

- Cálculo de deflexión por integración:

$$\frac{d^4 y}{dx^4} = -\frac{w(x)}{EI} \quad \leftarrow \text{Fza distribuida}$$

* Modelación de Fzas mediante función escalón ($v(x-x_0)$) para Fzas distribuidas, y delta de Dirac ($\delta(x-x_0)$) para Fzas puntuales

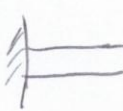
* Lo que ocurre en los extremos de la viga se reemplaza por condiciones de borde (Son necesarias 4 cond. de borde)

* Las cond. de borde son condiciones en la deflexión en los extremos, el ángulo de deflexión ($\frac{dy}{dx}$), el momento ($\frac{d^2 y}{dx^2}$) o de Fza de corte ($\frac{d^3 y}{dx^3}$)

$$\hookrightarrow \boxed{M = EI \frac{d^2 y}{dx^2}} \quad \wedge \quad \boxed{V = -EI \frac{d^3 y}{dx^3}}$$

* Algunos bordes:

 $\Rightarrow$ No hay deflexión $\rightarrow y(0) = 0$
No hay momento $\rightarrow \frac{d^2 y}{dx^2} = 0$

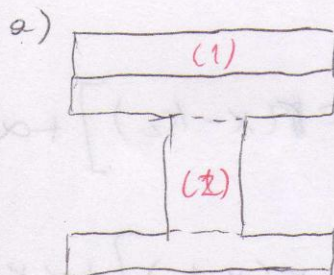
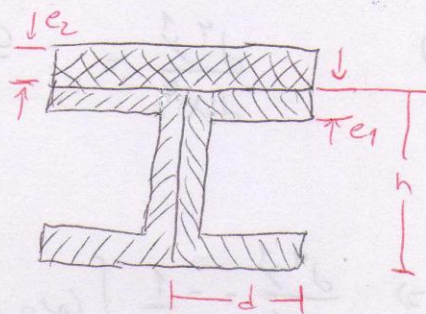
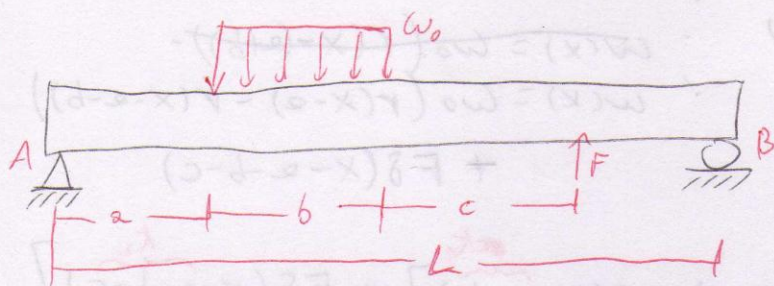
 $\Rightarrow$ No hay deflexión $\rightarrow y(0) = 0$
No hay ángulo $\rightarrow \frac{dy}{dx}(0) = 0$

 $\Rightarrow$ No hay momento $\rightarrow \frac{d^2 y}{dx^2}(0) = 0$

Balance de fzas $\rightarrow V(0) = -F = -k y(0)$

$\therefore -EI \frac{d^3 y}{dx^3}(0) = -k y(0)$

P1



$$\bar{y}_T = \frac{\sum \bar{y}_i \cdot A_i}{\sum A_i}$$

$$\bar{y}_1 = h + \frac{e_2}{2}; A_1 = 2de_2$$

$$\bar{y}_2 = \frac{h}{2}; A_2 = 4de_1 + 2e_1(h - 2e_1)$$

$$\Rightarrow \bar{y}_T = \frac{(h + \frac{e_2}{2}) \cdot 2de_2 + \frac{h}{2} (4de_1 + 2e_1(h - 2e_1))}{2de_2 + 4de_1 + 2e_1(h - 2e_1)}$$

$$\bar{y}_T = 0,177 \text{ m}$$

$$\Rightarrow \bar{I}_2 = \bar{I}_1 + \bar{I}_2$$

$$\hookrightarrow \bar{I}_1 = I_1 + (\bar{y}_1 - \bar{y}_T)^2 \cdot A_1 = \frac{2de_2^3}{12} + (\bar{y}_1 - \bar{y}_T)^2 \cdot 2de_2$$

$$\bar{I}_1 = 5,538 \cdot 10^{-5} \text{ [m}^4\text{]}$$

$$\hookrightarrow \bar{I}_2 = \left(\frac{2de_1^3}{12} + \left(\frac{e_1}{2} - \frac{h}{2} \right)^2 \cdot 2de_1 \right) + \left(\frac{2de_1^3}{12} + \left(h + \frac{e_1}{2} - \frac{h}{2} \right)^2 \cdot 2de_1 \right) + (\bar{y}_T - \bar{y}_2)^2 \cdot A_2$$

$$\bar{I}_2 = \left(\frac{2de_1^3}{12} + \left(\frac{e_1}{2} - \frac{h}{2} \right)^2 \cdot 2de_1 \right) + \left(\frac{2de_1^3}{12} + \left(h + \frac{e_1}{2} - \frac{h}{2} \right)^2 \cdot 2de_1 \right) + (\bar{y}_T - \bar{y}_2)^2 \cdot A_2$$

$$\Rightarrow \bar{I}_2 = 2 \left(\frac{dh^3}{12} - \frac{(d - e_1)(h - 2e_1)^3}{12} + (\bar{y}_T - \bar{y}_2)^2 \frac{A_2}{2} \right) = 1,701 \cdot 10^{-4} \text{ m}^4$$

$$\therefore \bar{I}_z = 7,259 = 2,255 \cdot 10^{-4} \text{ [m}^4\text{]}$$

$$b) \quad \frac{d^4 y}{dx^4} = - \frac{w(x)}{EI_z} \quad \begin{aligned} w(x) &= w_0 (r(x-a+b)) - \\ w(x) &= w_0 (r(x-a) - r(x-a-b)) \\ &\quad + F \delta(x-a-b-c) \end{aligned}$$

$$\Rightarrow \frac{d^4 y}{dx^4} = - \frac{1}{EI_z} \left[w_0 [r(x-a) - r(x-a-b)] + F \delta(x-a-b-c) \right]$$

$$\frac{d^3 y}{dx^3} = - \frac{1}{EI_z} \left[w_0 ((x-a)r(x-a) - (x-k_1)r(x-k_1)) - F r(x-k_2) \right] + \alpha$$

$$\frac{d^2 y}{dx^2} = - \frac{1}{EI_z} \left[w_0 \left(\frac{(x-a)^2}{2} r(x-a) - \frac{(x-k_1)^2}{2} r(x-k_1) \right) - F (x-k_2) \cdot r(x-k_2) \right] + \alpha x + \beta$$

$$\beta \frac{dy}{dx} = - \frac{1}{EI_z} \left[w_0 \left(\frac{(x-a)^3}{6} r(x-a) - \frac{(x-k_1)^3}{6} r(x-k_1) \right) - F \frac{(x-k_2)^2}{2} \cdot r(x-k_2) \right] + \frac{\alpha x^2}{2} + \beta x + \gamma$$

$$y(x) = - \frac{1}{EI_z} \left[w_0 \left(\frac{(x-a)^4}{24} r(x-a) - \frac{(x-k_1)^4}{24} r(x-k_1) \right) - F \frac{(x-k_2)^3}{6} \cdot r(x-k_2) \right] + \frac{\alpha x^3}{6} + \frac{\beta x^2}{2} + \gamma x + \delta$$

• Condiciones de Borde:

$$1-) \frac{d^2 y}{dx^2}(0) = 0 \Rightarrow \beta = 0; \quad 2-) y(0) = 0 \Rightarrow \delta = 0$$

$$3-) \frac{d^2 y}{dx^2}(L) = 0 \Rightarrow - \frac{1}{EI_z} \left[w_0 \left(\frac{(L-a)^2}{2} - \frac{(L-k_1)^2}{2} \right) - \frac{F(L-k_2)^2}{6} \right] + \frac{\alpha L^2}{2} + \gamma L = 0 \quad (1)$$

$$4-) y(L) = 0 \Rightarrow - \frac{1}{EI_z} \left[w_0 \left(\frac{(L-a)^4}{24} - \frac{(L-k_1)^4}{24} \right) - \frac{F(L-k_2)^3}{6} \right] + \frac{\alpha L^3}{6} + \gamma L = 0 \quad (2)$$

$$* D_e(1) \rightarrow \alpha = \frac{1}{LEI_z} \left[\omega_0 \left(\frac{(L-a)^2}{2} - \frac{(L-k_1)^2}{2} \right) - F \frac{(L-k_2)^3}{6} \right]$$

$$\alpha = \cancel{3,167 \cdot 10^{-4}} = -7,849 \cdot 10^{-5}$$

* $D_e(2)$

$$\gamma = \frac{1}{LEI_z} \left[\omega_0 \left(\frac{(L-a)^4}{24} - \frac{(L-k_1)^4}{24} \right) - \frac{F(L-k_2)^3}{6} \right] - \frac{\alpha L^2}{6}$$

$$\gamma = 1,402 \cdot 10^{-3}$$

* Una vez calculadas las ctes., calculamos la deflexión en $x = L/2$

$$\gamma(L/2) = -\frac{1}{EI_z} \left[\omega_0 \frac{(L/2-a)^4}{24} \right] + \frac{\alpha (L/2)^3}{6} + \gamma \cdot \frac{L}{2}$$

$$\gamma(L/2) = 5,563 \cdot 10^{-3} \text{ [m]}$$

* El est. normal ~~máximo~~ se calcula como:

$$\sigma_{\max} = -M(x) \cdot \gamma$$

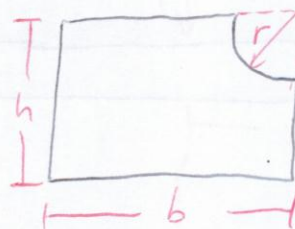
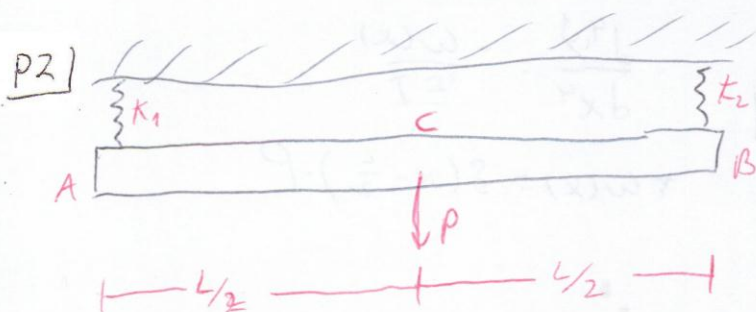
$$\sigma(x) = -\frac{M(x) \cdot \gamma}{I_z} \quad ; \quad M(x) = EI \frac{d^2 \gamma}{dx^2}$$

$$\therefore \sigma(x) = -\frac{d^2 \gamma}{dx^2} E \gamma$$

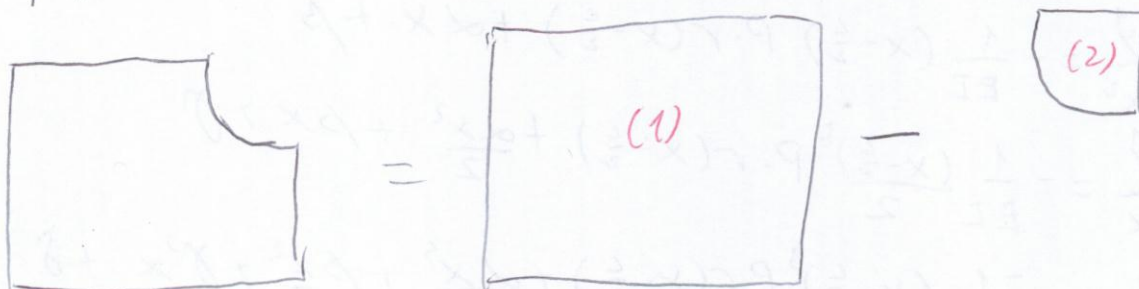
$$\hookrightarrow \frac{d^2 \gamma}{dx^2} \left(\frac{L}{2} \right) = -\frac{\omega_0}{2EI_z} \left(\frac{L}{2} - a \right)^2 + \alpha \cdot \frac{L}{2} = -4,279 \cdot 10^{-4}$$

$$\hookrightarrow \gamma_{\max} = -\bar{\gamma} = -0,177$$

$$\therefore \sigma(L/2) = \cancel{-25147466 \text{ [Pa]}} = 1,515 \cdot 10^7 \text{ [Pa]}$$



e) Propiedades de área:



$$(1): \bar{Y}_1 = \frac{h}{2} \quad \text{e} \quad I_1 = \frac{b \cdot h^3}{12}$$

$$(2): \left. \begin{array}{l} \bar{Y}_2 = \frac{4r}{3\pi} \quad \text{e} \quad I_2 = \frac{\pi r^4}{16} \end{array} \right\} \begin{array}{l} \text{Desde la base} \\ \bar{Y}_2 = h - \frac{4r}{3\pi} \end{array}$$

$$\Rightarrow \bar{Y}_T = \frac{\bar{Y}_1 \cdot A_1 - \bar{Y}_2 \cdot A_2}{A_1 - A_2} = \frac{\frac{h}{2} \cdot hb - \left(h - \frac{4r}{3\pi}\right) \cdot \frac{1}{4} \pi r^2}{hb - \frac{1}{4} \pi r^2} = 4,567 \text{ cm}$$

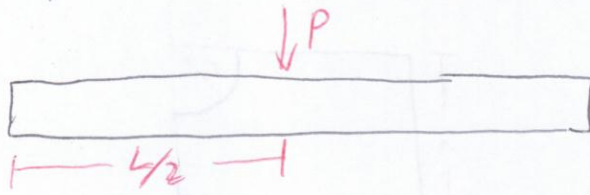
$$\boxed{\bar{Y}_T = 4,567 \text{ cm}}$$

$$\Rightarrow \bar{I}_1 = I_1 + S^2 A = \frac{bh^3}{12} + \left(\frac{h}{2} - \bar{Y}_T\right)^2 bh \Rightarrow \boxed{\bar{I}_1 = 1,278 \cdot 10^{-5} \text{ m}^4}$$

$$\bar{I}_2 = I_2 + S^2 A = \frac{\pi r^4}{16} + \left(\bar{Y}_T - \left(h + \frac{4r}{3\pi}\right)\right)^2 \cdot \frac{1}{4} \pi r^2 \Rightarrow \boxed{\bar{I}_2 = 3,38 \cdot 10^{-6} \text{ m}^4}$$

$$\therefore \bar{I}_T = \bar{I}_1 - \bar{I}_2 \Rightarrow \boxed{\bar{I}_T = 9,401 \cdot 10^{-6} \text{ m}^4}$$

b) Aplicando la ec. de la curva elástica



$$\frac{d^4 y}{dx^4} = -\frac{w(x)}{EI}$$

$$* w(x) = \delta(x - \frac{L}{2}) \cdot P$$

• Integrando:

$$\leadsto \frac{d^3 y}{dx^3} = -\frac{1}{EI} \cdot P \cdot r(x - \frac{L}{2}) + \alpha$$

$$\leadsto \frac{d^2 y}{dx^2} = -\frac{1}{EI} (x - \frac{L}{2}) \cdot P \cdot r(x - \frac{L}{2}) + \alpha x + \beta$$

$$\leadsto \frac{dy}{dx} = -\frac{1}{EI} \frac{(x - \frac{L}{2})^2}{2} P \cdot r(x - \frac{L}{2}) + \frac{\alpha x^2}{2} + \beta x + \gamma$$

$$\Rightarrow \hat{y}(x) = \frac{-1}{6EI} (x - \frac{L}{2})^3 P \cdot r(x - \frac{L}{2}) + \frac{\alpha x^3}{6} + \beta \frac{x^2}{2} + \gamma x + \delta$$

• Condiciones de Borde:

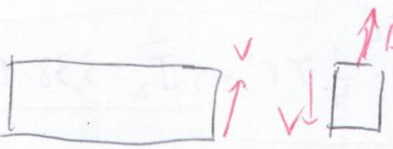
$$\hookrightarrow \boxed{\frac{d^2 y}{dx^2}(0) = 0 \text{ (1)} \quad \frac{d^2 y}{dx^2}(L) = 0 \text{ (2)}}$$

$\hookrightarrow$ Corte dif. en $x=0 \leadsto$  $\left. \begin{array}{l} V(0) + A = 0 \\ \Rightarrow V(0) = -A \end{array} \right\}$

$$* V(x) = -EI \frac{d^3 y}{dx^3}(x) \quad \Rightarrow \quad A = -k_1 \cdot \hat{y}(0)$$

$$\therefore -EI \frac{d^3 y}{dx^3}(0) = k_1 \hat{y}(0) \text{ (3)}$$

$\hookrightarrow$ Corte dif. en $x=L$

 $\left. \begin{array}{l} V(L) - B = 0 \Rightarrow V(L) = B \\ * B = -k_2 \hat{y}(L) \end{array} \right\}$

$$\therefore EI \frac{d^3 y}{dx^3}(L) = k_2 \hat{y}(L) \text{ (4)}$$

• Aplicando las cond. de Borde

$$(1) \frac{d^2 y}{dx^2}(0) = 0 \Rightarrow \boxed{\beta = 0}$$

$$(2) \frac{d^2 y}{dx^2}(L) = 0 \leadsto -\frac{1}{EI} \left(L - \frac{L}{2}\right) \cdot P + 2L = 0$$
$$\Rightarrow \boxed{\alpha = \frac{P}{2EI}}$$

$$(3) -EI \frac{d^3 y}{dx^3}(0) = K_1 y(0)$$

$$-2EI = K_1 \delta \leadsto \delta = -\frac{2EI}{K_1} = -\frac{P}{2EI} \cdot \frac{EI}{K_1} \Rightarrow \boxed{\delta = -\frac{P}{2K_1}}$$

$$(4) EI \frac{d^3 y}{dx^3}(L) = K_2 y(L)$$

$$-P + 2EI = K_2 \left(-\frac{1}{6EI} \left(L - \frac{L}{2}\right)^3 P + \frac{2L^3}{6} + \gamma L + \delta\right)$$

~~* Despejando~~

$$-P + \frac{P}{2} = K_2 \left(-\frac{L^3 P}{48EI} + \frac{2L^3}{6} + \frac{P}{2K_1}\right) + K_2 L \gamma$$

$$\therefore \gamma = \frac{1}{K_2 L} \left(-\frac{P}{2} + K_2 P \left(-\frac{L^3}{48EI} + \frac{L^3}{12EI} - \frac{1}{2K_1}\right)\right)$$

• Una vez calculadas todas las constantes, evaluemos $y\left(\frac{L}{2}\right)$ para encontrar el desplazamiento en el punto C.

$$y\left(\frac{L}{2}\right) = \alpha \frac{L^3}{6} + \gamma \frac{L}{2} + \delta = \alpha \frac{\left(\frac{L}{2}\right)^3}{6} + \gamma \frac{L}{2} + \delta$$

• Calculando los coef:

$$\alpha = 1,4 \cdot 10^{-3}$$
$$\gamma = -7,4 \cdot 10^{-5}$$
$$\delta = -8,3 \cdot 10^{-3}$$

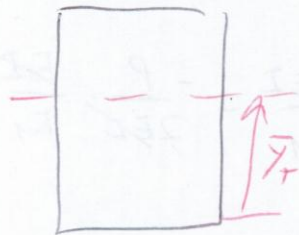
$$\therefore y\left(\frac{L}{2}\right) = -0,0241 \text{ m} = -2,41 \text{ cm}$$

a) $M = EI \frac{d^2 y}{dx^2} \Rightarrow M_{max} \sim \text{derivar } M(x) \text{ obtener } x \text{ para luego evaluar en } M(x)$

$\Rightarrow \frac{d^3 y}{dx^3} = 0 \Rightarrow \frac{1}{EI} P \cdot r(x - \frac{L}{2}) + \frac{P}{2EI}$

c) $M_{max} \sim \left(\frac{d^2 y}{dx^2} \right)_{max} \Rightarrow x = \frac{L}{2}$

$y_{max} \sim$



* $y = -\bar{y}_+ = -4,967 \text{ cm}$

* $y = h - \bar{y}_+ = 5,433 \text{ cm} \rightarrow M_{\text{max}} \text{ (Ext. sup)}$

$\therefore \sigma_{max} = -\frac{M}{I} \wedge M = EI \frac{d^2 y}{dx^2}$

$\Rightarrow \sigma_{max} = -E y_{max} \left(\frac{d^2 y}{dx^2} \right)_{max} \left\{ \frac{d^2 y}{dx^2} \left(\frac{L}{2} \right) = \frac{PL}{4EI} \right.$

$\therefore \sigma_{max} = -43,342 \text{ MPa}$