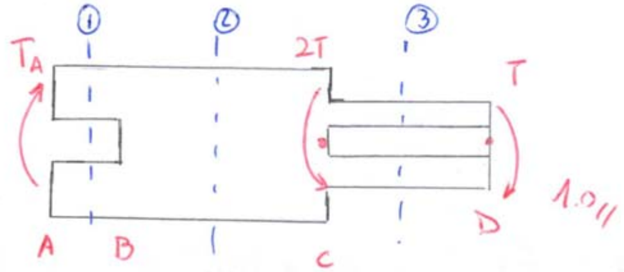
Pauta Control 2, P1

Total 25 ptos.

a) D.C.L.



$$\Sigma M = 0$$

$$\Rightarrow T_A = T \quad 1.0//$$

- Para $0 \leq x \leq 25$ (Entre A y B)

$$J = \frac{\pi}{32}\left(\phi_1^4 - \phi_2^4\right) = 1{,}63 \times 10^{-6} \, [m^4]$$

$$\tau_{máx} = \frac{T\, r}{J} = \frac{T\, \phi_1}{2J} = \tau_{máx \text{ del latón}} = 48 \, [MPa]$$

$$\therefore \quad T_{max} = \frac{48 \times 10^6 [Pa]}{0{,}064 \, [m]} \cdot 2 \cdot 1{,}63 \times 10^{-6} \, [m^4]$$

$$\underline{T_{máx} = 2447{,}1 \quad [Nm]} \quad 2.0//$$

- Para $25 < x \leq 50$ (Entre B y C)

$$J = \frac{\pi}{32} \phi_1^4 = 1{,}64 \times 10^{-6} \, [m^4]$$

$$\tau_{máx} = 48 \, [MPa] = \frac{T\, \phi_1}{2J}$$

$$\Rightarrow \quad T_{máx} = \frac{48 \times 10^6 \, [Pa]}{0{,}064} \cdot 2 \cdot 1{,}64 \times 10^{-6} \, [m^4]$$

$$T_{máx} = 2470{,}6 \quad [Nm] // \quad 2.0//$$

• Para $50 < x \leq 90$ (Entre C y D)

$$T = T_1 + T_2 \quad (1)$$

$$= \frac{G_1 \theta J_1}{L} + \frac{G_2 \theta J_2}{L} \quad 1.0$$

En este caso

$$G_1 = \frac{E_1}{2(1+\nu_1)} = 40\,[GPa]\,(\text{latón}) \qquad G_2 = \frac{E_2}{2(1+\nu_2)} = 80{,}8\,[GPa]\,(\text{acero})$$

$$J_1 = \frac{\pi}{32}\left(\phi_4^4 - \phi_3^4\right) = 3{,}5 \times 10^{-8}\,[m^4] \qquad J_2 = \frac{\pi}{32}\phi_3^4 = 2{,}8 \times 10^{-9}\,[m^4]$$

• Como se encuentran perfectamente unidos, en el pto. D ambos tienen igual ángulo de torsión.

$$\theta_1 = \frac{T_1 \cdot L}{G_1 J_1} = \theta_2 = \frac{T_2 L}{G_2 J_2}$$

$$\therefore T_1 = T_2 \frac{G_1}{G_2} \frac{J_1}{J_2} \quad (2) \quad 1.0$$

(1) y (2) $\Rightarrow T = T_2\left(\frac{G_1}{G_2}\frac{J_1}{J_2} + 1\right)$ ó $T_2 = \frac{T}{\left(1 + \frac{G_1}{G_2}\frac{J_1}{J_2}\right)}$

$$T_1 = \frac{T}{\left(1 + \frac{G_2}{G_1}\frac{J_2}{J_1}\right)}$$

En el caso del latón

$$\tau_{máx} = 48\,[MPa] = \frac{T_{máx}}{\left(1 + \frac{G_2}{G_1}\frac{J_2}{J_1}\right) \cdot J_1} \cdot \frac{\phi_4}{2} \Rightarrow T_{máx} = \frac{48}{\phi_4}[MPa] \cdot 2J_1\left(1 + \frac{G_2}{G_1}\frac{J_2}{J_1}\right)$$

$$T_{máx} = 158{,}2\,[N\,m] \quad 2.0$$

En el acero.

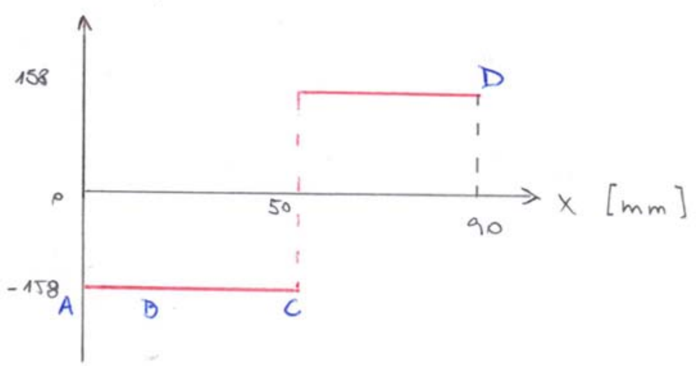
$$\tau_{máx} = 83\,[MPa] = \frac{T_{máx}}{\left(1+\frac{G_1}{G_2}\frac{J_1}{J_2}\right)\cdot J_2}\cdot \phi_3/2$$

$$\Rightarrow T_{máx} = \frac{83\times10^6\,[Pa]\cdot 2\cdot\left(1+\frac{G_1}{G_2}\frac{J_1}{J_2}\right)J_2}{\phi_3}$$

$$T_{máx} = 260,6\ [Nm]$$ // 2.0//

∴ El máximo valor del torque T admisible es 158,2 [Nm] // 2.0//

b) El diagrama del momento torsionante se realiza con el valor máximo de T obtenido en el punto anterior ($T = 158,2$ [Nm



3.0//

c) Ángulo de torsión en el pto. B relativo a A

$$\theta_{AB} = \frac{T\cdot L}{G\,J} = \frac{158,2\,[Nm]\cdot 25\,[mm]}{40\,[GPa]\cdot 1,63\times10^{-6}\,[m^4]}$$

$$\theta_{AB} = 6,06\times10^{-5}\,[rad]$$ // 1.0//

• Ángulo de torsión en el pto. C relativo a B

$$\theta_{BC} = \frac{T\cdot L}{G\,J} = \frac{158,2\,[Nm]\cdot 25\,[mm]}{40\,[GPa]\cdot 1,64\times10^{-6}\,[m^4]}$$ 1.0//

$$\theta_{BC} = 6\times10^{-5}\,[rad]$$

• En el punto D, relativo a C.

$$\theta_{CD} = \frac{T \cdot L}{[G_1 J_1 + G_2 J_2]} = \frac{-158,2\ [Nm] \cdot 0,04\ [m]}{\left(40\ [GPa] \cdot 3,5 \times 10^{-8}\ [m] + 80,8\ [GPa] \cdot 2,8 \times 10^{-9}\ [m^4]\right)}$$

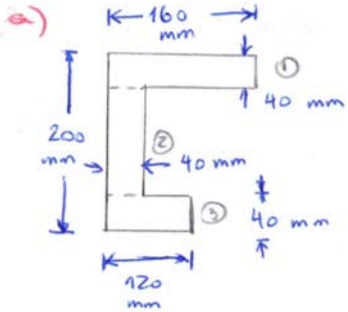
$$\theta_{CD} = -3,84 \times 10^{-3}\ [rad]$$ Z.0//

$$\therefore\ \theta_{AD} = \theta_{AB} + \theta_{BC} + \theta_{CD}$$

$$\theta_{AD} \approx -3,84 \times 10^{-3}\ [rad]$$ // Z.0//

Pauta P2 Total 29 ptos.

a)



Cálculo del eje neutro (posición)

$$\bar{y} = \frac{\int_A y' \, dA}{A} = \frac{\bar{y}_1 A_1 + \bar{y}_2 A_2 + \bar{y}_3 A_3}{A_1 + A_2 + A_3}$$ 1.0

$\bar{y}_1 = 180$ mm , $A_1 = 6400$ mm^2

$\bar{y}_2 = 100$ mm , $A_2 = 4800$ mm^2

$\bar{y}_3 = 20$ mm , $A_3 = 4800$ mm^2

$\Rightarrow \bar{y} = 108$ mm 2.0

Cálculo de I_z :

$I_1 = \dfrac{160 \cdot 40^3}{12} = 8{,}53 \times 10^{-7}$ [m^4] , $\delta_1 = (180 - 108) = 0{,}072$ [m]

$I_2 = \dfrac{40 \cdot 120^3}{12} = 5{,}76 \times 10^{-6}$ [m^4] , $\delta_2 = (100 - 108) = -0{,}008$ [m]

$I_3 = \dfrac{120 \cdot 40^3}{12} = 6{,}4 \times 10^{-7}$ [m^4] , $\delta_3 = (20 - 108) = -0{,}088$ [m]

1.0

$\therefore \; I_z = \sum I_{z_i} + \delta_i^2 A_i$

$I_z = 7{,}79 \times 10^{-5}$ [m^4] 1.0

$\cdot \ w(x) = 200(3-x) - \delta(x-2)R_B$ 1,0//

Del equilibrio estático se puede determinar que:

$\sum_A M_z = 0 \Rightarrow R_B = 450 \text{ N}$ // 1.0//

$\therefore \ w(x) = 200(3-x) - \delta(x-2)\cdot 450$

$$\therefore EI_z \frac{d^4 y}{dx^4} = -200(3-x) + \delta(x-2)\,450 \quad / \int(\%)$$

$$EI_z \frac{d^3 y}{dx^3} = 100(3-x)^2 + r(x-2)\,450 + \alpha$$ 1,0//

1^{era} Condición de borde $V(x=3) = 0 \Rightarrow \left.\frac{d^3 y}{dx^3}\right|_{x=3} = 0$ 1.0//

$$\therefore + \underbrace{r(3-2)}_{1}\cdot 450 + \alpha = 0$$

$$\therefore \ \alpha = -450 \ //$$ 1.0//

$$EI_z \frac{d^3 y}{dx^3} = 100(3-x)^2 + r(x-2)\,450 - 450 \quad / \int(\%)$$

$$EI_z \frac{d^2 y}{dx^2} = -\frac{100}{3}(x-3)^3 + (x-2)r(x-2)\,450 - 450x + \beta$$ 1.0//

2^{da} cond. borde $M(x=3) = 0 \Rightarrow \left.\frac{d^2 y}{dx^2}\right|_{x=3} = 0$ 1.0/

$$\therefore + 1\cdot\underbrace{r(1)}_{1}\cdot 450 - 450\cdot 3 + \beta = 0$$

$$\Rightarrow \beta = 900 \ //$$ 1.0//

$$E I_z \frac{d^2 \hat{y}}{dx^2} = -\frac{100}{3}(3-x)^3 + (x-2)\,r(x-2)\,450 - 450x + 900 \quad \Big/ \int($$

$$E I_z \frac{d\hat{y}}{dx} = \frac{100}{12}(3-x)^4 + \frac{1}{2}(x-2)^2\,r(x-2)\,450 - \frac{450}{2}x^2 + 900x + \gamma \quad \Big/ \int($$

1.0//

$$E I_z\, \hat{y}(x) = -\frac{5}{3}(3-x)^5 + \frac{1}{6}(x-2)^3\,r(x-2)\,450 - 75x^3 + 450x^2 + \gamma x + \delta$$

1.0//

Cond. borde $\hat{y}(x=0) = 0$ 1.0//

$$\therefore -\frac{5}{3}(3)^5 + \delta = 0 \Rightarrow \delta = 405 \;//$$ 1.0//

Cond. borde $\hat{y}(x=2) = 0$ 1.0//

$$\Rightarrow -\frac{5}{3} - 600 + 1800 + 405 + 2\gamma = 0$$

$$\Rightarrow \gamma = -801,\overline{6} \;//$$ 1.0//

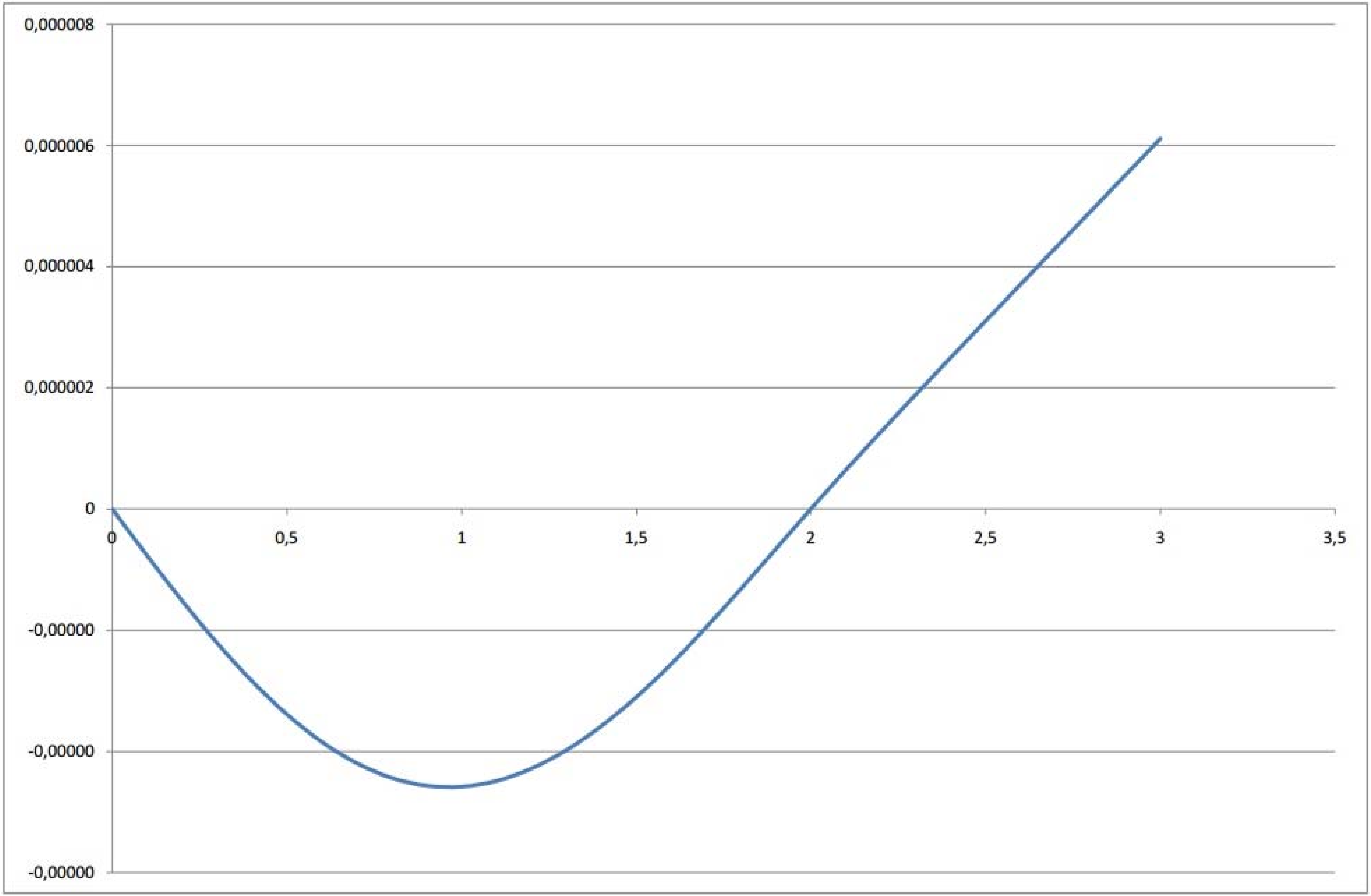
$$\therefore \hat{y}(x) = \frac{1}{E I_z}\left[-\frac{5}{3}(x-3)^5 + \frac{1}{6}(x-2)^3\,r(x-2)\,450 - 75x^3 - 450x^2 - 801,\overline{6}\,x + 405\right]$$

2.0//

2. ptos x gráfico

2 ptos x máximo, se ubica en $x = 3$ m

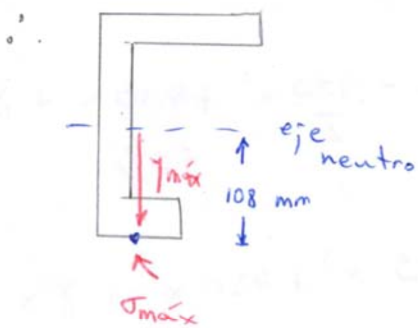
y es $\hat{y}_{máx} = 6,1 \times 10^{-6}$ [m]


0,000008
0,000006
0,000004
0,000002
0
-0,00000
-0,00000
-0,00000
0
0,5
1
1,5
2
2,5
3
3,5

b) El momento de flexión máximo se ubica en $x = 0{,}88$ 1.0

y es igual a $M_{máx} = 186{,}4 \ [N m]$ 1.0

$\therefore$

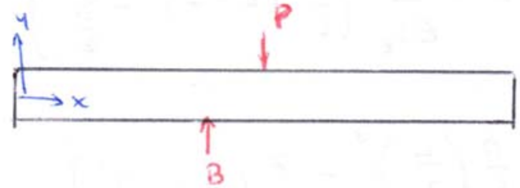


$y_{máx} = 108 \ mm$ 1.0

$\therefore \ \sigma_{máx} = \dfrac{186{,}4 \ [N m] \cdot 0{,}108 \ [m]}{7{,}79 \times 10^{-5} \ [m^4]}$

$\sigma_{máx} = 258 \ [kPa]$ 1.0

Parte P3.



total 23 ptos //

Condiciones de borde:

$\hat{y}(0) = 0$ 2.0 // $\hat{y}(L/2) = 0$ 1.0 // $\hat{y}(L) = 0$ 2.0 //

$\left.\frac{d^2\hat{y}}{dx^2}\right|_0 = 0$ $\left.\frac{d^2\hat{y}}{dx^2}\right|_L = 0$

deflexión:

$$\frac{d^4\hat{y}}{dx^4} = \frac{1}{EI_z}\left[B\,\delta(x - L/2) - P\,\delta(x-a)\right]$$ 1.0 //

$$\Rightarrow \frac{d^3\hat{y}}{dx^3} = \frac{1}{EI_z}\left[B\,r(x - L/2) - P\,r(x-a)\right] + C_3$$

$$\Rightarrow \frac{d^2\hat{y}}{dx^2} = \frac{1}{EI_z}\left[B(x-L/2)\,r(x-L/2) - P(x-a)\,r(x-a)\right] + C_3 x + C_2$$ 1.0 //

$$\frac{d\hat{y}}{dx} = \frac{1}{EI_z}\left[\frac{B}{2}(x-L/2)^2 r(x-L/2) - \frac{P}{2}(x-a)^2 r(x-a)\right] + C_3\frac{x^2}{2} + C_2 x + C_1$$

$$\hat{y} = \frac{1}{EI_z}\left[\frac{B}{6}(x-L/2)^3 r(x-L/2) - \frac{P}{6}(x-a)^3 r(x-a)\right] + C_3\frac{x^3}{6} + C_2\frac{x^2}{2} + C_1 x + C_0$$ 1.0 //

en $x=0$, $\frac{d^2\hat{y}}{dx^2} = 0 \Rightarrow C_2 = 0$ 1.0 //

$\hat{y}(0) = 0 \Rightarrow C_0 = 0$ 1.0 /

$$\frac{d^2\hat{y}}{dx^2}(L) = 0 \quad \Rightarrow \quad C_3 = \frac{1}{EI_z}\left[\frac{P}{L}(L-a) - \frac{B}{2}\right] \quad 1.0$$

$$\hat{y}(L) = 0 \quad \Rightarrow \quad \frac{1}{EI_z}\left[\frac{B}{6}\left(\frac{L}{2}\right)^3 - \frac{P}{6}(L-a)^3\right] + C_3 L^3/6$$

$$+ C_1 L = 0$$

$$\Rightarrow C_1 = \frac{1}{EI_z}\left[\frac{P}{6L}(L-a)^3 - \frac{BL^2}{48}\right]$$

$$C_1 = \frac{[3BL^3 - 8a(a-2L)(a-L)P]}{48 EI_z L} \quad 1.0$$

Apoyo rodillo en B

$$\Rightarrow \hat{y}(L/2) = 0 \quad \Rightarrow \quad \frac{C_3}{6}\left(\frac{L}{2}\right)^3 + C_1\frac{L}{2} = 0 \quad 2.0$$

Usando las expresiones para C_1 y C_3, se obtiene:

$$B = \frac{(a-L)(4a^2 - 8aL + L^2)P}{L^3} \quad 2.0$$

Finalmente se pide que $\hat{y}(L/4) = -\hat{y}(3L/4)$

$$\frac{C_3}{6}\left(\frac{L}{4}\right)^3 + C_1\frac{L}{4} = -\frac{1}{EI_z}\left[\frac{B}{6}\left(\frac{3L}{4} - \frac{L}{2}\right)^3 - \frac{P}{6}\left(\frac{3L}{4} - a\right)^3\right] - \frac{C_3}{6}\left(\frac{3L}{4}\right)^3$$

$$- C_1\frac{3L}{4} \quad 2.0$$

Reemplazando el valor de B, C_1 y C_2 se obtiene:

$$\frac{(11a - 10L)(L-2a)^2 P}{384 EI_z} = 0 \quad 3.0$$

$\underbrace{a = \frac{10}{11} L}_{\checkmark}$

1.0 //

$a = \frac{L}{2}$

$\Rightarrow d_1 = d_2 = 0$ //

1.0 //