{,}789\ [MPa]}$

- Eje 2: $\boxed{\tau_{max} = \frac{T_R}{k_1\, a b^2} = 50{,}87\ [MPa]}$