Peak-To-Average Power Ratio Reduction in OFDM Systems: A Survey And Taxonomy

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***Abstract*—The objective of this survey is to provide the readers and practitioners in the industry with a broader understanding of the high *peak-to-average power ratio* (PAPR) problem in *orthogonal frequency division multiplexing* (OFDM) systems and generate a taxonomy of the available solutions to mitigate the problem. Beginning with a description of OFDM systems, the survey describes the most commonly encountered impediment of OFDM systems, the PAPR problem and consequent impact on power amplifiers leading to nonlinear distortion. The survey clearly defines the metrics based on which the performance of PAPR reduction schemes can be evaluated. A taxonomy of PAPR reduction schemes classifies them into signal distortion, multiple signaling and probabilistic, and coding techniques with further classification within each category. We also provide complexity analyses for a few PAPR reduction methods to demonstrate the differences in complexity requirements between different methods. Moreover, the paper provides insights into the transmitted power constraint by showing the possibility of satisfying the constraint without added complexity by the use of companding transforms with suitably chosen companding parameters.**

The rapid growth in multimedia-based applications has triggered an insatiable thirst for high data rates and hence increased demand on OFDM-based wireless systems that can support high data rates and high mobility. As the data rates and mobility supported by the OFDM system increase, the number of subcarriers also increases, which in turn leads to high PAPR. As future OFDM-based systems may push the number of subcarriers up to meet the higher data rates and mobility demands, there will be also a need to mitigate the high PAPR that arises, which will likely spur new research activities. The authors believe that this survey will serve as a valuable pedagogical resource for understanding the current research contributions in the area of PAPR reduction in OFDM systems, the different techniques that are available for designers and their trade-offs towards developing more efficient and practical solutions, especially for future research in PAPR reduction schemes for high data rate OFDM systems.

***Index Terms*—Orthogonal frequency division multiplexing, peak-to-average power ratio, nonlinear power amplifier.**

I. Introduction

THE MODERN day phenomenon of increased thirst for more information and the explosive growth of new multimedia wireless applications have resulted in an increased demand for technologies that support very high speed transmission rates, mobility and efficiently utilize the available spectrum and network resources. OFDM is one of the best solutions to achieve this goal and it offers a promising choice for future high speed data rate systems [1], [2]. OFDM has been standardized as part of the IEEE 802.11a and IEEE 802.11g for high bit rate data transmission over wireless LANs [3]. It is incorporated in other applications and standards such as *digital audio broadcasting* (DAB), *digital video broadcasting* (DVB), the European HIPERLAN/2 and the Japanese *multimedia mobile access communications* (MMAC) [4], [5]. Also, OFDM is the transmission scheme of choice in the physical layer of the *worldwide interoperability for microwave access* (WiMAX) and *long term evolution* (LTE) standards. It has also been used by a variety of commercial applications such as *digital subscriber line* (DSL), *digital video broadcast-handheld* (DVB-H) and MediaFLO [6].

OFDM was first presented in the late 1950's and characterized in the mid 1960's [7], [8]. In OFDM modulation scheme, multiple data bits are modulated simultaneously by multiple carriers. This procedure partitions the transmission frequency band into multiple narrower subbands, where each data symbol's spectrum occupies one of these subbands. As compared to the conventional frequency division multiplexing (FDM), where such subbands are non-overlapping, OFDM increases spectral efficiency by utilizing subbands that overlap (Fig. 1). To avoid interference among subbands, the subbands are made orthogonal to each other, which means that subbands are mutually independent. By breaking the wide transmission band into narrower, multiple subbands, OFDM schemes effectively combat the effect of frequency-selective fading usually encountered in wireless channels. Frequency-selective fading is a consequence of the phenomenon called multipath propagation, where multiple copies of the transmitted signal traveling along different paths combine at the receiver [2]. To overcome the frequency-selective fading, each subband should be narrow enough such that its bandwidth B satisfies [3]

$$B < \frac{1}{2\pi\tau_{av}}, \tag{1}$$

where τ_{av} is the *average delay spread* defined as the average value of the exponentially distributed random variable used to model the incremental delays of the multiple received rays of the transmitted signal.

OFDM converts the frequency-selective fading channel into multiple flat-fading subchannels, thereby allows the use of simple frequency-domain equalizers to overcome the problem. However, OFDM introduces *inter-symbol interference* (ISI) and *inter-carrier interference* (ICI). ISI is the effect adjacent OFDM symbols exert on each other due to delay spread and ICI is the effect subcarriers exert on each other. Both of these

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problems can be reduced significantly by introducing a guard interval between OFDM symbols. This interval is a cyclic extension of the signal itself concatenated at the beginning of the OFDM symbol, called the *cyclic prefix* (CP). Detailed discussion of the problems of ISI and ICI and the mitigation techniques used to overcome them are beyond the scope of this survey and will not be discussed further.

In the future, OFDM systems are expected to assume greater importance in high speed wireless telecommunications systems, both fixed and mobile. The evolution of the physical layer of such high speed networks points to the use of OFDM systems with a large number of subcarriers with potentially high PAPR. Consequently, mitigation solutions are expected to gain increased interest and spur further research. Although the topic of PAPR reduction has been surveyed in the literature [9]–[12], this survey offers both deeper and wider coverage and includes the most recent literature related to the topic, compared with all the previous surveys. The paper also provides several original contributions via simulation results, complexity analyses and insights into the transmitted power constraint. Therefore, this survey is well-suited to serve as an all-in-one information source to the topic of PAPR reduction in OFDM systems. Its comprehensive and thorough treatment of the topic makes the paper a valuable tool to new researchers who wish to acquire wide knowledge as well as a categorized guide to extensive contributions available in the literature.

This rest of this survey is organized as follows: Section II reviews the basic concepts of conventional OFDM system. Section III presents the PAPR metric and other factors considered in evaluating the performance of PAPR reduction methods. Section IV presents the models commonly used in the literature to represent nonlinear power amplifiers. Section V is the heart of this survey where PAPR reduction methods available in the literature are classified and briefly presented with the most recent relevant bibliography. We demonstrate the computational complexity analysis for four typical PAPR reduction methods in Section VI. In Section VII, we give insights into the transmitted power constraint and use companding transforms as examples. A summary of the lessons learned and suggestions are provided in Section VIII. Finally, Section IX summarizes and concludes this survey.

II. OFDM System Model and Notation

OFDM can be generated using multiple modulated carriers transmitted in parallel. However, this method involves implementation problems and makes transmitters more complex and expensive. This problem can be avoided by the use of the *discrete Fourier transform* (DFT) technique [13].

Consider a data stream with rate R bps where bits are mapped to some constellation points using a digital modulation like the *quadrature amplitude modulation* (QAM). Let N of these constellation points be stored for an interval of $T_s = N/R$, referred to as the OFDM symbol interval. A serial-to-parallel converter is used to achieve this. Now, each one of the N constellation points is used to modulate one of the subcarriers, then, all modulated subcarriers are transmitted simultaneously over the symbol interval T_s [3]. The OFDM signal $x(t)$ can be expressed as

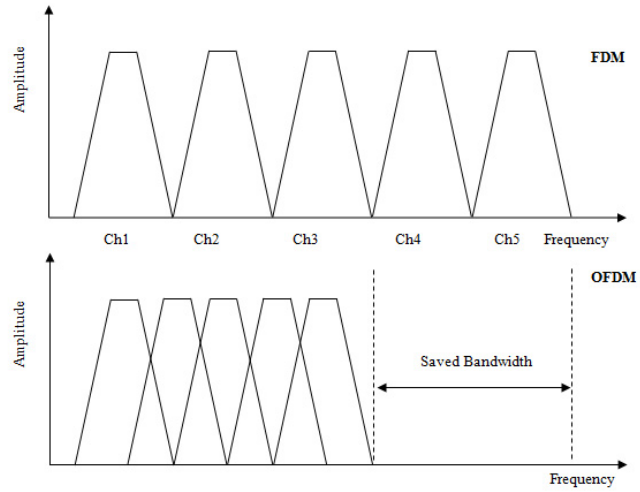


Fig. 1. Comparison of the spectral utilization efficiency between FDM and OFDM schemes.

$$\begin{aligned} x(t) &= \sum_{k=0}^{N-1} a_k \exp\left(j2\pi\left(f_c + k\,\Delta f\right)t\right) \\ &= \exp\left(j2\pi f_c\, t\right) \sum_{k=0}^{N-1} a_k \exp\left(j2\pi k\,\Delta f\, t\right) \\ &= \exp\left(j2\pi f_c\, t\right)\, a(t), \end{aligned} \tag{2}$$

where a_k, $0 \le k \le N-1$, are complex-valued constellation points representing data and $f_k = f_c + k\Delta f, 0 \le k \le N-1$, is the k^{th} subcarrier, with f_c being the lowest subcarrier frequency. Δf is the frequency spacing between adjacent subcarriers, chosen to be $1/T_s$ to ensure that the subcarriers are orthogonal. If $a(t)$ is sampled at rate R samples per second, where t is replaced by $nT_s/N, n = 0, \ldots, N-1$, then $a(t)$ is represented by the sampled function $a[n]$ expressed as

$$a[n] = \sum_{k=0}^{N-1} a_k \exp\left(j2\pi kn/N\right). \tag{3}$$

This equation takes exactly the same form as the *inverse discrete Fourier transform* (IDFT) and can be implemented efficiently using the *inverse fast Fourier transform* (IFFT) algorithm [14]. Equations (2) and (3) demonstrate that OFDM can be generated by modulating the IFFT of the sequence $\{a[n],\ 0 \le n \le N-1\}$ by a single carrier of frequency f_c instead of by modulating N constellation points by subcarriers. IFFT reduces the computational complexity in comparison to IDFT. DSP chip implementations of IFFT and FFT are readily available. The above observation leads us to conclude that IFFT should be the preferred choice for OFDM systems implementation. After demodulating the received signal, the receiver carries out the reverse process of that of the transmitter during each OFDM symbol interval by employing FFT, a parallel-to-serial converter, and a demapping to recover the desired data bit stream [3]. Figure 2 shows a conventional FFT-based OFDM system, where IFFT and FFT are used at the transmitter and receiver respectively.

An interesting alternative of implementing the OFDM scheme, though less popular than the IFFT approach, is the use of wavelet filter banks. Orthogonality property of some

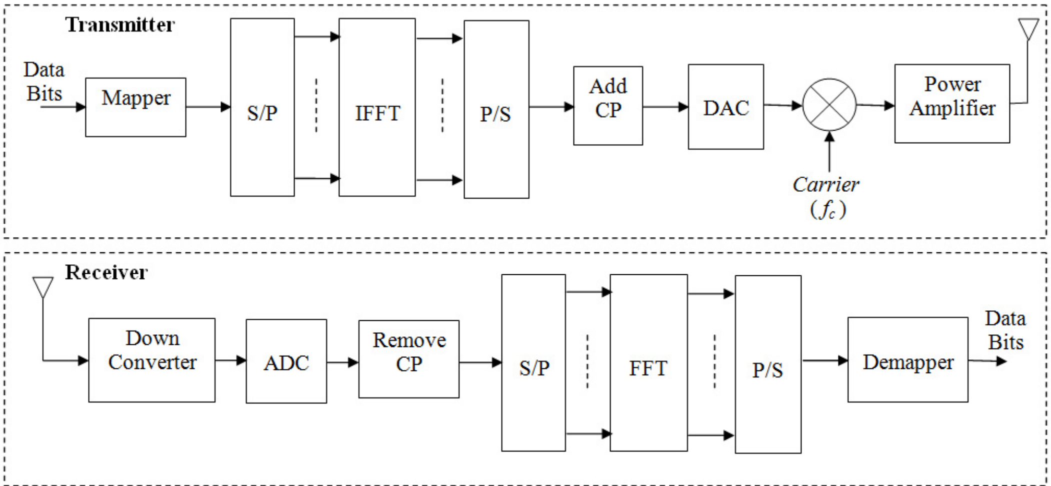


Fig. 2. Conventional FFT-based OFDM transmitter and receiver.

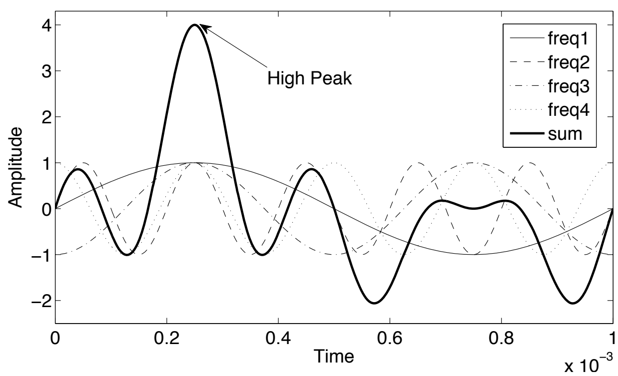


Fig. 3. High peaks in OFDM signal generated by summing multiple sinusoids.

wavelet bases makes them suitable to be used as coefficients of a set of orthogonal digital filter banks. However, such implementation also faces the problem of high PAPR and hence, solutions for mitigating these problems need to be included. Many references address issues that arise in using wavelet filter banks for implementing OFDM systems, compare IFFT and wavelets approaches, and propose methods to reduce high PAPR in wavelet-based OFDM systems [15]–[26].

III. Peak-to-Average Power Ratio and System Performance

A major disadvantage that arises in multicarrier systems like OFDM is the resulting non-constant envelope with high peaks [27]. When the independently modulated subcarriers are added coherently, the instantaneous power will be more than the average power.

Consider the OFDM signal $x(t)$ defined in (2) where N subcarriers are added together. If N is large enough, then, based on *central-limit theorem* (CLT), the resulting signal $x(t)$ will be close to a complex Gaussian process [28]. This means that both of its real and imaginary parts are Gaussian distributed and its envelope and power follows Rayleigh and exponential distributions respectively. The PAPR for the continuous-time signal $x(t)$ is the ratio of the maximum instantaneous power to the average power. For the discrete-time version $x[n]$, PAPR is expressed as

$$\text{PAPR}\left(x\left[n\right]\right) = \max_{0\leq n\leq N-1} \frac{\left|x\left[n\right]\right|^2}{E\left[\,|x[n]|^2\,\right]}\ , \tag{4}$$

where $E[.]$ is the expectation operator. It is worth mentioning here that PAPR is evaluated per OFDM symbol. Figure 3 illustrates how a high peak is obtained by adding four sinusoidal signals with different frequencies and phase shifts coherently. The resulting signal's envelope exhibits high peaks when the instantaneous amplitudes of the different signals have high peaks aligned at the same time. Such high peaks will produce signal excursions into nonlinear region of operation of the *power amplifier* (PA) at the transmitter, thereby leading to nonlinear distortions and spectral spreading [29]. Since IFFT is used to generate the OFDM signal, the resulting discrete-time OFDM signal samples are obtained at the Nyquist-rate. The peak value computed using these samples may not coincide with the peak value of the continuous-time OFDM signal [30]. Hence, oversampling by a factor greater than 1 is used to increase the accuracy. It is found that the PAPR of the oversampled discrete-time signal offers an accurate approximation of the PAPR of the continuous-time OFDM signal if the oversampling factor is at least 4 [31]. References [32] and [33] provide a detailed discussion about the relationship between the oversampled OFDM signal's PAPR and the continuous signal's PAPR.

Although PAPR is the classical and most widely used metric to quantify the envelope fluctuations, another metric known as *cubic metric* (CM) has been proposed and adopted by the *third generation partnership project* (3GPP) [34], [35]. This metric was considered in some recent contributions [36]–[38]. The motivation behind the CM lies in the fact that a major part of the distortion introduced by the nonlinearity of the PA is due to the third order intermodulation product, which can be expressed as the convolution of the signal and the third order nonlinearity of the PA model. While PAPR considers only the main peak of power, CM accounts for the secondary peaks of power that affect the PA performance due to the cubic term

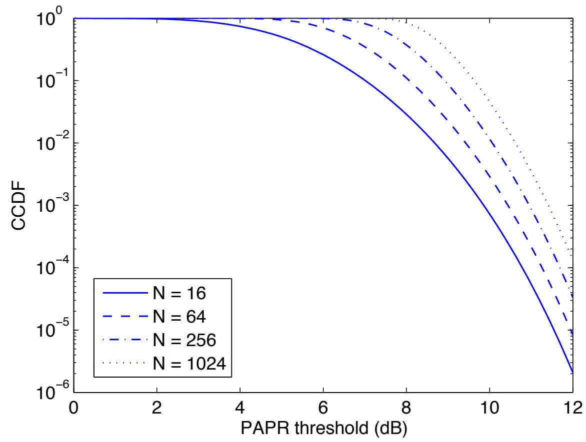


Fig. 4. CCDF for various values of N ($\alpha = 1$).

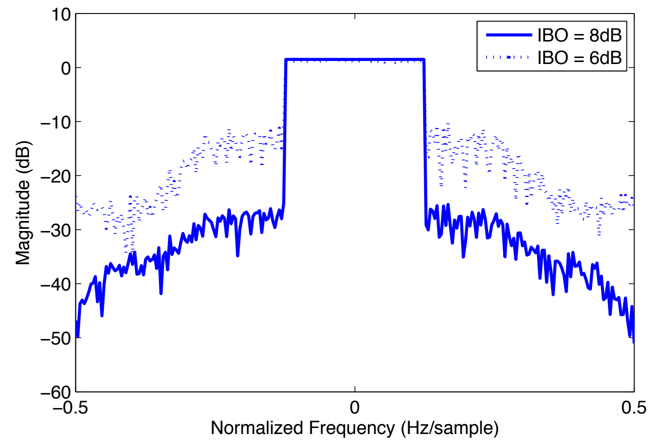


Fig. 5. Spectral spreading effect.

in the PA gain characteristic function defined as [34]

$$y(t) = G_1 \left(x(t) + G_3 \left| x(t) \right|^3 \right) , \tag{5}$$

where $x(t)$, $y(t)$, G_1 and G_3 are, respectively, the input, the output, the linear gain parameter and the nonlinear gain parameter of the PA.

Cubic metric is defined as $CM|_{dB} = (RCM|_{dB} - RCM_{ref}|_{dB})/K$ [35], where RCM is the raw cubic metric defined as

$$RCM\left(x(t)\right)|_{dB} = 20 \log \left(rms \left(\frac{|x(t)|}{rms(x(t))} \right)^3 \right) , \tag{6}$$

and rms is the root mean square value. The terms RCM_{ref} and K are the reference RCM of the wideband code-division multiple-access voice reference signal and the empirical slope factor, respectively. These two terms are constants for each multicarrier system. For example, in downlink LTE $RCM_{ref}|_{dB} = 1.52dB$ and $K = 1.56$ [36]. Since PAPR is by far more popular and widely used in the literature compared to CM, CM will not be discussed further here.

The performance of a PAPR reduction scheme is usually demonstrated by three main factors: the *complementary cumulative distributive function* (CCDF), *bit error rate* (BER), and *spectral spreading*. While CCDF is independent of the characteristics of the PA used at the transmitter, the other two factors are considerably affected. There are also other factors to be considered such as transmitted signal power, computational complexity, bandwidth expansion and data rate loss. These factors are explained next.

A. Complementary Cumulative Distributive Function

In practice, the empirical CCDF is the most informative metric used for evaluating the PAPR. PAPR reduction capability is measured by the amount of CCDF reduction achieved. CCDF provides an indication of the probability of the OFDM signal's envelope exceeding a specified PAPR threshold within the OFDM symbol and is given by

$$CCDF[PAPR\left(x^n\left(t\right)\right)] = \text{prob}\left[PAPR\left(x^n\left(t\right) > \delta\right)\right] , \tag{7}$$

where $PAPR(x^n(t))$ is the PAPR of the n^{th} OFDM symbol and δ is some threshold. Based on the CLT, the envelope of the OFDM signal follows the Rayleigh distribution and consequently its energy distribution becomes an exponential, or equivalently, a central chi-square distribution with two degrees of freedom and zero mean with a CDF given by

$$CDF\left(\delta\right) = \left(1 - e^{-\delta}\right) . \tag{8}$$

The probability that the PAPR of the OFDM signal with N subcarriers is below a threshold δ is the probability that all the N samples are below the threshold. Assuming that the OFDM samples are mutually independent, this probability can be given as [39], [40]

$$\begin{aligned} \text{prob}\left(PAPR < \delta\right) &= CDF\left[PAPR\left(x^n\left(t\right)\right)\right] \\ &= \left(1 - e^{-\delta}\right)^N . \end{aligned} \tag{9}$$

This result is an approximation that is valid for the peak OFDM samples but not necessarily for the maximum peak of the continuous OFDM signal. Oversampling could be used to solve this problem and accurately estimate the peak of the continuous OFDM. However, for the oversampled OFDM, the assumption that OFDM samples are mutually independent is no longer valid. An empirical approximation is proposed by [39], where the distribution of the oversampled OFDM signal of N subcarriers is approximated by the distribution of OFDM signal of αN subcarriers without oversampling. As a result, the CDF of the PAPR is approximated by

$$CDF\left[PAPR\left(x^n\left(t\right)\right)\right] = \left(1 - e^{-\delta}\right)^{\alpha N} . \tag{10}$$

While reference [39] shows that $\alpha = 2.8$ is a practical approximation, other references like [31] state that $\alpha = 4$ is a good choice. Then, the probability of PAPR of the n^{th} OFDM symbol with N subcarriers exceeding a threshold δ is expressed by the CCDF as [39]

$$CCDF\left[PAPR\left(x^n\left(t\right)\right)\right] = 1 - \left(1 - e^{-\delta}\right)^{\alpha N} . \tag{11}$$

Other closed form approximations for the distribution of the PAPR were developed in [40] based on level-crossing rate analysis and in [41] based on extreme value theory. Figure 4 shows the approximated CCDF of the PAPR given by (11) for different values of N and $\alpha = 1$. It is shown that as N increases, CCDF and hence PAPR increases too.

Consequently, high PAPR is a more severe problem for OFDM systems which support high transmission rates and higher mobility, where large N is required. This relationship between N and PAPR motivates the search for new solutions.

B. Bit Error Rate

The performance of a modulation technique can be quantified in terms of the required *signal-to-noise ratio* (SNR) to achieve a specific *bit error rate* (BER). Although the main focus of PAPR reduction techniques is to reduce the CCDF, this is usually achieved at the expense of increasing the BER. Clipping the high peaks of the OFDM signal by the PA causes a substantial in-band distortion that leads to higher BER. Other techniques may require that side information be transmitted as well. If the side information is received incorrectly at the receiver, the whole OFDM symbol is recovered in error and the BER performance degrades.

C. Spectral Spreading

Due to the limit imposed on the maximum peak of the OFDM signal by the PA, an increase is encountered in both the in-band and out-of-band distortions. The second causes undesirable increase in the power of the side lobes of the *power spectral density* (PSD) of the OFDM signal. This effect is referred to as spectral spreading or spectral regrowth. As demonstrated in Fig. 5, when the nonlinearity of the PA is higher, IBO is smaller, and the spectral spreading is higher. Spectral spreading leads to higher interference between the subbands of the OFDM signal, unless the frequency separation between adjacent subcarriers is also increased to maintain orthogonality. However, this solution has the disadvantage of lowering the spectral efficiency.

D. Transmitted Signal Power

Some PAPR reduction techniques require that the average power of the transmitted signal be increased. If the linear region of the PA is not stretched to accommodate the new signal, the signal will traverse the nonlinear region leading to higher distortions and degraded BER performance. However, this solution increases the hardware cost.

E. Computational Complexity

Generally, techniques with increased complexity have better PAPR reduction capability with less undesirable effects than simple ones. However, complex techniques require additional hardware, processing power and time. In practice, both hardware and processing complexity should be as minimum as possible to support real-time system operations and minimize cost.

F. Data Rate Loss

Some PAPR reduction techniques cause some data rate loss due to extra bandwidth required to send side information. Other techniques may require some non-information symbols to be dedicated for controlling PAPR. If the information data rate is required to be the same as that prior to applying the technique, a bandwidth expansion will be a direct result.

G. Other Factors

PAPR reduction techniques should take into consideration the effect of nonlinear devices in the transmitter such as the DAC, mixer, transmit filter and PA. The nonlinearity introduced by these devices and their cost are two important factors in the system design process.

IV. Nonlinearity and Power Amplifier Models

Multicarrier modulated signals like the OFDM signal are more sensitive to the nonlinearities encountered in the transceivers than constant envelope signals. The sources of nonlinearity include: nonlinearity in the FFT and IFFT blocks due to the limited binary word length, signal clipping and quantization errors due to the digital-to-analog and analog-to-digital conversions, and nonlinearity of the PA. However, due to the high PAPR in multicarrier modulations, the nonlinearity of the PAs have the dominant effect. Therefore, precise models for the characteristics of the PAs must be defined.

In general, modeling PAs is complicated, but a common approach is to model them as memoryless nonlinearities with frequency-nonselective response [42]. If the input of the PA is given by

$$x(t) = |x(t)|\, e^{j\,\phi(t)}\,, \tag{12}$$

where $|x(t)|$ and $\phi(t)$ are the amplitude and phase of the input signal, respectively, then the output is given by

$$y(t) = G[\,|x(t)|\,]\, e^{j\,\{\phi(t)+\Phi[\,|x(t)|\,]\}}\,, \tag{13}$$

where $G[.]$ and $\Phi[.]$ are known as the amplitude/amplitude (AM/AM) and amplitude/phase (AM/PM) conversions, respectively. $G[.]$ shows the effect of nonlinearity on the amplitude $|x(t)|$, and $\Phi[.]$ shows the effect of nonlinearity on the phase $\phi(t)$.

There are two PA models commonly used in the literature, the *solid state power amplifier* (SSPA) model [43] and the *traveling wave tube amplifier* (TWTA) model [44]. The SSPA model is expressed as

$$G\,[\,|x(t)|\,] = \frac{g_0\ |x(t)|}{\left[1+\left(\frac{|x(t)|}{x_{sat}}\right)^{2p}\right]^{1/2p}}\,, \tag{14}$$

and

$$\Phi\,(\,|x\,(t)|\,) = 0\,, \tag{15}$$

where g_0 is the amplifier gain, x_{sat}, the saturation level of the PA, and p, a parameter that controls the AM/AM sharpness of the saturation region as shown in Fig. 6(a). As $p \longrightarrow \infty$, the AM/AM characteristics curve becomes similar to the nonlinearity of a soft-limiter.

The TWTA model is expressed as [45]

$$G\,[\,|x(t)|\,] = \frac{\alpha_a\ |x\,(t)|}{1+\beta_a\ |x\,(t)|^2}\,, \tag{16}$$

and

$$\Phi\,[\,|x\,(t)|\,] = \frac{\alpha_\phi\ |x\,(t)\,|}{1+\beta_\phi\ |x\,(t)|^2}\,, \tag{17}$$

where α_a, β_a, α_ϕ and β_ϕ are parameters that control the characteristics of AM/AM and AM/PM conversions. These parameters are chosen such that the *root mean square* (RMS)

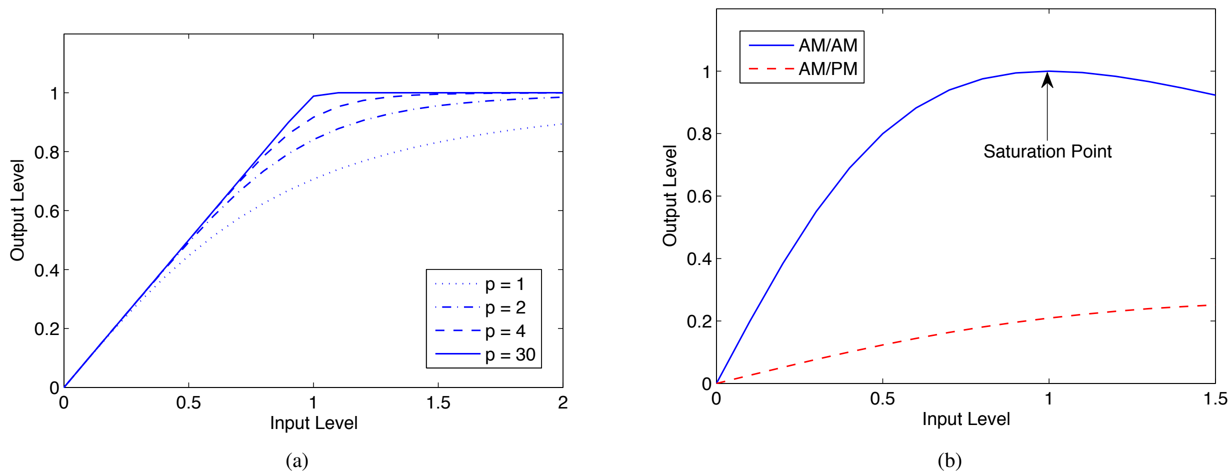


Fig. 6. (a) AM/AM curves for the SSPA for different values of p; (b) AM/AM and AM/PM curves for the TWTA.

error between the model and the TWTA experimental data is minimized. A common choice for the above parameters is

$$\alpha_a = 2\,x_{sat}\,,\ \beta_a = \frac{1}{x_{sat}^2}\,,\ \alpha_\phi = \frac{\pi}{12}\ \text{ and }\ \beta_\phi = 0.25\,, \quad (18)$$

where x_{sat} is the saturation level of the PA. Figure 6(b) shows the AM/AM and AM/PM characteristic curves of the TWTA model with the suggested parameter values as in Eq. (18). Notes that the AM/PM characteristic curve of the TWTA model is non-zero and therefore the TWTA model incorporates increased nonlinearity compared to the SSPA model.

The most efficient operating point for a PA is at the saturation level. However, high peaks encountered in OFDM signals can drive the PA into saturation. Therefore, *input back-off* (IBO) is required to shift the operating point to the left as shown in Fig. 7. The IBO factor is defined as the ratio between the saturation power of the PA and the average power of the input signal. In *decibel* (dB) scale, IBO is given by

$$\begin{aligned} \text{IBO} &= 10\log_{10}\left(\frac{P_{sat}}{P_{av}}\right) \\ &= 10\log_{10}\left(\frac{x_{sat}^2}{E\left[\left|x\left(t\right)\right|^2\right]}\right) \\ &= [P_{sat}]_{dB} - [P_{av}]_{dB}\,, \end{aligned} \quad (19)$$

where $[P_{sat}]_{dB}$ and $[P_{av}]_{dB}$ are the saturation and average powers in dB, respectively. To ensure that the amplified peaks of the OFDM signal do not exceed the saturation level, IBO should be at least equal to PAPR. However, such solution forces the PA to work at a reduced efficiency.

V. PAPR Reduction Techniques

A large PAPR would drive PAs at the transmitter into saturation, producing interference among the subcarriers that degrades the BER performance and corrupts the spectrum of the signal. To avoid driving the PA into saturation, the average power of the signal may be reduced. However, this solution reduces the signal-to-noise ratio and, consequently, the BER performance. Therefore, it is preferable to solve the problem of high PAPR by reducing the peak power of the signal. Many PAPR reduction techniques have been proposed in the literature. These techniques can be broadly classified into three main categories: Signal distortion techniques, multiple signaling and probabilistic techniques, and coding techniques. Fig. 8 illustrates the various techniques under these three categories discussed in this survey. We will review some key methods under each category and point out the main advantages and disadvantages of each.

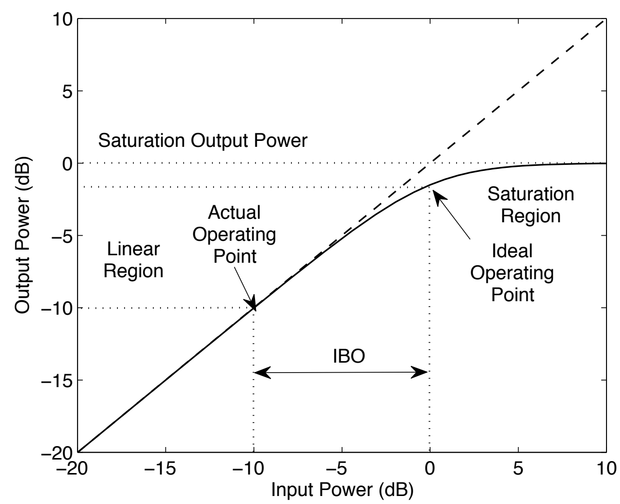


Fig. 7. Typical input power versus output power characteristics curve for a SSPA in dB scale.

A. Signal Distortion Techniques

Signal distortion techniques reduce the PAPR by distorting the transmitted OFDM signal before it passes through the PA. The most well-known signal distortion techniques are clipping and filtering [46]–[53], peak windowing [54], companding [55]–[68], and peak cancellation [69], [70]. These techniques reduce the PAPR significantly but they introduce both in-band and out-of-band distortion, leading to increase in BER. Although the OFDM transmitted signal may

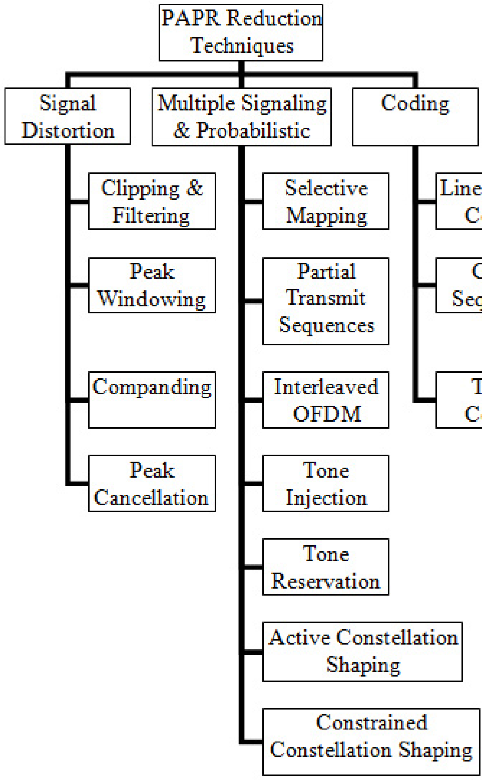


Fig. 8. Taxonomy of PAPR reduction techniques.

have a high PAPR, the high magnitude peaks occur rarely and most of the signal power will depend on low amplitude samples. Therefore, it is possible to remove the high peaks without significantly distorting the signal. Hence, PAPR may be reduced at the expense of some tolerable increase in BER.

1) Clipping and Filtering: One of the simplest signal distortion methods is the method of clipping the high peaks of the OFDM signal prior to passing it through the PA. This method employs a clipper that limits the signal envelope to a predetermined *clipping level* (CL) if the signal exceeds that level; otherwise, the clipper passes the signal without change [71], as defined by

$$T(x[n]) = \begin{cases} x[n] & \text{if } |x[n]| \leq CL \\ CL\, e^{j\angle x[n]} & \text{if } |x[n]| > CL \end{cases}, \tag{20}$$

where $x[n]$ is the OFDM signal, CL is the clipping level and $\angle x[n]$ is the angle of $x[n]$. Clipping is a nonlinear process that leads to both in-band and out-of-band distortions [48]. While the latter one causes spectral spreading and can be eliminated by filtering the signal after clipping, the former can degrade the BER performance and cannot be reduced by filtering [50]. However, oversampling by taking longer IFFT can reduce the in-band distortion effect as portion of the noise is reshaped outside of the signal band that can be removed later by filtering. Filtering the clipped OFDM signal can preserve the spectral efficiency by eliminating the out-of-band distortion and, hence, improving the BER performance but it can lead to peak power regrowth. References [29], [51], [52] propose various repeated clipping-filtering procedures to reduce the overall peak power regrowth. In [53], the authors investigate the effect of clipping on the performance of OFDM systems for a frequency selective fading channel. The impact of clipping on PAPR reduction and channel capacity is studied in [50]. Reference [72] presented a modified repeated clipping and filtering scheme which limits the distortion on each tone of the OFDM to achieve both low PAPR and low BER with fast convergence. In [73], the authors developed an optimized repeated clipping and filtering method which determines an optimal frequency response filter for each iteration using convex optimization. The filter is designed to minimize signal distortion such that the PAPR is below a specified threshold. The authors claimed that the method achieves a desired PAPR reduction after only 1 or 2 iterations, whereas the conventional clipping and filtering method requires about 8 to 16 iterations to achieve a similar PAPR reduction.

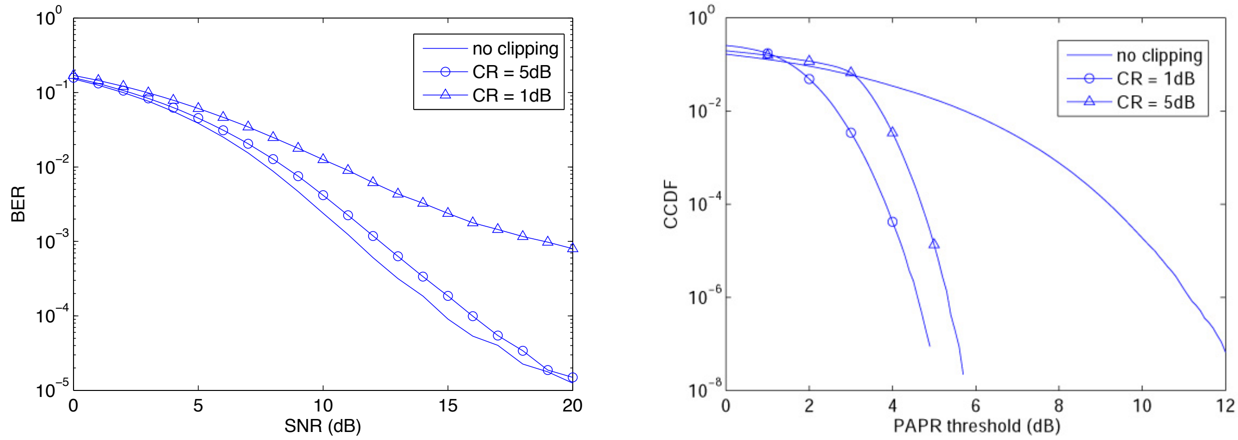
To demonstrate the effect of clipping on BER, we have conducted computer simulations, the results from which are shown in Fig. (9). OFDM symbols of 1024 subcarriers are considered in our simulations with a symbol structure that follows the WiMAX standard in the *down link partial use subcarrier* (DL-PUSC) mode. Data subcarriers are modulated with QPSK data symbols and no coding or any other form of diversity is considered. The SSPA model is used with $p = 2$ and $x_{sat} \approx max|x[n]|$. The *Stanford university interim* (SUI)-1 fading channel model [74] is used and *additive white Gaussian noise* (AWGN) is added. At the receiver, perfect channel estimation is assumed. The simulations are conducted for the OFDM signal without clipping and when clipping is used with a *clipping ratio* (CR) of 1dB and 5dB. The CR is related to the clipping level by the expression

$$CR = 20\,\log_{10}\left(\frac{CL}{E[x[n]]}\right), \tag{21}$$

where $E[x[n]]$ is the average of the OFDM signal $x[n]$. The results presented in Fig. 9 show that as the CR is reduced, the CL is lowered down and more parts of the OFDM signal are clipped and hence, the BER is increasing and the empirical CCDF is decreasing.

2) Peak Windowing: Unlike peak clipping where the peaks that exceed a predetermined threshold are hard-limited, peak windowing limits such high peaks by multiplying them by a weighting function called a window function. Many window functions can be used in this process as long as they have good spectral properties [54]. The most commonly used window functions include Hamming, Hanning and Kaiser windows. To reduce PAPR, a window function is aligned with the signal samples in such a way that its valley is multiplied by the signal peaks while its higher amplitudes are multiplied by lower amplitude signal samples around the peaks. This action attenuates signal peaks in a much smoother way compared to hard clipping, resulting in reduced distortion.

3) Companding Transforms: Companding transforms are typically applied to speech signals to optimize the required number of bits per sample. Since OFDM and speech signals behave similarly in the sense that high peaks occur infrequently, same companding transforms can also be used to



(a) (b)

Fig. 9. (a) BER without clipping and with clipping for different values of CR; (b) empirical CCDF without clipping and with clipping for different values of CR.

reduce the OFDM signal's PAPR [55], [56]. Besides having relatively low computational complexity compared to other PAPR reduction techniques, companding complexity is not affected by the number of subcarriers. Also, companding does not require side information and hence does not reduce bit rate. Their simplicity of implementation and the advantages they offer make companding transforms an attractive PAPR reduction technique. The PAPR reduction obtained by companding transforms comes though with the price of increasing the BER.

Companding transforms can be generally classified into four classes: *linear symmetrical transform* (LST), *linear asymmetrical transform* (LAST), *nonlinear symmetrical transform* (NLST) and *nonlinear asymmetrical transform* (NLAST). Figure 10 depicts the profiles of these four classes. The LST companding transform, denoted by C_{LST}, is given by

$$C_{LST}\left(x[n]\right) = a\,x[n] + b\,, \tag{22}$$

where $0 < a < 1$ is the slope parameter and $b > 0$ is the bias parameter. The LAST companding transform, denoted by C_{LAST}, is defined piecewise by

$$C_{LAST}(x[n]) = \begin{cases} \frac{1}{u}\,x[n] & \text{if } |x[n]| \leq v \\ u\,x[n] & \text{if } |x[n]| > v \end{cases} \tag{23}$$

where $0 < v < \max|x[n]|$ is the threshold level and u is the piecewise slope parameter. For nonlinear companding transforms, many nonlinear functions, for which the inverse exists, can be used. Many companding transforms, which fall under the four classes mentioned above, are presented and studied in the literature.

The use of the μ-law companding transform to reduce PAPR is studied extensively in [57]–[59]. Using the μ-law companding, $x[n]$ is distorted before the PA and the resulting companded signal $x_c[n]$ is expressed as [57]

$$x_c\left[n\right] = \frac{A\ \mathrm{sgn}\left(x\left[n\right]\right)\ \log\left[1+\mu\left|x[n]/A\right|\right]}{\log\left(1+\mu\right)}, \tag{24}$$

where A is a normalization constant such that $0 \leq |x[n]/A| \leq 1$, μ is the companding parameter and $\mathrm{sgn}(x[n])$ denotes

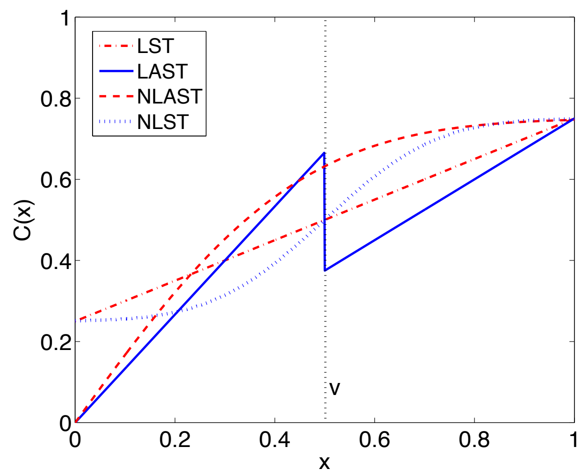


Fig. 10. Profiles of the four companding transform classes

the sign of $x[n]$. If companding transform is applied to the envelope of a complex signal, the sign function of the envelope is always equal to 1. Figure 11 shows the uncompanded envelope of an OFDM signal $x[n]$ (modulated using QPSK with 16 subcarriers and an oversampling factor of 4) and the μ-law companded envelope corresponding to it, with $A = \max|x[n]|$ and $\mu = 8$. The figure shows that μ-law companding transform preserves the high peaks and enhance the low amplitudes of the signal. This process keeps the peak power unchanged and increases the average power, hence reduces the PAPR. At the receiver side, the received OFDM signal $r[n]$ is expanded to retrieve the original signal prior to demodulation. The expanded received OFDM envelope $r_e[n]$ is found according to [57]

$$r_e\left[n\right] = A\,\frac{\exp\left[\frac{r[n]}{A\ \mathrm{sgn}(r[n])}\ \log\left(1+\mu\right)\right] - 1}{\mu\ \mathrm{sgn}\left(r\left[n\right]\right)}. \tag{25}$$

The authors in [57] investigate the effect of companding on the BER performance of the OFDM system in the presence of AWGN and demonstrate that a reasonable symbol error rate

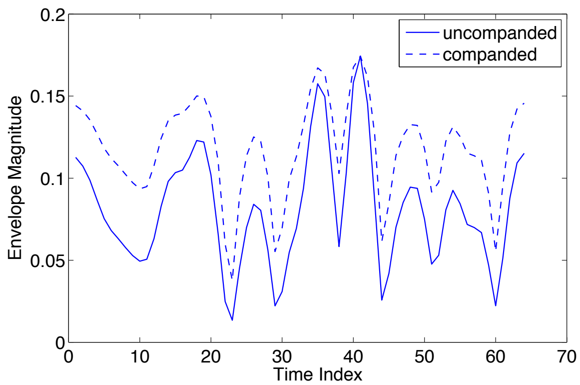


Fig. 11. Uncompanded and companded using μ-law OFDM envelopes.

is obtained by properly choosing companding coefficients. In [60], the symbol error rate due to companding is theoretically analyzed and compared to that of the original uncompanded OFDM. Reference [61] proposes a NLAST to reduce PAPR using the error function transformation given by

$$x_c\left[n\right] = k_1 \,\mathrm{erf}\left(k_2\, x\left[n\right]\right)\,, \tag{26}$$

where k_1 and k_2 are properly chosen coefficients based on the statistics of the transmitted OFDM signal. A proper choice for k_1 and k_2 should project the high peaks of the signal envelope into the nonlinear region of the companding function, while the lower magnitudes are projected onto the linear region. This means that low values will be enhanced, while high peaks are relatively attenuated. The error function transforms the Gaussian distributed OFDM signal into a quasi-uniform distributed one. This action increases the mean power and reduces the peak power, and, consequently, reduces PAPR. A similar NLAST, which uses the error function is proposed in [62] to transform the Rayleigh distributed envelope or the exponentially distributed power of the original OFDM signal into a uniform distribution. These two techniques provide different trade-offs between PAPR reduction and BER performance. Similar work is presented in [75], where a nonlinear companding transform is proposed to transform the OFDM signal into a trapezium distribution. The proposed transform has more flexibility compared to the one in [62] and can perform variably to satisfy the different performance requirements for the system. In fact, the uniformly distributed transform proposed in [62] is a special case of the more general transform in [75]. The trade-off between PAPR reduction and BER performance is controlled by setting the value of a control parameter. Besides the error function, similar NLASTs such as the exponential [63], [64], logarithmic [65], and hyperbolic tangent [66], [67], with proper companding coefficients, are used in the literature to reduce PAPR.

While all the nonlinear functions mentioned above are examples of NLASTs, the nonlinear companding transform presented in [76] assumes a transition point $c\,\sigma$ and a cut-off point A. In the interval $[0, c\,\sigma]$, the PDF of the companded signal is intended to be similar to that of the original OFDM signal x (i.e. Rayleigh distribution); in the interval $[c\,\sigma, A]$ it is intended to have a uniform distribution. From the definition of the PDF $\int_0^\infty f_x(x)dx = 1$, it is found that $A = (c + 1/2c)\,\sigma$. Keeping a constant average power after companding implies that $c = 1/\sqrt{6}$. Hence the companding transform in [76] is given by

$$C(x)=\begin{cases} x & \text{, if } |x| \le \frac{\sigma}{\sqrt{6}} \\ \mathrm{sgn}(x).\sqrt{6}\sigma\left(\frac{2}{3}-\frac{1}{2}\exp\left(\frac{1}{6}-\frac{|x|^2}{\sigma^2}\right)\right)e^{\angle x} & \text{, if } |x| > \frac{\sigma}{\sqrt{6}}, \end{cases} \tag{27}$$

where $\angle x$ is the angle of x. A similar but more flexible scheme is proposed in [77], which transform the statistics of the OFDM signal into a specified distribution while keeping the average transmitted power unchanged. By properly adjusting the transform parameters, more design flexibility in companding form can be achieved to satisfy various system requirements. This scheme can achieve significant PAPR reduction and improved BER performance. The schemes described in [63], [75], [76] can be considered as special cases of the general scheme proposed in [77]. Reference [78] proposed a companding scheme based on a smooth function, called the airy special function. This function was proposed based on the conclusion that companding introduces minimum amount of out-of-band radiation if the companding function is infinitely differentiable. The functions that meet this condition are the smooth functions.

The work presented in [68] proposed a general companding transform design criteria for all the four classes of companding transforms, that facilitate an effective trade-off between PAPR reduction and BER performance. The LAST is characterized by the inflection point and the slopes of the two different segments. Figure 10 shows a LAST function with one inflection point at $x = 0.5$. In [79], the authors claimed that LAST with two inflexion points provides a higher degree of freedom and outperforms the basic LAST with one inflexion point.

To demonstrate the effect of companding transforms on BER, we have conducted computer simulations with similar parameters as in the clipping and filtering simulations, the results for which are shown in Fig. 12. The nonlinear transform, $C(x[n]) = k_1\, tanh(k_2\, x[n])$ with $k_1 = 1/k_2$ is used. It is shown that smaller values of k_1 perform deeper companding, thereby reduce PAPR but increase BER. Reference [80] examines the relative BER performance of the LST and nonlinear companding transforms. It determines the transform that yields the lower BER by comparing the slope parameter of the LST and the derivative of the nonlinear companding transform. Similar work is presented in [81], where the BER performance of LST and LAST are compared and a sufficient condition is derived to ensure the superiority of LAST over LST.

4) Peak Cancellation: In this technique, a peak cancellation waveform is appropriately generated, scaled, shifted and subtracted from the OFDM signal at those segments that exhibit high peaks. The generated waveform is band limited to certain peak cancellation tones that are not used to transmit data [69], [70]. Peak cancellation can be carried out after the IFFT block of the OFDM transmitter as shown in Fig. 13 by subtracting the peak cancellation waveform from the OFDM signal whenever a potential peak higher than a certain threshold is detected. Fig. 14(a) illustrates a segment of an OFDM signal with 256 subcarriers. Figure 14(b) shows the

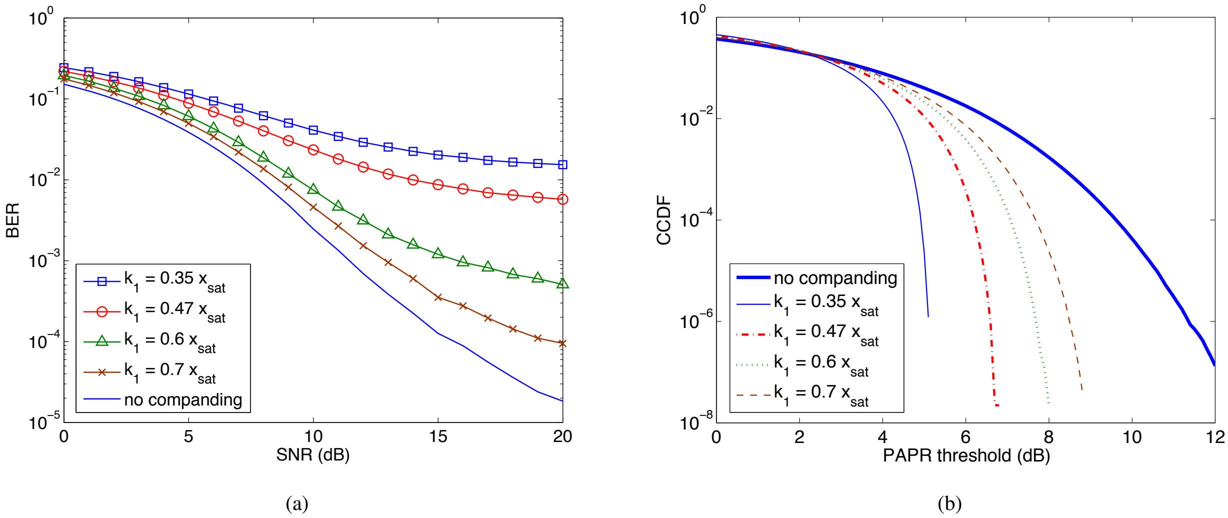


Fig. 12. (a) BER without companding and with companding for different values of k_1 and $k_2 = 1/k_1$; (b) empirical CCDF without companding and with companding for the same values of k_1 and k_2 as in (a).

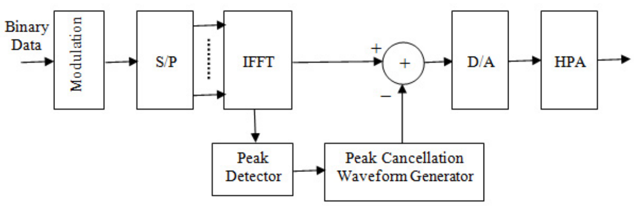


Fig. 13. Peak cancellation in OFDM transmitter.

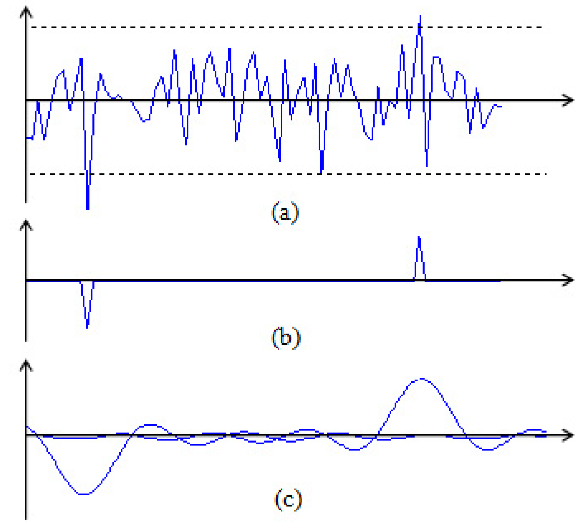


Fig. 14. Peak Cancellation; (a) OFDM signal, (b) identified peaks, (c) scaled and shifted peak cancellation waveform.

detected potential peaks of the signal and Fig. 14(c) shows a randomly chosen $sinc$ function as the peak cancellation waveform. While performing the peak cancellation process, care should be taken not to create new peaks.

B. Multiple Signaling and Probabilistic Techniques

These techniques work in one of two ways. One way is to generate multiple permutations of the OFDM signal and transmit the one with minimum PAPR. The other way is to modify the OFDM signal by introducing phase shifts, adding peak reduction carriers, or changing constellation points. The modification parameters are optimized to minimize PAPR.

1) Selective Mapping: Selective mapping (SLM) is a relatively simple approach to reduce PAPR. The basic idea is to generate a set of sufficient different OFDM symbols $x_{(m)}$, $0 \leq m \leq M-1$, each of length N, all representing the same information as the original OFDM symbol x, then transmit the one with the least PAPR [82], [83]. Mathematically, the transmitted OFDM symbol $\tilde{x}$ is represented as

$$\tilde{x} = \underset{0 \leq m \leq M-1}{\arg\min} \left[PAPR\left(x_{(m)}\right)\right]. \quad (28)$$

The OFDM symbols set can be generated by multiplying the original data block $X = [X_1\ X_2\ \ldots\ X_N]$, element-by-element, by M different phase sequences p_m, each of length N, prior to performing IDFT. These phase sequences are represented as

$$p_m = \left[e^{j\varphi_{m,1}}\ e^{j\varphi_{m,2}}\ \ldots\ e^{j\varphi_{m,N}}\right],\ 0 \leq m \leq M-1, \quad (29)$$

where $\varphi_{m,k}$ takes values between 0 and 2π, excluding 2π, i.e., $\varphi_{m,k} \in [0, 2\pi)$ for $k = 1, 2, \ \ldots\ , N$. Then the modified OFDM symbol $x_{(m)}$, $0 \leq m \leq M-1$, is the IDFT of the element-by-element multiplication of X and p_m

$$x_{(m)} = IDFT\left[X_1\, e^{j\varphi_{m,1}}\ X_2\, e^{j\varphi_{m,2}}\ \ldots\ X_N\, e^{j\varphi_{m,N}}\right]. \quad (30)$$

If QAM symbols are used as input to the OFDM system, this multiplication has the effect of rotating data symbols within the QAM constellation. A block diagram of the SLM technique is depicted in Fig. 15. For implementation simplicity, the phase sequences p_m can be set to $\{\pm 1, \pm j\}$ as these values can be implemented without multiplication. The extent of PAPR reduction achieved depends on the number of generated phase sequences M and the design of these sequences [9].

Information about the selected phase sequence should be transmitted to the receiver as side information to allow the recovery of original symbol sequence at the receiver, which

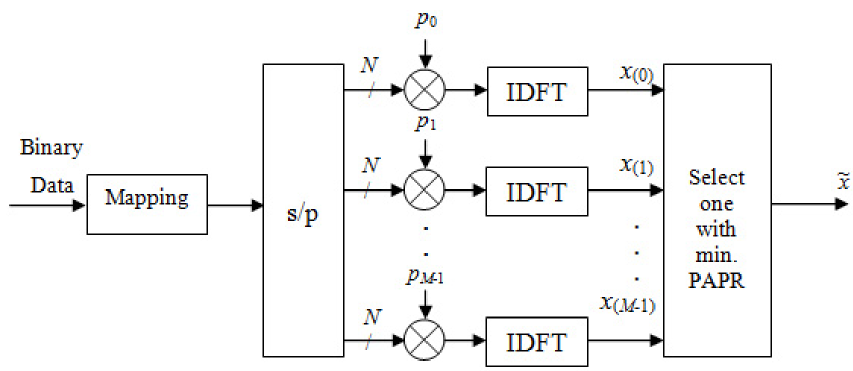


Fig. 15. Block diagram of OFDM transmitter with SLM.

reduces the data transmission rate. SLM needs to transmit $\lfloor \log_2 M \rfloor$ bits as side information, where $\lfloor y \rfloor$ denotes the smallest integer that does not exceed y, and M IDFT operations for each data block. Moreover, the phase sequences p_m, $0 \leq m \leq M-1$, need to be stored at both the transmitter and receiver. An erroneous detection of the side information causes the whole OFDM symbol to be recovered incorrectly. Therefore, strong protection of the side information is required resulting in more loss of data transmission rate. To avoid the need for transmitting side information, several blind SLM schemes have been studied [84]–[88]. Among these, the maximum likelihood decoder is derived for the scheme in [87], which shows the same BER performance as the conventional SLM scheme assuming perfect side information recovery but causes large decoding complexity at the receiver. In [89], a blind SLM scheme with low decoding complexity was proposed in which the side information is embedded into each phase sequence by giving the phase offset to the elements of the phase sequence, which are determined by the biorthogonal vectors for the partitioned subblocks. A maximum likelihood decoder with low decoding complexity was derived for the proposed scheme, which reduces the decoding complexity by $(M-2)/M$ compared with the conventional blind SLM scheme in [87]. Also, it was shown that for QPSK and 16-QAM that the BER of the scheme is almost the same as that of the conventional blind PTS in [87].

The optimization process of selecting the best out of M OFDM signals may not be computationally feasible if the size of the OFDM blocks is large, and more importantly, if the number of phase sequences M is increased, which is required to achieve a substantial PAPR reduction [9]. Many attempts have been made to address the problem of increased computational complexity incurred by the conventional SLM when a substantial PAPR reduction is required. Reference [90] proposed two reduced-complexity SLM schemes. The first scheme substitutes the M-IFFT blocks with one IFFT block and a conversion matrix, to produce the $M-1$ permutations of the OFDM signal from the output of the single IFFT block. The second uses two IFFT blocks with a conversion matrix. Considering an OFDM system with LN-point IFFT block, where L is the oversampling factor, the first scheme reduces the computational complexity required for the original $M-1\,LN$-IFFT blocks to $(M-1) \times 3LN$ complex additions. The second scheme reduces the computational complexity required for the original $M-2\,LN$-IFFT blocks to $(M-2) \times 3LN$ complex additions. This gain in complexity is achieved at the cost of a slight degradation in PAPR reduction for the first scheme, and almost a negligible degradation for the second scheme. One further refinement was presented in [91] to ensure that the elements of the phase rotation vectors, composing the conversion matrix in [90], have an equal magnitude by giving them the form of a perfect sequence. If the elements of the phase rotation vectors all have the same magnitude, the periodic autocorrelation function (PACF) of the corresponding conversion vectors has the form

$$\sum_{m=0}^{N-1} g[m].\, g^*[(m-n)_N] = E.\,\delta[n], \quad 0 \leq n \leq N-1. \quad (31)$$

where $g[m]$ is the mth element of the conversion vector, $*$ is the complex conjugate operation, $(.)_N$ denotes the modulo N operation, E is a constant and $\delta[n]$ is the delta function. Sequences which satisfy the above condition, are defined as perfect sequences. The perfect sequences adopted are compositions of certain base vectors and their cyclically shifted equivalents. To reduce the computational complexity of the conversion process, two constraints are imposed: First, the maximum number of non-zero elements in the base vectors is limited to 4; Second, the non-zero elements in the base vectors must belong to the set $\{\pm 1, \pm j, \pm 1 \pm j\}$. Performing an exhaustive search, three classes of perfect sequence of length LN were identified. However, more perfect sequences can be obtained, if the above constraints are removed. Using these three perfect sequences, three reduced-complexity SLM schemes are proposed and proved to achieve a substantial gain in complexity reduction with PAPR reduction performance loss no more than $0.2dB$. In [92], a pilot phase sequence enabling data recovery without side information and a low complexity decoding scheme are proposed. The BER performance of the proposed scheme is approximately the same as that of the conventional SLM scheme with side information and considerably better than that of the Maximum Likelihood decoding scheme. In addition, the computational complexity of the proposed decoding scheme is much lower, as compared to the maximum likelihood decoding scheme. In [93], the authors proposed a low-complexity SLM scheme

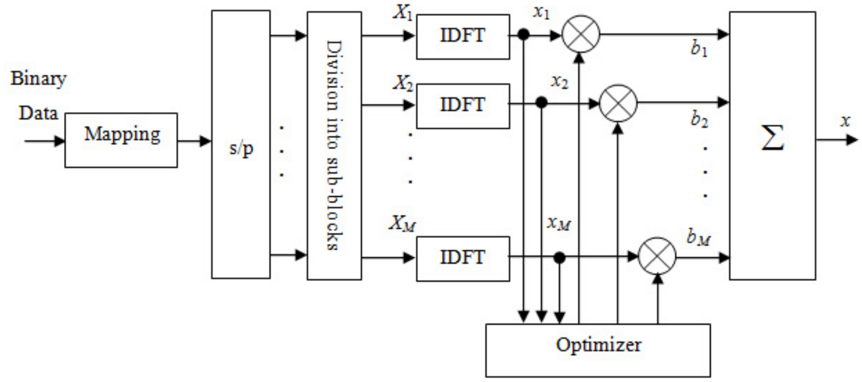


Fig. 16. Block diagram of OFDM transmitter with PTS.

which generates alternative OFDM signals by adding mapping sequences to the OFDM signal in time domain. The proposed scheme considerably reduces the computational complexity without sacrificing BER and PAPR reduction performance. In [94], the authors developed a set of conversion matrices for the SLM scheme based on some periodic properties of the IFFT matrix. Candidate signals are obtained via multiplying the time-domain OFDM signal by the conversion matrices. To reduce the complexity, both real and imaginary parts of the conversion matrices were restricted to the values $\{0, \pm 1\}$. Therefore, the generation of candidate signals involves no complex multiplications. For an OFDM system with N sub-carriers and L oversampling factor, the scheme uses one LN-point IFFT and $3LN$ complex additions to generate other candidate signals. Unfortunately, the number of valid candidate signals is restricted to 12, leading to a strictly limited PAPR reduction performance. This scheme is modified in [95] by dividing the frequency-domain signals into multiple sub-blocks to increase the number of the valid conversion matrices, and thus more candidate signals are available for PAPR reduction. By applying this scheme, the number of candidate signals can be increased from 12 to 28 and 128 for the two-subblock and four-subblock cases, respectively.

Another possible solution is the IFFT manipulation technique based on Radix-2 decimation in time IFFT [96], [97]. This method divides the decimation in time based IFFT into a common part and a remaining part, where the sum of the number of stages in both part is $\log_2 N$. In [98], a similar decimation in frequency IFFT scheme is proposed to reduce the computational complexity. However, the problem with these methods is the trade-off between the computational complexity and the PAPR reduction capability. Reference [99] also suggested decimation in frequency IFFT scheme together with an interleaver and butterfly ensemble to generate multiple candidates. Orthogonality in the frequency domain is kept and better PAPR reduction performance is achieved without additional complex multiplications. In [100], additional alternative OFDM symbols are generated through linear combinations of other alternative OFDM symbols. Therefore, the additional computational complexity due to the IFFT operations can be reduced while keeping the PAPR reduction performance similar to that of the conventional SLM scheme.

The authors in [101] proposed a scheme which produces OFDM sequences by rotating the symbol phase using multiple all-pass filters instead of the multiple complex multiplication modules and IFFT modules employed in the conventional SLM scheme. This scheme avoids using the multiple IFFT modules that incur a heavy computational burden at the transmitter, thereby reducing the computational complexity. The reduction in complexity is however achieved at the cost of a slight degradation in PAPR reduction performance. For example, the proposed scheme with 8 first order all-pass filters for 2048 subcarriers OFDM system reduces the number of required multiplications by 69.2% and additions by 63.1% at a sacrifice of only 0.25 dB PAPR increase compared to the conventional SLM scheme with 8 IFFT modules [101].

2) Partial Transmit Sequence: In *partial transmit sequence* (PTS), an input data block of length N is partitioned into a number of disjoint sub-blocks. The IDFT for each one of these sub-blocks is computed separately and then weighted by a phase factor. The phase factors are selected in such a way as to minimize the PAPR of the combined signal of all the sub-blocks [102]–[107]. Figure 16 shows a block diagram of the OFDM transmitter with PTS technique. Let an input data block, $X = [X_1 \; X_2 \; \ldots \; X_N]$ be partitioned into M disjoint sub-blocks, $X_m = [X_{m,1} \; X_{m,2} \; \ldots \; X_{m,N}]$, $1 \leq m \leq M$, such that any two of these sub-blocks are orthogonal and X is the combination of all the M sub-blocks

$$X = \sum_{m=1}^{M} X_m. \tag{32}$$

Then the IDFT for each sub-block, x_m, $1 \leq m \leq M$, is computed and weighted by a phase factor $b_m = e^{j\varphi_m}$, where $\varphi_m = [0, 2\pi)$, $1 \leq m \leq M$. The objective now is to select the set of phase factors, b_m's that minimizes the PAPR of the combined time domain signal x, where x is defined as

$$x = \sum_{m=1}^{M} b_m \, x_m. \tag{33}$$

In the process of selecting the optimum phase factors, search is usually limited to a finite number of elements to reduce search complexity [9]. Assume that the set of allowed phase factors is defined as $b_m = e^{\frac{j2\pi k}{W}}$, where $k = 0, 1, \ldots, W-1$, and W is the number of allowed phase factors. The first phase factor b_1 can be set to 1 without any loss of performance, therefore, $M-1$ phase factors are to be found by an exhaustive search [9]. Hence, W^{M-1} sets of phase factors are searched to find the optimum one. The reduction in PAPR attainable depends on M and W. On one hand, the larger is the number of sub-blocks M, the greater is the reduction in PAPR. On the other hand, the search complexity is increasing exponentially with M. In addition, M IDFT blocks are needed to implement the PTS scheme, requiring $\lfloor \log_2 W^{(M-1)} \rfloor$ bits of side information to be transmitted [9]. Another factor that affects PAPR is the type of partitioning employed. Three kinds of partitioning schemes are prevalent: adjacent, interleaved, and pseudo-random partitioning [103]. Of these, pseudo-random partitioning has been found to be the best choice.

In the literature, various techniques are suggested to reduce the computational complexity of the PTS scheme and yet maintain a substantial reduction in PAPR. Reference [106] proposes a scheme that updates the set of phase factors iteratively till PAPR drops below a specified threshold. In [104], a simple iterative flipping algorithm is proposed to reduce the complexity of the PTS method by converging to a sub-optimal choice of the phase factors. The phase factors are initially set to 1 and b_1 remains 1 while the values of the other phase factors are chosen among all W possible values. In the first iteration, the second factor b_2 is changed and the PAPR is computed. Then, the value of b_2 which achieves the lowest PAPR is chosen as part of the final set of phase factors. The algorithm continues to work in the same manner till all phase factors are explored. In [107], algorithms are described for combining partial transmit sequences with reduced complexity and very little performance degradation. In [108] a gradient descent search for phase factors is proposed, which reduces search complexity at the expense of some performance degradation too. In [109], the authors proposed a PTS scheme based on listing the phase factors into multiple subsets table and utilizing the correlation among phase factors in each subset, in order to reduce the computational complexity. Reference [110] proposed a sign selection technique where a set of subcarrier signs is selected to significantly reduce the PAPR statistics for OFDM signals. To determine a good set of subcarrier signs with improved PAPR statistics while reducing the computational complexity of an exhaustive search over all combinations of sign patterns, the use of the *quantum-inspired evolutionary algorithm* (QEA) was proposed. QEA is an effective population-based search algorithm that solves various combinatorial optimization problems. Other combinatorial optimization algorithms such as the artificial bee colony algorithm [111] and parallel tabu search algorithm [112] have been used to efficiently search a good subset of phase rotating vectors for the PTS scheme to reduce the complexity. An optimal search has been proposed in [113], where the computational complexity of the conventional PTS scheme is reduced by restricting the search among the alternative sequences inside a sphere by using sphere decoding algorithm.

A PTS scheme with low computational complexity is proposed in [114], where two search steps are employed to find a subset of phase rotating vectors with good PAPR reduction performance. In the first step, sequences with low correlation such as Kasami sequences [115] or quaternary sequences of family A [116] are used as initial phase rotating vectors for PTS scheme. In the second step, local search is performed based on the initial phase vectors to find additional phase rotating vectors with good PAPR reduction performance.

In [117], the authors propose a W-way tree based PTS scheme with low complexity, where the nodes in the tree correspond to phase factors and layers correspond to subblocks. The calculation of candidate signals utilizes the structure of the tree by combining layers and weighting factors on the paths from the root to the leaves. The scheme reduces complexity dramatically, whereas the PAPR reduction capability is kept as that achievable by the conventional PTS.

Reference [118] proposed a PTS system based on selecting a phase sequence that maximizes the similarity between the input and output of the power amplifier model using the cross correlation as an optimized metric. A similar approach was proposed in [119] in which the distortion introduced by the nonlinearity of the power amplifier is predicted and then used to select the optimal phase sequences for the PTS or SLM methods. The adopted distortion metrics were the *distortion-to-signal power ratio* (DSR) and the *peak interference-to-carrier ratio* (PICR), which are predicted at the transmitter side after the IFFT.

3) Interleaved OFDM: One way to generate multiple OFDM signals that carry the same information is to use interleavers [120]–[125]. This technique is similar to SLM but interleavers are used instead of phase sequences. An interleaver is a device that operates on a block of symbols and permutes or reorders them in a specific way. To achieve a substantial decrease in PAPR, multiple interleavers are used to generate a set of sufficient different permutations from the original data block. Permutations can be performed on symbols or bits. The IDFT is computed for each one of the different permutations separately to generate multiple OFDM signals. Then the OFDM signal with the smallest PAPR is chosen for transmission. Fig. 17 shows a block diagram of the interleaved OFDM transmitter. To include the original data block in the PAPR comparison of M different OFDM signals, $M-1$ interleavers and M IDFT blocks are required. Also $\lfloor \log_2 M \rfloor$ side information bits have to be transmitted to the receiver since the receiver must know which interleaver was used to generate the selected signal for transmission. The permutation indices have to be stored in both the transmitter and receiver. If the different permutations of the data block are uncorrelated, the CCDF of the interleaved OFDM signal can be derived based on (11) as [125]

$$CCDF\left[PAPR\left(x^n\left(t\right)\right)\right] = \left(1 - \left(1 - e^{-\delta}\right)^{\alpha N}\right)^M, \quad (34)$$

where $x^n(t)$ is the n^{th} OFDM symbol, α is the oversampling factor, and N is the size of the OFDM symbol. Equation (34) shows smaller CCDF for interleaved OFDM signal compared to original OFDM signal defined in (11).

4) Tone Injection: In this technique the constellation size is increased so that each point in the original complex plane con-

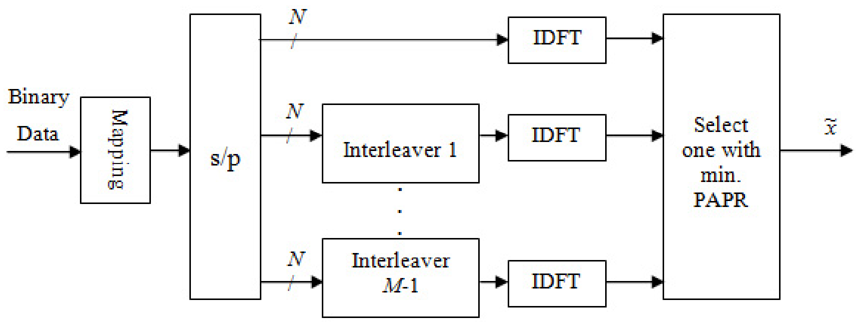


Fig. 17. Block diagram of Interleaved OFDM transmitter.

stellation is mapped onto several other points in the expanded constellation prior to IDFT processing [126]–[129]. This extra degree of freedom facilitates a reduction in PAPR. Substituting a point in the original constellation for one in the expanded one is equivalent to injecting a tone with proper frequency and phase to the OFDM signal [9], and hence the name of this method. If a square QAM constellation is used with the original constellation size as M, and its points are spaced by d, then in order not to degrade the BER performance of the OFDM signal by using tone injection technique, the spacing between each original point in the original constellation and its equivalent points in the expanded constellation should be

$$D = \rho\, d\sqrt{M}, \tag{35}$$

with $\rho \geq 1$. The k^{th} QAM symbol on a single subcarrier with multiple constellation points is represented as

$$\tilde{X}_k = X_k + p_k\, D + q_k\, D, \tag{36}$$

where X_k is the k^{th} original QAM symbol, p_k and q_k are integer numbers used to change the real and imaginary parts of X_k respectively, and they are chosen to reduce the PAPR [128]. Figure 18 graphically illustrates the tone injection procedure for 16-QAM constellation, where the original point labeled A maps to one of A_i's, $i = 1, \ldots, 8$, from which possible values of p and q can be derived. Each one of these points is spaced by a distance D from A, where D is known to both transmitter and receiver. One of these redundant points A_1 to A_8 is chosen for transmission in order to reduce PAPR of the transmitted signal. The PAPR reduction attainable using this method depends on the value of ρ in (35) and the number of modified symbols in the data block [9]. At the receiver, the symbol recovery process is carried out by applying a modulo-D operation to the received modulation symbol, which is then followed by the decoding process. Tone injection technique requires no side information at all and, consequently, there is no loss of bit rate. Also the complexity added at the receiver is negligible since only two modulo-D operations are required for the real and imaginary parts of the received symbol. Despite the advantages this method offers, it is weighed down by the increased complexity of the transmitter. Tone injection technique can reduce PAPR significantly at the expense of some increase in average signal power due to the use of enlarged signal constellation. It is possible to minimize this increase in power by appropriately remapping the signaling points or by carefully choosing the redundant constellation. References [130] and [131] proposes a tone injection technique with hexagonal constellation to achieve PAPR reduction with only a small power increase compared to the QAM constellation. It is possible to pack more regularly spaced signaling points using a hexagonal constellation than can be done with a QAM constellation of the same area. This extra degree of freedom can be exploited to reduce PAPR. At the same time, the power increase is less than that of the QAM constellation since signaling points in the hexagonal constellation will have average magnitude smaller than the corresponding average of the QAM constellatation. A factor that influences the power increase is D. If D increases, the spacing between the redundant and original constellations increases, and hence the average transmit power increases. On the other hand, if D decreases, the original and redundant constellations move closer to each other, but not closer than a certain minimum distance separation between signaling points. Unfortunately, in this case the outermost points of the original constellation may have nearest neighbors that differ in more than one bit position. Consequently, the symbol error rate and hence the BER will increase. This undesirable effect can be alleviated by increasing D so that a sufficient separation between the original and redundant constellations is kept to ensure nearest neighbor points differ in just a single bit. This solution though increases the average transmit power.

5) Tone Reservation: Tone reservation (TR) is a technique in which a subset of tones is reserved for PAPR reduction. Due to their low SNR, these tones carry no information data. A structured time domain vector c is added to the OFDM signal x to change its statistical distribution to help reduce PAPR [9], [126]–[128]. If the added frequency domain vector is denoted as C, then the new OFDM signal is defined as [11]

$$\tilde{x} = x + c = IDFT\left(X + C\right), \tag{37}$$

where $X = [X_0\; X_1\; \ldots\; X_{N-1}]$ and $C = [C_0\; C_1\; \ldots\; C_{N-1}]$ are restricted to lie in disjoint frequency subspaces. The problem now is to find c that minimizes the maximum peak value of the new OFDM signal, i.e.

$$\min_c \|x + c\|_\infty = \min_C \|x + IDFT\left(C\right)\|_\infty. \tag{38}$$

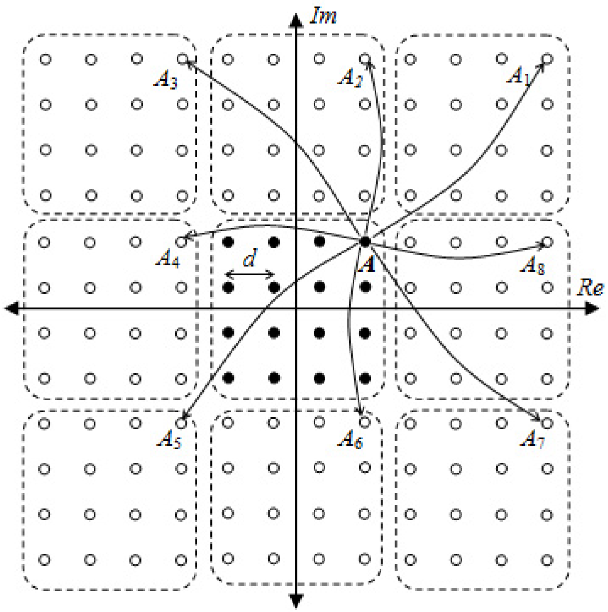


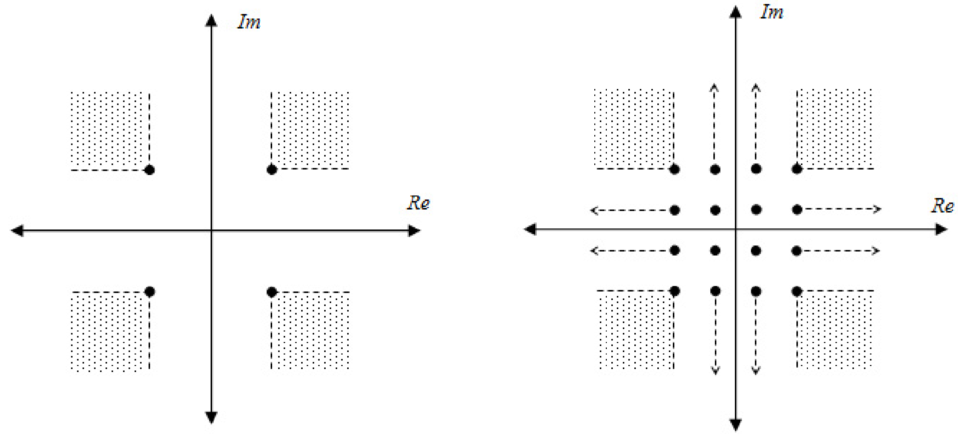
Fig. 18. Tone injection technique for 16-QAM constellation.

If there are L unused tones in the OFDM signal, and correspondingly a set of L nonzero subcarriers in C, i.e. $C_n \neq 0$, $n \in \{i_1, i_2, \ldots, i_L\}$, then $X_n = 0$, $n \in \{i_1, i_2, \ldots, i_L\}$. The L nonzero positions in C are called *peak reduction carriers* (PRCs). Since subcarriers are orthogonal, these PRCs have no distortion effect on the information bearing subcarriers. The values of C_n, $n \in \{i_1, i_2, \ldots, i_L\}$ are determined through a convex optimization, which can be cast as a *linear programming* (LP) problem of complexity $O\left(LN^2\right)$, which is very high [9], [126], [128]. The extent of PAPR reduction in TR technique depends on the number of reserved tones, their locations in the frequency domain vector C, and the complexity of optimization. The locations of PRCs need to be known by the receiver and, therefore, are transmitted as overhead information. For OFDM systems with small N, the number of reserved tones will not be negligible leading to loss in data rate. Reference [127] shows that by optimizing the signal-to-clipping noise ratio instead of PAR and by using a gradient algorithm with optimization done on the time domain vector c, it is possible to attain the same performance as the general LP with a reduced complexity of $O(N)$. Reference [132] proposes a simple and computationally efficient TR algorithm in which a few frequency domain tones are reserved to generate a time domain Gaussian pulse to be used as a peak cancellation signal while minimizing the occurrence of secondary peaks. Another scheme is proposed in [133], where one of the PRCs should have a phase close to one of the four possibilities $\{\varphi, \varphi + \pi/2, \varphi + \pi, \varphi - \pi/2\}$ at the peak time sample location and φ is the phase of the peak time domain sample. This results in no complex multiplications or divisions. In addition to modifying the PRCs, it is also possible to modify the information bearing subcarriers [134]. However, the optimization procedure is more complex since all the N tones are permitted to change. Reference [135] develops an efficient computational algorithm for this case. Reference [136] exploits the use of unused subcarriers as well as the phase information of pilot subcarriers in OFDM systems to reduce the PAPR. This scheme is extended in [137] to include the use of phase information of frequency domain signal to further reduce PAPR. Reference [138] exploits the use of *pseudo noise* (PN) code based PRCs selection sequence to minimize the amount of overhead information sent between the transmitter and receiver, while achieving PAPR reduction. The sequence is generated by applying starting points determined by bit reversing and gradually-increasing offsets. PN code or maximal length sequence is generated by *linear feedback shift registers* (LFSR). This scheme required virtually no overhead since that the only overhead information sent between the transmitter and receiver is the initial state of the LFSR.

In [139] the authors proposed a TR design with the objective to reduce the envelope fluctuations of the OFDM time-domain signal. The method mainly reduces both in-band and out-of-band distortion after the nonlinearity of the power amplifier. However, the method also reduces the PAPR since it is very likely that OFDM signals with high peaks have high variances. The problem was defined as a convex optimization problem with equality constraint, which can be solved iteratively with fast convergence.

Reference [140] implemented the clipping control TR method, which could obtain a moderate PAPR reduction with little degradation in BER performance. An improved version called the adaptive scaling TR algorithm [141] was proposed later. The idea is to generate peak cancelation signals by time-domain clipping and frequency-domain filtering until the desired PAPR reduction is achieved. However, this method required many iterations with increased computational complexity. Faster convergence was achieved later in [142], where the original clipping control TR is modified by implementing least squares approximation in the optimization process. Therefore, with almost two iterations, peak-cancelation signals are found that achieve comparable PAPR reduction to the original clipping control TR scheme. Although TR-based clipping is attractive for practical implementations, determining an optimal clipping level is difficult and cannot be determined in the initial stage. To overcome this problem, [143] proposed an efficient scheme based on genetic algorithm with low computational complexity for searching a suboptimal PRC set. An adaptive amplitude clipping algorithm was also developed to obtain good PAPR reduction performance regardless of the initial target clipping level. Reference [144] proposed efficient approximate algorithms and their implementation structures for implementing the TR method. In the proposed structure, a binary search algorithm was proposed for the PRC-based Clipping, and the FFT is replaced with an approximate DFT in which complex multiplications are replaced with shift operations. In addition, the IFFT is minimized by reducing the *first input first output* (FIFO) size and the corresponding twiddle-factor lookup table. Compared to the conventional structure, the proposed structure reduced hardware complexity and power consumption by approximately 62.2% and 58.4%, respectively, while achieving almost the same performance as that of the conventional structure.

Inspired by the efficiency of the *cross-entropy* (CE) method [145] for finding near-optimal solutions in huge search spaces,



(a) (b)

Fig. 19. Active constellation extension technique; (a) QPSK constellation, (b) 16-QAM constellation.

[146] proposed the application of the CE method to search the optimal PRC set to reduce the computational complexity. The main idea of CE is to maintain a distribution of possible solutions rather than one simple solution candidate per step as do most optimization algorithms. Then, the method updates the parameters of the distribution to produce better samples in the next iteration.

Reference [147] proposed a computationally efficient PAPR reduction scheme, which reduces the peak sample of each parabolic pulse using the truncated kernel signal generated from the IFFT of the shaped PRC set. The scheme repeats peak canceling in the time domain only without iteratively performing IFFT and FFT. The application of this scheme to ACE can reduce the number of required iterations and therefore the computational complexity. The scheme improves both the BER and out-of-band radiation while maintaining the PAPR reduction performance.

6) Active Constellation Extension: Active constellation extension (ACE) or *active set extension* (ASE) is a PAPR reduction technique where the modulation constellation over active subcarriers in the OFDM data block is modified or predistorted such that the PAPR of the data block is reduced without significantly degrading the BER performance [148], [149]. In this modification process, some of the outer constellation points are dynamically extended toward the outside of the original constellation. Let us examine the two cases of QPSK and 16-QAM modulation constellations as shown in Fig. 19. For the QPSK case, the constellation points are located at the corners of the shaded regions. These shaded regions are called feasible regions because if a conventional constellation point is reassigned to another location inside the corresponding feasible region, the minimum Euclidean distance between the newly assigned constellation point and any other constellation point in other feasible regions is guaranteed not to be less than the minimum distance between the conventional constellation points. Also, the increase in average transmit power due to constellation modification is fairly small and, consequently, there will be no significant degradation in BER performance [148]. For the 16-QAM constellation, the exterior corner constellation points have their corresponding feasible regions, while for the exterior non-corner constellation points the feasible regions are straight lines starting at the point itself and extend to infinity [150]. Unlike [148] and [149] which use clipping, the algorithm proposed in [151] predistorts a set of input frequency-domain symbols per block using a simple metric to reduce PAPR. The metric measures the contribution of each symbol to the output samples of the IFFT block with large values. Then, symbols with largest positive metrics are predistorted by scaling only the amplitude or the real and/or the imaginary parts. Similar predistortion was proposed in [152] by minimizing the peak value of the multicarrier signal over the signs and amplitudes of the subcarriers.

ACE technique simultaneously decreases BER slightly while substantially reducing PAPR. Furthermore, there is no side information required and therefore no data rate loss. However, the method increases the average transmitted signal power and finds limited applicability to modulation schemes with a large constellation size [9].

7) Constrained Constellation Shaping: In this method, the modulating constellation points over the data subcarriers in the OFDM symbol are modified within an allowed error to reduce the PAPR. The price paid is some acceptable degradation in the BER performance. An interesting feature in this solution is the possibility to formulate it as a convex optimization problem. The authors in [153] show that the OFDM signal with global minimum PAPR, subject to constraints on the allowed constellation error and the free subcarriers power, can be efficiently computed using convex optimization.

Let $c_0 \in \mathbb{C}^N$ be an ideal constellation and $c \in \mathbb{C}^N$ be the actual transmitted constellation as demonstrated in Fig. 20 for a 16-QAM constellation. The average *error vector magnitude* (EVM) of c is a function of data subcarriers and is defined as

$$EVM = \sqrt{\frac{\sum_{i=1}^{d} \parallel c_i - c_{0i} \parallel}{d\, P_0}}, \tag{39}$$

where d is the number of data subcarriers and P_0 is the average power of the modulation scheme. The EVM constraint can be

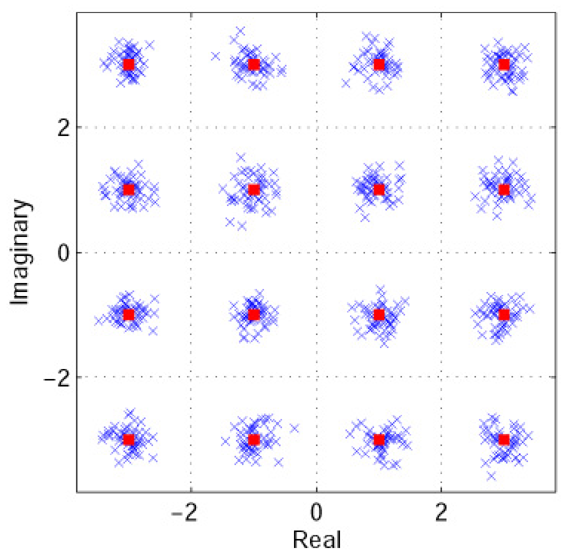


Fig. 20. Ideal (■) and modified (×) 16-QAM constellation.

expressed as

$$\| S\ (c - c_0)\ \| \leq EVM_{max}\ \sqrt{d\,P_0}\ , \tag{40}$$

where EVM_{max} is the maximum allowed EVM and $S \in \mathbb{R}^{N \times N}$ is a diagonal matrix with $S_{ii} = 1$ for data subcarriers and $S_{ii} = 0$ for other subcarriers. EVM_{max} is empirically determined based on the desired BER and is usually specified in standards. If the transmitted signal satisfies the EVM constraint, then the received data will have the desirable BER.

C. *Coding Techniques*

The inherent capability to provide error detection and correction for some coding schemes makes them a desirable choice to perform PAPR reduction by modifying these coding schemes to provide both functions with an acceptable extra complexity. We describe here some of the PAPR reduction coding techniques available in the literature.

1) Linear Block Coding: Instead of dedicating some bits of the codeword to enhance BER performance, these bits are now dedicated to reduce PAPR. The goal is to choose the codewords with low PAPR for transmission. A simple *linear block coding* (LBC) scheme was proposed in [154], where 3 bits are mapped into 4 bits by adding a parity bit. In [155], a simple rate-$3/4$ cyclic code is used for any number of subcarriers that is a multiple of 4 to reduce PAPR by more than 3 dB. Similar performance with less complexity was obtained in [156] using the proposed *sub-block coding* (SBC) scheme, where long information sequences are divided into sub-blocks, and an odd parity bit is added to each sub-block. The position of the added parity bit is optimized to further reduce PAPR. Moreover, instead of one coding scheme, two coding shemes can be used to encode each sub-block, and the combination of the coded sub-blocks can be optimized to lower PAPR. Both methods require the transmission of side information to the receiver to indicate the locations of the parity bits or the coding schemes used to encode each sub-block. This means that the bandwidth efficiency of the system will be reduced. In [157], a combined $(8, 4)$ LBC is used to provide error control capability and reduce PAPR of a multicarrier modulation by 4 dB. In [158], another simple LBC is proposed based on the observation that regardless of the number of subcarriers, codewords with equal odd and even bit values have high PAPR. Therefore, PAPR can be reduced easily by eliminating these codewords by adding a simple bit code. Reference [159] proposes a rate-$1/2$ linear coding scheme to achieve the minimum PAPR and provide error correction capability for an OFDM system utilizing QPSK modulation and four subcarriers. The idea is to use a generator matrix to encode 4 bits into codewords of 8 bits (rate $1/2$), and then multiply the resulting 8 bits by a phase rotator vector of length 8 to have the encoded codewords. Both the generator matrix and the phase rotator vector are chosen to reduce PAPR. Similar precoding schemes using proper generator matrices to reduce the PAPR of OFDM signals has been presented in the literature [160]. However, precoding schemes increase the error probability of OFDM systems. By assuming that the transmitter has the knowledge of the channel information and the receiver filter, the authors in [161] proposed a systematic procedure to choose a precoding matrix for achieving the required PAPR reduction performance at the minimum error probability over AWGN channels.

A low complexity *complement block coding* (CBC) scheme is proposed in [162], [163], where few complement bits are inserted in the middle of the information bits to form a codeword with reduced PAPR. In [164], standard arrays of linear block codes are used for PAPR reduction. This scheme may be regarded as a modified version of SLM, in which the coset leaders of a linear code are used for scrambling, hence no side information is required to be transmitted and the received signal can be decoded by syndrome decoding. Another study in [165] combines the use of weighting functions such as Gaussian or raised cosine, introduced by [166], and the product codes based on linear BCH block codes, introduced by [167], to jointly optimize PAPR and BER of OFDM signals. The use of *low-density parity-check* (LDPC) codes to mitigate the high PAPR problem was proposed in [168].

In [169], the authors describe a coding method to reduce the PAPR in coded OFDM systems. For an alphabet of size q, the scheme adds α symbols to a k-symbol data word and the resulting message word is encoded using $(n, k + \alpha)$ linear block code. For each message word of length k, there will be q^α new message sequences of length $k + \alpha$. Then, the coded sequence with minimum PAPR is used. This scheme requires q^α IFFT blocks. To reduce the hardware required and allow the use of only one IFFT block, an adaptive sequential search is used. One combination of the appended α symbols is tested at a time, and kept if the PAPR is below a specific threshold. Otherwise, a different combination is used at a time till the PAPR threshold is met.

In [170] the authors proposed the use of fountain codes [171] to control the PAPR of OFDM signals. The first practical realization of fountain codes was the LT codes [172] and later a further enhancement was proposed by the Raptor codes [173]. The motivation behind this scheme was the fact that the best fountain-coded OFDM packets can be generated with a low PAPR. The basic idea is to encode a sequence of k OFDM symbols into a potentially limitless stream of parity-check symbols. A receiver can reconstruct the original data once it receives a sufficient number of packets. This code works well in practice since its encoding and decoding cost is linear over k and the coding overhead is small.

2) Golay Complementary Sequences: Golay Complementary Sequences [174] can be used as codewords to modulate the subcarriers of the OFDM system, yielding a signal of PAPR with an upper bound of 2 [175], [176]. Golay Complementary Sequences satisfy the property that their out of phase autocorrelation function is zero. Let $a = [a_0\ a_1\ \ldots\ a_{N-1}]$ be a bipolar sequence such that $a_i \in \{+1, -1\}$. Define the aperiodic autocorrelation function of the sequence a as follows

$$\rho_a(k) = \sum_{i=1}^{N-k-1} a_i\, a_{i+k}\ ,\ 0 \le k \le N-1. \tag{41}$$

Let b be another bipolar sequence similar to a. Then the pair (a, b) is called a Golay Complementary Pair if it satisfies

$$\rho_a(k) + \rho_b(k) = 0\ ,\ \forall k \neq 0. \tag{42}$$

Each member in this pair is called *Golay complementary sequence* (GCS). Large set of binary length 2^m Golay pairs can be found from certain second-order cosets of the first-order Reed Muller codes [177]–[179]. Schemes that combine the block coding approach and the use of GCSs provide a powerful way for incorporating both the capabilities of error correction and control over PAPR. However, the usefulness of such approach is limited to OFDM systems with small number of subcarriers. For OFDM systems with large number of subcarriers, this approach results in transmission rate loss and increased computational complexity due to the exhaustive nature of the search required to find good codes.

Several recent papers studied the use of Golay sequences and Golay complementary pairs with different constellations to reduce PAPR. The construction of Golay sequences based on STAR-16-QAM constellation is studied in [180] and an upper bound for PAPR is derived and found to be 2. The construction of STAR-16-QAM constellation is done using scaled set sum of two QPSK constellations. Another constellation, called the Asterisk 16-QAM is proposed in [181] to reduce PAPR when the data is encoded using Golay sequences and Golay complementary pairs. This constellation maintains the same PAPR upper bound of the STAR-16-QAM while enhancing the mean symbol error rate. A generalized paradigm for the Asterisk 16-QAM constellation is presented in [182], where a new constellation family controlled by a single parameter is proposed. In [183], Golay sequences are used as the building blocks to construct a new family of 64-QAM sequences that are not necessarily Golay sequences and that could outperform other existing OFDM sequences in terms of both PAPR and code rate.

3) Turbo Coding: One way to exploit turbo codes for PAPR reduction is to implement the SLM approach with the candidates generated by a turbo encoder with various interleavers [184]–[186]. The selected candidate for transmission is the one with the least PAPR. Since no side information is required, this method avoids the BER performance degradation resulting from incorrect recovery of side information in conventional SLM approaches. Beside PAPR reduction benefit, it is possible to make use of the error correction capabilities of Turbo coding.

Another approach was proposed in [187] based on the dual *bose-ray-chaudhuri* (BCH) codes. Dual BCH code has some appealing PAPR properties [188]. Specifically, [189] shows that the IFFT of the codewords of this code exhibits low envelope fluctuations and therefore low PAPR. However, the potential of this code for PAPR reduction is restricted due to the lack of a practical decoder and the large performance gap to the Shannon limit. To solve these problems, the proposed scheme in [187] constructed a new code with favorable PAPR properties based on dual BCH codes, exploited this code in a turbo structure to obtain an adequately low BER and reduce the gap between the performance and Shannon limit, and developed the associated decoder based on the *maximum a posteriori* (MAP) criterion. The bounded PAPR of the coded OFDM symbol using this scheme is guaranteed. Also, it was shown that among the dual BCH codes only those with one-bit error correction capability have a PAPR considerably lower than that of the other error correcting codes.

VI. Complexity Analyses

To offer a better understanding of the complexity requirements for PAPR reduction methods, detailed computational complexity analyses are carried out for four PAPR reduction schemes: one signal distortion scheme (clipping and filtering), two multiple signaling schemes (SLM and PTS) with suboptimal solutions and one optimization-based scheme (CCS) with optimal solution. To demonstrate the differences, we chose schemes which fall into three groups in terms of computational complexity: simple, moderate and complex. The computational complexity is quantified by the number of real multiplications and additions. Since the computational complexities of multiplication and addition operations are almost equivalent to those of division and subtraction, respectively, we refer here to divisions as multiplications and to subtractions as additions.

IFFT is used at the transmitter and sometimes multiple IFFT blocks are required, hence we start by calculating the complexity of IFFT block. For a radix-2 decimation-in-time IFFT or FFT implementations, $\frac{N}{2}\log_2(N)$ complex multiplications and $N\log_2(N)$ complex additions are required for N-points complex data sequence. To quantify these complex operations in terms of real multiplications and additions, note that, each complex multiplication requires four real multiplications and two real additions and each complex addition requires two real additions. With simple calculations, it is possible to show that IFFT or FFT block requires $2N\log_2(N)$ real multiplications and $3N\log_2(N)$ real additions.

A. Clipping and Filtering

To implement clipping, the power of the time-domain OFDM signal is compared to a threshold. This comparison requires N subtractions. Calculating the power of N complex-valued samples requires $2N$ real multiplications and N real additions. After clipping, filtering is necessary to eliminate the out-of-band radiation. Assuming a *finite impulse response* (FIR) filter of length L is used the filtering process requires approximately $NL+L^2+L$ complex multiplications and complex additions. For an OFDM system with large N, $L << N$, and the complexity is further approximated by NL. This is equivalent to $4NL$ real multiplications and real additions. The total sum will be $4NL + 2N$ real multiplications and real additions for each time clipping and filtering are carried out. For iterative clipping and filtering, this computational complexity is multiplied by the number of iterations.

B. Selective Mapping

For implementation simplicity, the M phase sequences, each of length N, are set to $\{\pm 1, \pm j\}$ as these values can be implemented in hardware without multiplication. Therefore, MN additions are required to apply the phase sequences. Then, M-IFFT blocks are required, adding $2MN \log_2(N)$ real multiplications and $3MN \log_2(N)$ real additions. PAPRs for the M permutations of the OFDM signal are computed by $M(2N+1)$ real multiplications and $M(3N-2)$ real additions. Finally, $(M-1)$ subtractors/adders are necessary to find the minimum PAPR. Total complexity is $2MN(1+\log_2 N)+M$ real multiplications and $3MN(1+\log_2 N)+M(N-1)-1$ real additions.

C. Partial Transmit Sequences

Generally, using exhaustive search to find optimal phase factors which give minimum PAPR in PTS methods is expensive and unsuitable for real-time implementations. There are many PTS algorithms available in the literature that yield a suboptimal solution with reduced complexity. We consider the simple iterative flipping PTS scheme proposed in [104] with M partial sequences for the computational complexity analysis.

First, M-IFFT blocks are required with a complexity of $2MN \log_2(N)$ real multiplications and $3MN \log_2(N)$ real additions. Second, the next phase factor (the first phase is kept 1 and iterative flipping starts from the second) is flipped and the partial sequences are combined. This requires $N(2M-1)$ real additions. Third, PAPR is computed and compared with the previous one to keep the phase factor that achieves the minimum. These steps require $(2N+1)$ real multiplications and $(3N-1)$ real additions. The second and third steps are repeated $(M-1)$ times, where by the end, a suboptimal solution is reached. Therefore, the total computational complexity in this case is $2MN \log_2(N)+2N+1$ real multiplications and $3MN \log_2(N)+(M-1)[2N(M+1)-1]$ real additions.

D. Constrained Constellation Shaping

The CCS PAPR reduction scheme proposed in [153] uses convex optimization to converge to a PAPR global minimum. The *interior-point method* (IPM) complexity per iteration is approximately four IFFT blocks and the solution of a system of N linear equations. For large N, the computational complexity required for solving the linear equations will dominate all other computations. Solving the system of N linear equations using Gaussian elimination requires $\frac{N^3}{3}+N^2-\frac{N}{3}$ real multiplications and $\frac{N^3}{3}+\frac{N^2}{2}-\frac{5N}{6}$ real additions. Simulation results presented in [153] show that two iterations are sufficient to get a significant reduction in PAPR. The obtained solution produces an OFDM signal with a CCDF performance that is close to the performance at the global minimum PAPR. Therefore, the total computational complexity for the CCS method for two iterations is almost $\left(16N \log_2(N)+\frac{2N^3}{3}+2N^2-\frac{2N}{3}\right)$ real multiplications and $\left(24N \log_2(N)+\frac{2N^3}{3}+N^2-\frac{5N}{3}\right)$ real additions. It is possible to use Cholesky factorization to solve the system of N linear equations. This can approximately cut the solution cost by half as compared to the use of Gaussian elimination.

TABLE I
SUMMARY OF THE DERIVED COMPUTATIONAL COMPLEXITIES FOR SOME PAPR REDUCTION METHODS QUANTIFIED BY THE NUMBER OF REAL MULTIPLICATIONS AND ADDITIONS

Method	Complexity
Clipping and Filtering	$4NL+2N$ multiplications $4NL+2N$ additions
SLM	$2MN(1+\log_2 N)+M$ multiplications $3MN(1+\log_2 N)+M(N-1)-1$ additions
PTS	$2MN \log_2(N)+2N+1$ multiplications $3MN \log_2(N)+(M-1)[2N(M+1)-1]$ additions
CCS	$\left(16N \log_2(N)+\frac{2N^3}{3}+2N^2-\frac{2N}{3}\right)$ multiplications $\left(24N \log_2(N)+\frac{2N^3}{3}+N^2-\frac{5N}{3}\right)$ additions

Table I summarizes the derived computational complexities for the four PAPR reduction methods above, quantified by the number of real multiplications and additions. Figure 21 shows the computational complexity quantified by the number of real multiplications for the four PAPR reduction methods. The complexity curves are plotted according to the analyses presented in this Section with $M = 16$ and $L = 7$. The scenarios considered for the SLM and PTS in this Section are special cases with minimum possible complexity and can not be considered as a general result. For example, if the phase factors are allowed to have values from a larger set, the computational complexity will increase exponentially. Generally, for OFDM systems with large values of N like the WiMAX and LTE, signal distortion methods have the lowest complexity compared to multiple signaling and probabilistic or coding methods. On the other hand, signal distortion methods introduce more BER degradation.

VII. Insights Into Transmitted Power Constraint

Although some PAPR reduction methods are simple and provide a desirable performance, they change the average transmitted power. Active constellation extension is an example of a PAPR reduction method that increases the average

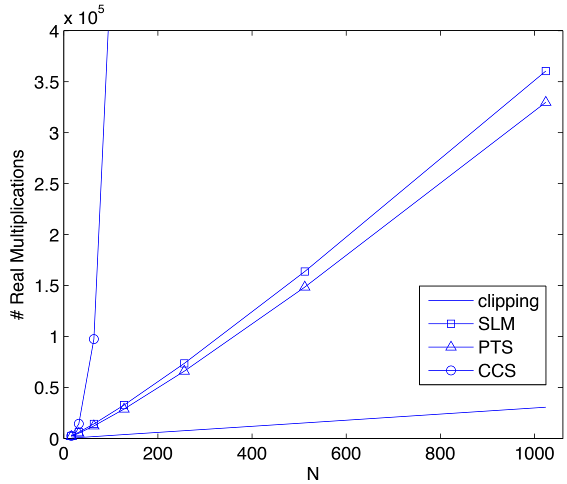


Fig. 21. Computational complexity for four PAPR reduction methods quantified by the number of real multiplications.

transmitted power, while clipping and filtering is an example that reduces it. If the change in power is not negligibly small, the power amplifier's gain must be dynamically adjusted, which will increase the hardware cost. Therefore, it is desirable to include modifications in some PAPR reduction methods to maintain the average power constant. It is desirable to satisfy this additional consideration with minimum extra complexity. We consider companding transforms as a case study and derive mild sufficient conditions that keep the average power unchanged after companding.

Invoking the CLT, it is shown that both the real and imaginary parts of the complex time-domain OFDM signal $x[n]$, $n = 0, \ldots, N-1$, are asymptotically independent and identically distributed Gaussian random variables, if the number of subcarriers (N) is large . Consequently, the envelope of the OFDM signal $x[n]$, denoted by x_e, follows the Rayleigh distribution with the probability density function [49]

$$f_{x_e}(x_e) = \frac{x_e}{\sigma^2} \exp\left(\frac{-x_e^2}{2\sigma^2}\right), \tag{43}$$

where σ is a strictly positive adjustable Rayleigh parameter. The mean, variance and second moment of a Rayleigh distributed random variable are given by $\sigma\sqrt{\pi/2}$, $\frac{4-\pi}{2}\,\sigma^2$ and $2\,\sigma^2$, respectively. To keep the average power of the OFDM signal unchanged after companding, the second moment must remain unchanged after companding, i.e.

$$\begin{aligned} E\left[x_e^2\right] &= E\left[C^2(x_e)\right] \Leftrightarrow \\ 2\,\sigma^2 &= E^2\left[C(x_e)\right] + \mathit{var}\left[C(x_e)\right]. \end{aligned} \tag{44}$$

Since the distribution of x_e is known and C is a known function, it is possible to use the approximate expressions for the mean and variance of $C(x_e)$ given by [190]

$$E\left[C(x_e)\right] \approx C\left[E(x_e)\right], \tag{45}$$

$$\mathit{var}\left[C(x_e)\right] \approx \left[\frac{d}{d\,x_e} C\left[E(x_e)\right]\right]^2 \mathit{var}(x_e)\,. \tag{46}$$

For some NLASTs such as the error or the hyperbolic tangent functions, Eqs. (45) and (46) are substituted in Eq. (44), then we solve for k_1 to find a mathematical expression that relates k_1 and k_2 such that the average power is kept unchanged. The following conditions are found

$$k_1^{\tanh} \approx \sqrt{\frac{2\,\sigma^2/\tanh^2(1.25\,\sigma\,k_2)}{1 + 0.43\,\sigma^2 k_2^2 \left[\frac{1-\tanh^2(1.25\,\sigma\,k_2)}{\tanh(1.25\,\sigma\,k_2)}\right]^2}}\,, \tag{47}$$

$$k_1^{\mathrm{erf}} \approx \sqrt{\frac{2\,\sigma^2/\,\mathrm{erf}^{\,2}(1.25\,\sigma\,k_2)}{1 + 0.546\left[\frac{\sigma\,k_2\,e^{-(1.25\,\sigma\,k_2)^2}}{\mathrm{erf}\,(1.25\,\sigma\,k_2)}\right]^2}}\,. \tag{48}$$

For an OFDM signal with 1024 subcarriers and QPSK modulation, we found that the ensemble average value of $\hat{\sigma}$ is $E\left[\hat{\sigma}\right] \approx 0.03$. For practical values of k_2, $(1.25\,\sigma\,k_2) << 1$. Hence to the first order Taylor approximation, we have $\mathrm{erf}(1.25\,\sigma\,k_2) \approx 1.25\,\sigma\,k_2$, $\tanh(1.25\,\sigma\,k_2) \approx 1.25\,\sigma\,k_2$ and $e^{-(1.25\,\sigma\,k_2)^2} \approx 1$. Therefore, Eqs. (47) and (48) can be both simplified, respectively, as

$$k_1^{tanh} \approx \sqrt{\frac{2\,\sigma^2/\,(1.25\,\sigma\,k_2)^2}{1 + 0.43\,\sigma^2 k_2^2 \left[\frac{1-(1.25\,\sigma\,k_2)^2}{(1.25\,\sigma\,k_2)}\right]^2}} \approx \frac{1}{k_2}\,, \tag{49}$$

and

$$k_1^{erf} \approx \sqrt{\frac{2\,\sigma^2/\,(1.25\,\sigma\,k_2)^2}{1 + 0.546\left[\frac{1}{(1.25\,\sigma\,k_2)}\right]^2}} \approx \frac{1}{k_2}\,. \tag{50}$$

Besides the above conditions that relates k_1 and k_2 together, k_1 must not exceed the maximum value of the OFDM envelope and must not drop severely. We found from computer simulations that $2\sigma^2$ is a good lower bound for k_1 to ensure that the average power of the companded OFDM signal does not drop significantly compared to the power of the original OFDM signal. Hence, k_1 is bounded by

$$2\,\sigma^2 \le k_1 \le \max_{n=1,\cdots,N} x_e\,. \tag{51}$$

In [67], the hyperbolic tangent companding function is used with the two companding parameters k_1 and k_2 chosen such that $k_1 = 1/k_2$. In [61], the authors used the error function with the values $k_1 = 1$ and $k_2 = 0.7071$, hence $k_1 k_2 = 0.7071$. While the first choice totally agrees with the condition derived in Eq. (49), the second is close to satisfying Eq. (50).

Following the same procedure for both the LST and LAST companding transforms, gives the conditions

$$a^2\left(2\,\sigma^2\right) + a\left(2\,\sigma\,b\sqrt{\frac{\pi}{2}}\right) + \left(b^2 - 2\,\sigma^2\right) = 0\,, \tag{52}$$

and

$$2\left(\frac{\sigma}{u}\right)^2\left(1 - \exp\left(\frac{-v^2}{2\sigma^2}\right)\right) + 2\,(\sigma u)^2 \exp\left(\frac{-v^2}{2\sigma^2}\right) = 2\,\sigma^2\,, \tag{53}$$

respectively. The quadratic formula in Eq. (52) can be used to find a set of solutions for a in terms of b and σ. Given the conditions $b > 0$ and $0 < a < 1$, the only nontrivial solution for a is obtained when b satisfies the bound $0 < b < \sqrt{2}\,\sigma$.

These examples show that the average transmitted power can be maintained unchanged after companding by properly choosing the companding parameters with no extra complexity

TABLE II
SUMMARY OF THE CONDITIONS REQUIRED TO MAINTAIN THE AVERAGE POWER OF THE OFDM SIGNAL UNCHANGED AFTER COMPANDING

Companding Transform	**Condition**
LST	$a^2\left(2\,\sigma^2\right)+a\left(2\,\sigma\,b\,\sqrt{\frac{\pi}{2}}\right)+\left(b^2-2\,\sigma^2\right)=0$
LAST	$2\left(\frac{\sigma}{u}\right)^2\left(1-\exp(\frac{-v^2}{2\sigma^2})\right)+2\,(\sigma u)^2\exp(\frac{-v^2}{2\sigma^2})=2\,\sigma^2$
erf	$k_1 \approx 1/k_2$
tanh	$k_1 \approx 1/k_2$

needed to achieve this goal. Table II summarizes the derived conditions to maintain the average power unchanged after using different companding transforms. If similar approaches can be followed for other PAPR reduction methods, the extra complexity for adding a separate block or modifying the amplifier's gain to do the job will be eliminated.

VIII. Lessons Learned and Suggestions

The topic of PAPR reduction in OFDM systems continues to attract the attention of researchers and still remains an active area of research. Different methods achieve PAPR reduction at the expense of other performance factors. Table III summarizes some of the typical PAPR reduction techniques and performance impacting factors discussed in Section III. It was shown earlier that no PAPR reduction technique achieved the best trade-off in all situations and the proper technique should be selected based on performance constraints and available resources.

Modern OFDM-based wireless standards which support high data rates and mobility achieve such performance by using a high number of subcarriers N. For example, fixed WiMAX, mobile WiMAX, LTE and DVB-T standards support up to 512, 2048, 2048 and 8192 subcarriers, respectively. It is shown in Section V and demonstrated in Section VI, that the computational complexity increases rapidly in a nonlinear manner with N for most PAPR reduction methods. Although many methods may look elegant, their high computational complexity renders most of them impractical for real-time implementations. Therefore, it is noticeable that the repeated clipping and filtering method was implemented by most commercial products for its simplicity and low computational complexity. Since high peaks in the OFDM signal occur with a low probability, the increase in BER caused by repeated clipping and filtering is tolerable by most systems.

However, some recent contributions in the literature tackled the complexity problem by proposing modifications to the conventional methods such as low-complexity blind SLM [89], [93], [101] or PTS [114], [117]–[119] schemes. Other contributions suggested TR [142]–[144] or repeated clipping and filtering [73] schemes with fast convergence where desired PAPR is achieved with much fewer iterations compared to the conventional implementations. Such attempts have the potential to lead to viable solutions for practical implementations with minimum cost and seem to be the most promising direction to pursue further research in the area of PAPR reduction for OFDM systems.

TABLE III
COMPARISON BETWEEN PAPR REDUCTION TECHNIQUES

PAPR Reduction Method	BER Increase	Bit Rate Loss	Implementation Complexity	Power Increase
Clipping	yes	no	low	no
Companding	yes	no	low	no
SLM	no	yes	high	no
PTS	no	yes	high	no
Interleaving	no	yes	high	no
TI	no	no	high	yes
TR	no	yes	high	yes
ACE	no	no	high	yes
CCS	yes	no	high	no
LBC	no	yes	low	no
GCS	no	yes	high	no

On the other hand, many proposed methods in the literature such as most of the coding-based schemes failed to translate their potential into practical implementations, mainly because of their high implementation complexity. This problem limited their use to OFDM systems with low N, where high PAPR is not a severe problem in the first place. Therefore, if practical potential is sought rather than just seeking possible theoretical solutions, a wise starting point in future attempts would be to give higher priority to the complexity factor by considering the simplest possible solutions first. This makes sense since other undesirable effects can be mitigated by other modules already available in wireless system. For example, the extra errors in recovered data resulting from clipping and filtering high peaks at the transmitter can be detected and mostly corrected by the channel coding techniques already available in wireless systems regardless of the PAPR reduction scheme employed.

IX. Conclusions

OFDM is an efficient multicarrier modulation technique for both wired and wireless applications due to its high data rates, robustness to multipath fading and spectral efficiency. Despite these advantages, it has the major drawback of generating high PAPR, which drives the transmitter's PA into saturation, causing nonlinear distortions and spectral spreading. The literature is rich with PAPR reduction techniques, which decrease PAPR substantially at the expense of increased BER, increased transmitted power, reduced bit rate, or increased complexity. This survey has discussed many important aspects of PAPR reduction techniques and the impact of these techniques on a number of critical design factors. Some absolutely essential mathematical formulations were presented including the statistics of PAPR and the distribution of the OFDM signal. We demonstrated that no single technique is the best under all circumstances and the proper technique should be selected based on system requirements and available resources. For example, in OFDM systems with a large number of subcarriers ($N \geq 256$), signal distortion techniques and specifically clipping and filtering are the least demanding in terms of computational complexity, while achieving good PAPR reduction.

The subject of PAPR reduction assumes increased importance due to the fact that future wireless systems are likely to apply OFDM structures with higher number of subcarriers than present ones in order to achieve higher data rates and mobility. This implies that the problem of developing PAPR reduction schemes for OFDM systems that are capable of mitigating the problem with best performance trade-offs, including minimum complexity and cost, is a rich subject with exciting possibilities for conducting further research.

Besides providing an extensive set of references to the subject of PAPR reduction techniques, this survey brings up to date previously available surveys with a treatment of most recent research as well as provides original contributions with simulations, complexity analyses, and a treatment of the topic under transmitted power constraint.

The authors strongly believe that this survey will serve as a valuable pedagogical resource to researchers, OFDM system architects, designers, and developers by providing them an understanding of the current research contributions in the area of PAPR reduction in OFDM systems, the different available techniques and their trade-offs towards developing more efficient and practical solutions.

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