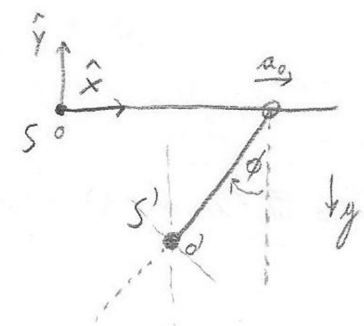
Pauta Aux 18

P1



a)

• sistema fijo en O, cartesianas:

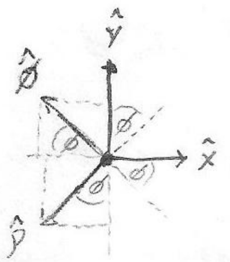
$$S(\hat{x}\hat{y}) \rightarrow \vec{R} = x\hat{x},\ \vec{V} = \dot{x}\hat{x},\ \vec{A} = a_0\hat{x}$$

• sistema móvil en O', polares:

$$S'(\hat{\rho},\hat{\phi}) \rightarrow \vec{r}\,' = L\hat{\rho},\ \vec{v}\,' = L\dot{\phi}\hat{\phi}$$

$$\vec{a}\,' = -L\dot{\phi}^2\hat{\rho} + L\ddot{\phi}\hat{\phi} \quad (1)$$

• Relación entre los vectores unitarios:



$$\Rightarrow \left.\begin{array}{l} \hat{x} = -\,\mathrm{sen}\phi\,\hat{\rho} - \cos\phi\,\hat{\phi} \\ \hat{y} = -\cos\phi\,\hat{\rho} + \mathrm{sen}\phi\,\hat{\phi} \end{array}\right\} (2)$$

• Ecuación "corregida" de newton: $m\vec{a}\,' = \vec{F} + \vec{F}_{ficticias}$ (3)

$$\vec{F} = \vec{P}_{eso} + \vec{T} = mg(\cos\phi\,\hat{\rho} - \mathrm{sen}\phi\,\hat{\phi}) - T\hat{\rho}$$

$$\vec{F}_{ficticias} = \vec{F}_A + \underbrace{\cancel{\vec{F}_c + \vec{F}_{co} + \vec{F}_T}}_{0 \text{ porque } \vec{\Omega} = \vec{0}} = -m\vec{A} = -ma_0\hat{x} \overset{(2)}{=} +ma_0(\mathrm{sen}\phi\,\hat{\rho} + \cos\phi\,\hat{\phi}) \quad (4)$$

(1) y (4) en (3) nos da 1 ec. vectorial que podemos separar en componentes $\hat{\rho}$, $\hat{\phi}$.

$$\hat{\rho}\,\big| \quad -mL\dot{\phi}^2 = mg\cos\phi - T + ma_0\,\mathrm{sen}\phi \quad (5)$$

$$\hat{\phi}\,\big| \quad -mL\ddot{\phi} = -mg\,\mathrm{sen}\phi + ma_0\cos\phi \Rightarrow \ddot{\phi} = \frac{-g}{L}\,\mathrm{sen}\phi + \frac{a_0}{L}\cos\phi$$

integramos con: $\ddot{\phi} = \dot{\phi}\dfrac{d\dot{\phi}}{d\phi}$

$$\int_0^{\dot\phi} \dot\phi\, d\dot\phi = \int_0^{\phi} \left(\frac{-g}{L} \sin\phi + \frac{a_0}{L} \cos\phi \right) d\phi$$

* parte del reposo:

$\phi_0 = 0$

$\dot\phi_0 = 0$

$$\Rightarrow \frac{\dot\phi^2}{2} - 0 = \frac{+g}{L} \left| \cos\phi \right|_0^{\phi} + \frac{a_0}{L} \left| \sin\phi \right|_0^{\phi} = \frac{g}{L}(\cos\phi - 1) + \frac{a_0}{L} \sin\phi \quad (6)$$

· El máximo ángulo se logra en $\dot\phi = 0$

$$\Rightarrow \frac{g}{L}\cos\phi - \frac{g}{L} + \frac{a_0}{L}\sin\phi = 0 \Rightarrow \cos\phi = 1 - \frac{a_0}{g}\sin\phi \quad /(\ )^2$$

$$\Rightarrow \cos^2\phi = 1 - \sin^2\phi = 1 - \frac{2a_0}{g}\sin\phi + \frac{a_0^2}{g^2}\sin^2\phi$$

$$\Rightarrow \sin\phi \left[\sin\phi \left(\frac{a_0^2}{g^2} + 1 \right) - \frac{2a_0}{g} \right] = 0 \quad \Rightarrow \sin\phi = 0 \Rightarrow \phi = 0$$

es solución, pero no interesa el ϕ máximo.

$\phi \neq 0$

$$\Rightarrow \sin\phi = \frac{\frac{2a_0}{g}}{\frac{a_0^2}{g^2} + 1} \Rightarrow \boxed{\phi_{máx} = \arcsin\left(\frac{2a_0 g}{a_0^2 + g^2} \right)} \quad (7)$$

b) Para calcular la tensión máxima reemplazamos (6) en (5):

$$\Rightarrow T = mg\cos\phi + ma_0\sin\phi + 2mg\cos\phi - 2mg + 2a_0 m \sin\phi$$

$$\Rightarrow T = 3mg\cos\phi + 3ma_0\sin\phi - 2mg$$

· maximizamos: $\frac{dT}{d\phi} = 0 \Rightarrow \frac{dT}{d\phi} = 3m(-g\sin\phi + a_0\cos\phi) = 0$

$$\Rightarrow g\sin\phi = a_0\cos\phi \Rightarrow \boxed{\tan\phi = \frac{a_0}{g}} \begin{cases} \sin\phi = \dfrac{a_0}{\sqrt{g^2 + a_0^2}} \\ \cos\phi = \dfrac{g}{\sqrt{g^2 + a_0^2}} \end{cases}$$

$$\Rightarrow T = \frac{3m}{\cancel{\sqrt{g^2 + a_0^2}}} \left(g^2 + a_0^2 \right)^{\cancel{}\,1/2} - 2mg \Rightarrow \boxed{T = 3m\sqrt{g^2 + a_0^2} - 2mg}$$

tensión máxima