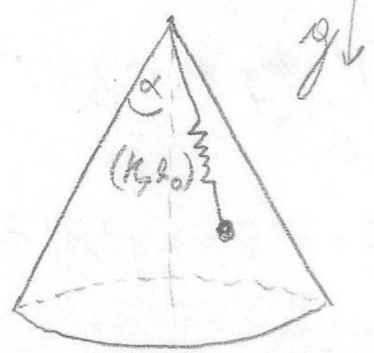
Aux 9

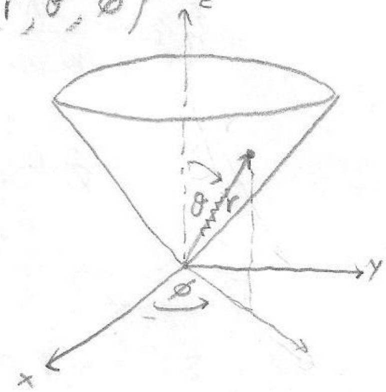
P1



¿Coordenadas?

- Cartesianas es mucho notación.
- Cilíndricas es difícil porque varía la altura, el radio polar y el ángulo polar.
- Intrínsecas es malo porque no sabemos la trayectoria

$\Rightarrow$ Esféricas! $(\hat{r}, \hat{\theta}, \hat{\phi})$



a) • En esféricas con origen en el vértice del cono:

$\Rightarrow$ r variable, ϕ variable, θ constante (cono)

$$\Rightarrow \quad \theta = \alpha \Rightarrow \dot{\theta} = \ddot{\theta} = 0$$

• Condiciones iniciales: $\vec{r}(0) = a\hat{r}$

$$\vec{v}(0) = \dot{r}(0)\hat{r} + \underbrace{\cancel{r\dot{\theta}}}_{0}\hat{\theta} + \underbrace{r(0)}_{a}\dot{\phi}(0)\sin\alpha\,\hat{\phi} = v_0\,\hat{\phi}$$

$$\Rightarrow \quad \dot{r}(0) = 0 \quad , \quad \dot{\phi}(0) = \frac{v_0}{a\sin\alpha} \quad , \quad r(0) = a \quad (1)$$

Ec. de torque: $\frac{d\vec{L}}{dt} = \sum \vec{\tau}$ con $\vec{L} = m\vec{r}\times\vec{v}$

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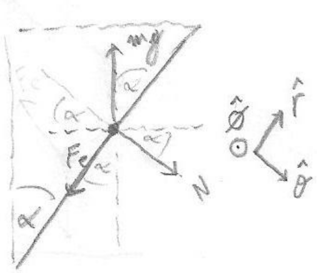
$$\Rightarrow \sum \vec{\tau} = \vec{\tau}_N + \vec{\tau}_{mg} + \vec{\tau}_{F_e} \qquad (\vec{\tau} = \vec{r}\times\vec{F})$$



$$\vec{\tau}_{F_e} = r\hat{r}\times k(l_0 - r)\hat{r} = 0$$

$$\vec{\tau}_N = rN(\hat{r}\times\hat{\theta}) = rN\hat{\phi}$$

$$\vec{\tau}_{mg} = r\hat{r}\times(mg\cos\alpha\,\hat{r} - mg\sin\alpha\,\hat{\theta}) = -r\,mg\sin\alpha\,\hat{\phi}$$

momento angular: $\vec{L} = m\vec{r}\times\vec{v} = mr\,\hat{r}\times(\dot{r}\hat{r} + \underset{0}{\cancel{r\dot{\theta}}}\hat{\theta} + r\sin\alpha\,\dot{\phi}\hat{\phi})$

$$\Rightarrow \vec{L} = -mr^2\dot{\phi}\sin\alpha\,\hat{\theta}$$

Ec. de Torque: $\underbrace{\frac{d}{dt}(-mr^2\dot{\phi}\sin\alpha\,\hat{\theta})} = r\hat{\phi}(N - mg\sin\alpha)$ (2)

$$-m\sin\alpha\frac{d}{dt}(r^2\dot{\phi}\hat{\theta}) = -m\sin\alpha\left(2r\dot{r}\dot{\phi}\hat{\theta} + r^2\ddot{\phi}\hat{\theta} + r^2\dot{\phi}[\dot{\phi}\cos\theta\hat{\phi} - \underset{0}{\dot{\theta}}\hat{r}]\right)$$

$r\neq 0 \Rightarrow$ $\frac{(2)}{-mr\sin\alpha}$ $\left(g - \frac{N}{m\sin\alpha}\right)\hat{\phi} = r\dot{\phi}^2\cos\theta\,\hat{\phi} + (2\dot{r}\dot{\phi} + r\ddot{\phi})\hat{\theta}$

$$\Rightarrow \boxed{r\dot{\phi}^2\cos\alpha = g - \frac{N}{m\sin\alpha}} \quad (3)$$

$$\Rightarrow 2\dot{r}\dot{\phi} + r\ddot{\phi} = 0 \quad \cdot r \Rightarrow 2r\dot{r}\dot{\phi} + r^2\ddot{\phi} = \frac{d}{dt}(r^2\dot{\phi}) = 0$$

$\Rightarrow$ $\boxed{r^2\dot{\phi} \text{ es cantidad conservada, por C.I tenemos:}}$

b) $$r^2\dot{\phi} = r(0)^2\dot{\phi}(0) \overset{(1)}{=} a^2\frac{v_0}{a\sin\alpha} = \frac{av_0}{\sin\alpha} \Rightarrow \dot{\phi} = \frac{av_0}{r^2\sin\alpha} \quad (4)$$

Ec. de mov en $\hat{r}$: $\hat{r}]\ m(\ddot{r} - \underset{0}{r\dot{\theta}^2} - r\dot{\phi}^2\sin^2\alpha) = mg\cos\alpha + k(l_0 - r)$

$\frac{1}{m} \Rightarrow \boxed{\ddot{r} = \frac{a^2v_0^2}{r^3} + g\cos\alpha + \frac{k}{m}(l_0 - r)}$ ec. diferencial para r. (5)

c) Con el cambio: $\ddot{r} = \dot{r}\frac{d\dot{r}}{dr}$ podemos resolver la EDO y tener $\dot{r}(r)$.

$v_f = v_i \Rightarrow \dot{r}_i = \dot{r}_f = 0$

$(5) \Rightarrow \quad \dot{r}\frac{d\dot{r}}{dr} = \frac{a^2 v_0^2}{r^3} + g\cos\alpha + \frac{k}{m}(l_0 - r) \qquad r_f = \frac{a}{2}$

$$\Rightarrow \int_0^0 \dot{r}\,d\dot{r} = \int_a^{\frac{a}{2}} \frac{a^2 v_0^2}{r^3}\,dr + \int_a^{a/2} g\cos\alpha\,dr + \int_a^{a/2} \frac{k}{m} l_0\,dr - \int_a^{a/2} \frac{k}{m} r\,dr$$

$$\Rightarrow \quad 0 = -2a^2 v_0^2\left(\frac{4}{a^2} - \frac{1}{a^2}\right) + \left(g\cos\alpha + \frac{k l_0}{m}\right)\left(\frac{a}{2} - a\right) - \frac{k}{2m}\left(\left(\frac{a}{2}\right)^2 - a^2\right)$$

$$\Rightarrow 0 = -6 v_0^2 + -\frac{a}{2}\left(g\cos\alpha + \frac{k l_0}{m}\right) - \frac{k a^2}{2m}\underbrace{\left(\frac{1}{4} - 1\right)}_{-\frac{3}{4}}$$

$$\Rightarrow \quad \boxed{\frac{3ka^2}{8m} - \frac{a}{2}\left(g\cos\alpha + \frac{k l_0}{m}\right) - 6 v_0^2 = 0}$$

Con esto relacionamos los parámetros $(k, a, m, \alpha, v_0, l_0, g)$

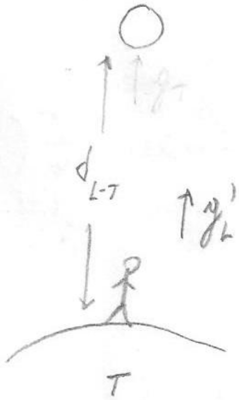
P2

a) aceleración en la Tierra: $g_T = \dfrac{G M_T}{R_T^2} \Rightarrow G = \dfrac{g_T R_T^2}{M_T}$ (1)

aceleración en la Luna: $g_L = \dfrac{G M_L}{R_L^2} \overset{(1)}{\Rightarrow} g_L = g_T \dfrac{M_L}{M_T}\left(\dfrac{R_T}{R_L}\right)^2 \simeq 10 \cdot 10^{-2} (4)^2 \frac{m}{s^2} \simeq 1.6 \ \frac{m}{s^2}$

$\Rightarrow \boxed{g_L \simeq 1.6 \ \frac{m}{s^2}}$

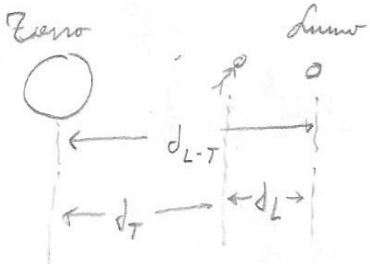
b)



$$g'_L = \frac{G M_L}{d_{L-T}^2} = g_T \frac{M_L}{M_T}\left(\frac{R_T}{d_{L-T}}\right)^2 \simeq 10 \cdot 10^{-2} \left(\frac{64 \cdot 10^2}{38 \cdot 10^4}\right)^2$$

$$\simeq 10^{-1}(1.6 \cdot 10^{-2})^2 \simeq 2.56 \cdot 10^{-5} \Rightarrow \boxed{g'_L \simeq 2.56 \cdot 10^{-5} \ \frac{m}{s^2}}$$

c)



Para sentir la misma gravedad: $g'_L = g'_T$

$$\Rightarrow \frac{G M_T}{d_T^2} = \frac{G M_L}{d_L^2} \Rightarrow \frac{M_T}{M_L} = \left(\frac{d_T}{d_L}\right)^2 \simeq 10^2$$

$\Rightarrow d_T = 10\, d_L$ y $d_T + d_L = d_{L-T} = 3.8 \cdot 10^5$ km

$\Rightarrow d_L \cdot 11 = 3.8 \cdot 10^5$ km $\Rightarrow \boxed{d_L \simeq 3.4 \cdot 10^4 \text{ km}}$

$\Rightarrow \boxed{d_T = 3.46 \cdot 10^5 \text{ km}}$