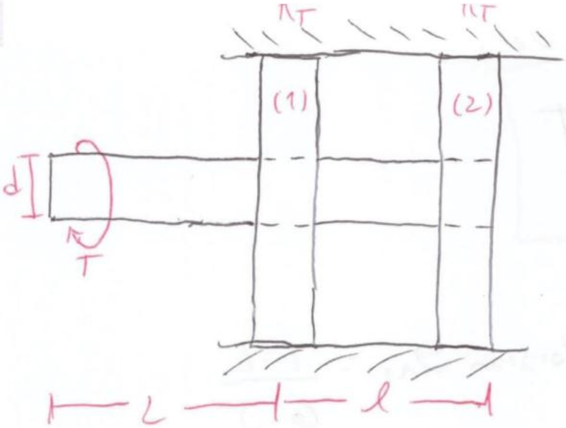
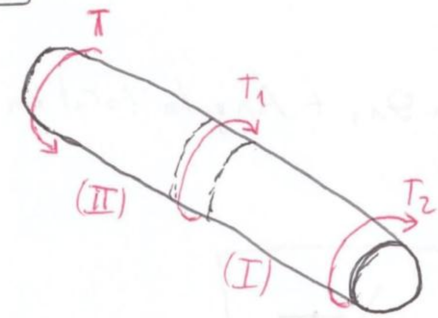
P1)



$$\left.\begin{aligned} T_1 &= k_T \theta_1 \\ T_2 &= k_T \theta_2 \end{aligned}\right\} \boxed{\begin{aligned} \theta_1 &= \frac{T_1}{k_T} \\ \theta_2 &= \frac{T_2}{k_T} \end{aligned}} \quad (*)$$

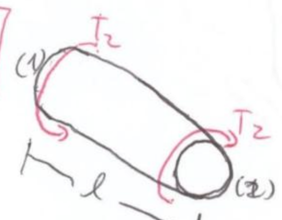
DCL)



$$\boxed{T = T_1 + T_2} \quad (**)$$

Hacemos un corte justo antes de T_1 y justo después de T_1, y se hace DCL de cada una de las secciones

I)



Ángulo de torsión $\theta_{12} = \frac{T_2 \cdot l}{GJ}$; con $J = \frac{\pi d^4}{32}$

$\Rightarrow$ Ángulo total en (1) = Ángulo de torsión θ_{12} + Ángulo de desplazamiento en (2)

$$\theta_1 = \theta_{12} + \theta_2 \quad \text{/Haciendo uso de } (*)$$

$$\frac{T_1}{k_T} = T_2\left(\frac{l}{GJ} + \frac{1}{k_T}\right) \quad \text{/Haciendo uso de } (**)\text{, se tiene } T_1 = T - T_2$$

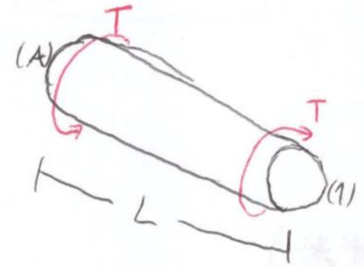
$$\Rightarrow T - T_2 = T_2\left(\frac{k_T l}{GJ} + 1\right)$$

$$\boxed{\therefore T_2 = \frac{T}{2 + \frac{k_T l}{GJ}}}$$

* Reemplazando en la ec. del ángulo ϑ_1:

$$\boxed{\vartheta_1 = \left(\frac{1/k_T + l/GJ}{2 + \frac{k_T\, l}{GJ}} \right) T}$$

$\boxed{\text{II}}$)



Ángulo de torsión $\vartheta_{A1} = \dfrac{T \cdot L}{G \cdot J}$

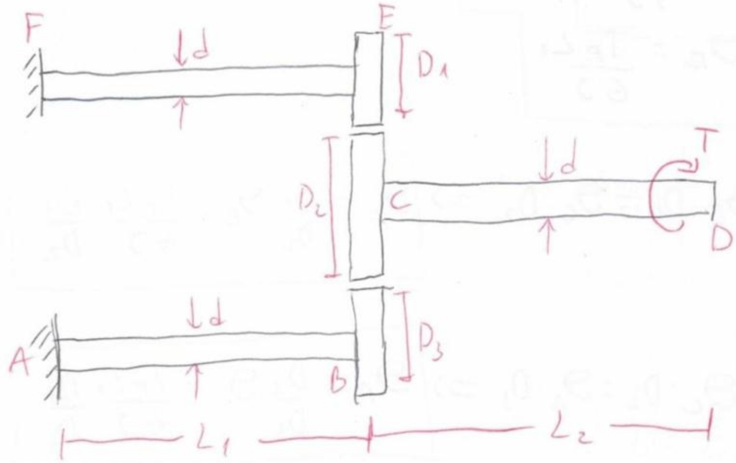
$\Rightarrow$ Ángulo total en (A) = Ángulo de torsión ϑ_{A1} + Ángulo total en (1)

$$\vartheta_T = \vartheta_{A1} + \vartheta_1$$

$$\boxed{\therefore\ \vartheta_T = \left(\frac{1/k_T + \frac{l}{GJ}}{2 + \frac{k_T\, l}{GJ}} + \frac{L}{GJ} \right) T}$$

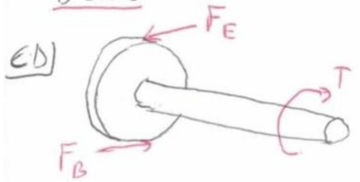
//

P2



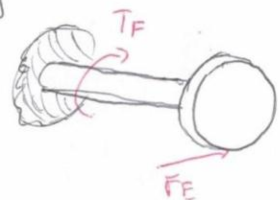
• DCLs

ED



$$\Rightarrow \boxed{T = \frac{D_2}{2}(F_E + F_B)} \quad (*)$$

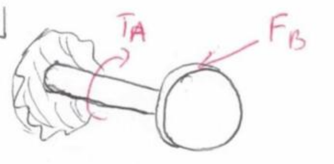
FE



$$\Rightarrow T_F = \frac{D_1}{2} F_E \rightarrow \boxed{F_E = \frac{2}{D_1} T_F}$$

Reemplazando en (*)

AB



$$\Rightarrow T_A = \frac{D_3}{2} F_B \rightsquigarrow \boxed{F_B = \frac{2}{D_3} T_A}$$

$$T = \frac{D_2}{2}\left(\frac{2}{D_1} T_F + \frac{2}{D_3} T_A\right) = D_2 \left(\frac{D_3 T_F + D_1 T_A}{D_1 \cdot D_3}\right)$$

$$\boxed{\therefore D_1 T_A + D_3 T_F = \frac{D_1 D_3}{D_2} T} \quad (\star)$$

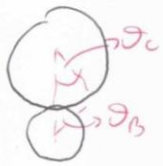
* Como el sistema es estáticamente indeterminado, necesitamos encontrar una relación entre ángulos (o torques)

FE| $T_F = \frac{GJ}{L_1}\theta_E \Rightarrow \boxed{\theta_E = \frac{T_F L_1}{GJ}}$

Engranajes E y C|

$$\theta_E \cdot D_1 = \theta_C \cdot D_2 \Rightarrow \boxed{\theta_C = \frac{D_1}{D_2}\theta_E = \frac{T_F L_1}{GJ}\frac{D_1}{D_2}}$$

Engranajes C y B|



$$\theta_C \cdot D_2 = \theta_B \cdot D_3 \Rightarrow \boxed{\theta_B = \frac{D_2}{D_3}\theta_C = \frac{T_F L_1}{GJ}\frac{D_1}{D_3}}$$

AB| $T_A = \frac{GJ}{L_1}\theta_B = \frac{\cancel{GJ}}{\cancel{L_1}} \cdot \frac{T_F \cancel{L_1}}{\cancel{GJ}}\frac{D_1}{D_3}$

$$\boxed{\therefore T_A = \frac{D_1}{D_3}T_F}$$

• Reemplazando en (☆)

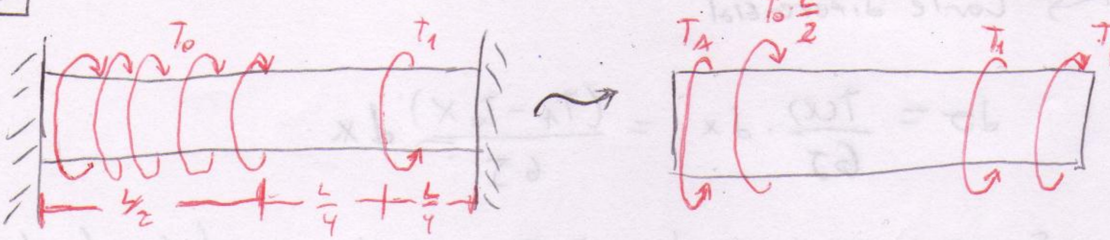
$$\frac{D_1 \cdot D_1}{D_3}T_F + D_3 T_F = \frac{D_1 D_3}{D_2}T$$

$$\boxed{\therefore T_F = \frac{D_1 D_3^2}{D_2(D_1^2 + D_3^2)}T}$$

$$\boxed{T_A = \frac{D_1^2 D_3}{D_2(D_1^2 + D_3^2)}T}$$

$$\boxed{T_F = 22,154\ Nm \ \wedge \ T_A = 14,769\ Nm}$$

P3



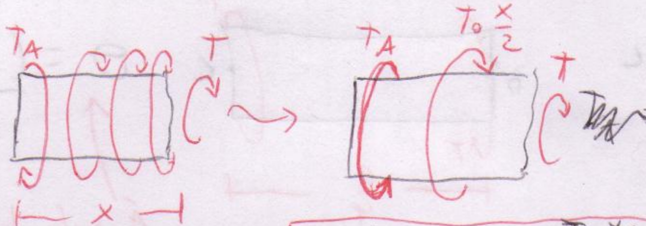
$\underline{DCL} \sim \boxed{T_A - T_0 \frac{L}{2} + T_1 - T_H = 0} \; (*)$

* Cada uno de los segmentos se deforma en ángulos diferentes.

↳ Es necesario calcular estos ángulos y relacionarlos

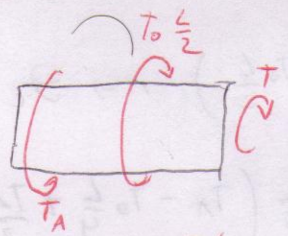
* Determinemos los torques que actúan en cada segmento y luego los ángulos de torsión:

• Corte $0 \leq x \leq \frac{L}{2}$



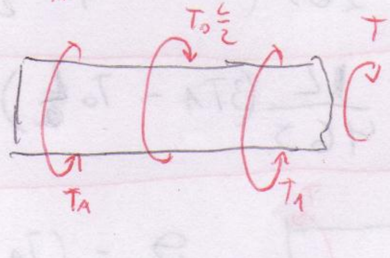
$\boxed{T(x) = T_A - T_0 \frac{x}{2}}$

• Corte $\frac{L}{2} \leq x \leq \frac{3L}{4}$



$\boxed{T(x) = T_A - T_0 \frac{L}{2}}$

• Corte $\frac{3}{4}L \leq x \leq L$



$\boxed{T(x) = T_A + T_1 - T_0 \frac{L}{2}}$

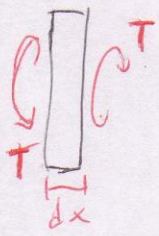
(Por (*))

$\boxed{T(x) = T_H}$

* Ángulos de torsión:

$T = \frac{GJ\vartheta}{L} \sim \vartheta = \frac{TL}{GJ} \sim d\vartheta = \frac{T}{GJ}dx$

• $0 < x < \frac{L}{2}$ ~> Corte diferencial



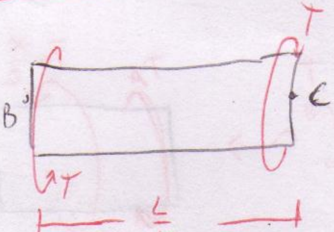
$$d\theta = \frac{T(x)}{GJ} \cdot dx = \frac{(T_A - T_0 x)}{GJ} dx$$

• Si integramos hasta el punto B, obtendremos el ángulo de torsión en el punto B

$$\sim \int d\theta = \theta_B = \frac{1}{GJ}\int_0^{L/2} (T_A - T_0 x)\, dx = \frac{1}{GJ}\left(T_A \cdot \frac{L}{2} - T_0 \frac{\left(\frac{L}{2}\right)^2}{2}\right)$$

$$\boxed{\therefore \theta_B = \frac{L}{2GJ}\left(T_A - T_0 \frac{L}{4}\right)}$$

• $\frac{L}{2} < x < \frac{3}{4}L$



$$\theta = \frac{\left(T_A - T_0 \cdot \frac{L}{2}\right) \cdot \frac{L}{4}}{GJ}$$

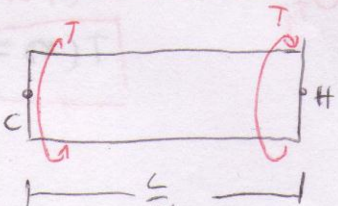
Ángulo de torsión relativo entre B y C

$$\sim \theta_{BC} = \frac{L}{4GJ}\left(T_A - T_0 \frac{L}{2}\right) \sim \theta_C = \theta_B + \theta_{BC}$$

$$\therefore \theta_C = \frac{L}{2GJ}\left(T_A - T_0 \frac{L}{4} + \frac{T_A}{2} - T_0 \frac{L}{4}\right)$$

$$\boxed{\theta_C = \frac{L}{4GJ}\left(3T_A - T_0 L\right)}$$

• $\frac{3}{4}L < x < L$



$$\theta_{CH} = \frac{\left(T_A + T_1 - T_0 \frac{L}{2}\right) \cdot \frac{L}{4}}{GJ}$$

$$\sim \theta_{CH} = \frac{L}{4GJ}\left(T_A + T_1 - T_0 \frac{L}{2}\right) \sim \theta_H = \theta_C + \theta_{CH}$$

$$\rightsquigarrow \vartheta_H = \frac{L}{4GJ}\left(3T_A - T_0 L\right) + \frac{L}{4GJ}\left(T_A + T_1 - T_0 \frac{L}{2}\right)$$

$$= \frac{L}{4GJ}\left(\cancel{3}T_A - T_0 L + \cancel{T_A} + T_1 - T_0 \frac{L}{2}\right)$$

$$\boxed{\therefore\ \vartheta_H = \frac{L}{4GJ}\left(4T_A - 3T_0 \frac{L}{2} + T_1\right)}$$

* Pero como H es un punto empotrado $\rightsquigarrow \vartheta_H = 0$

$$\Rightarrow 4T_A - 3\frac{T_0 L}{2} + T_1 = 0$$

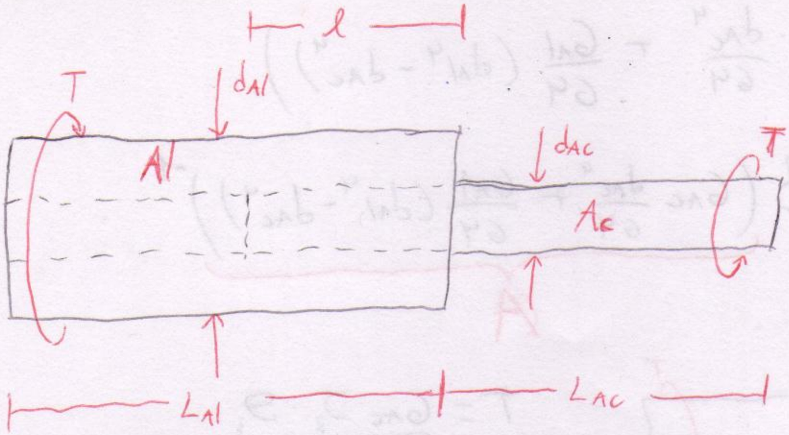
$$\boxed{\therefore\ T_A = \frac{3T_0 L}{8} - \frac{T_1}{4}}$$

*) Reemplazando en (*)

$$T_H = T_A - T_0 \frac{L}{2} + T_1 = \frac{3}{8} T_0 L - \frac{T_1}{4} - T_0 \frac{L}{2} + T_1$$

$$\boxed{\therefore\ T_H = -\frac{1}{8} T_0 L + \frac{3}{4} T_1}$$

P4 /



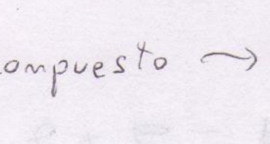
- Torsión en eje macizo (Acero) $\rightarrow$ $T = \dfrac{GJ\vartheta}{L}$; $J = \dfrac{\pi}{32} d_{Ac}^4$

- Torsión en eje hueco (Aluminio) $\rightsquigarrow$ $T = \dfrac{GJ\vartheta}{L}$; $J = \dfrac{\pi}{32}\left(d_{Al}^4 - d_{Ac}^4\right)$

- Torsión en eje compuesto $\rightarrow$ $T = \dfrac{2\pi\vartheta}{L}\left(G_{Ac}\dfrac{d_{Ac}^4}{64} + \dfrac{G_{Al}}{64}\left(d_{Al}^4 - d_{Ac}^4\right)\right)$

* Separamos el sistema en 3. Cada uno tendrá su propio ángulo de torsión. Luego, el ángulo total de torsión del sistema será la suma de los ángulos de torsión de los 3 segmentos.

1-) Aluminio $\longleftrightarrow$

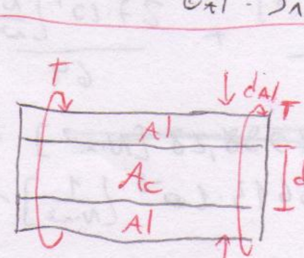


$J_1 = \dfrac{\pi}{32}\left(d_{Al}^4 - d_{Ac}^4\right) = 746{,}03\ cm^4$

$\boxed{J_1 = 7{,}4603 \cdot 10^{-6}\ [m^4]}$

$\Rightarrow T = \dfrac{G_{Al} \cdot J_1 \cdot \vartheta_1}{(L_{Al} - \ell)} \rightsquigarrow \boxed{\vartheta_1 = \dfrac{T \cdot (L_{Al} - \ell)}{G_{Al} \cdot J_1}}$ (*)

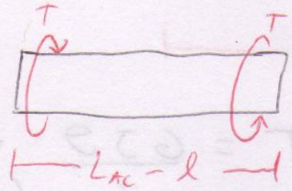
2-) Eje Compuesto $\rightsquigarrow$



$$\Rightarrow T = \frac{2\pi \cdot \vartheta_2}{l}\left(G_{AC} \cdot \frac{d_{AC}^4}{64} + \frac{G_{Al}}{64}\left(d_{Al}^4 - d_{AC}^4\right)\right)$$

$$\therefore \vartheta_2 = \frac{T \cdot l}{2\pi}\underbrace{\left(G_{AC}\frac{d_{AC}^4}{64} + \frac{G_{Al}}{64}\left(d_{Al}^4 - d_{AC}^4\right)\right)^{-1}}_{A}$$

3-) Acero $\rightarrow$



$$T = \frac{G_{AC}\, J_3 \cdot \vartheta_3}{(L_{AC} - l)}$$

$$J_3 = \frac{\pi}{32} d_{AC}^4 = 2{,}357 \cdot 10^{-6}\ [m^4]$$

$$\boxed{\therefore \vartheta_3 = \frac{T\,(L_{AC} - l)}{G_{AC} \cdot J_3}}$$

* Ángulo total $\vartheta_T = 7° = 0{,}1222$ rad $= \vartheta_1 + \vartheta_2 + \vartheta_3$

$$\rightsquigarrow \frac{T\,(L_{Al} - l)}{G_{Al} \cdot J_1} + A\,\frac{T\,l}{2\pi} + \frac{T\,(L_{AC} - l)}{G_{AC} \cdot J_3} = 0{,}1222$$

$$\boxed{\therefore T = 0{,}1222 \cdot \left(\frac{(L_{Al} - l)}{G_{Al} \cdot J_1} + \frac{A \cdot l}{2\pi} + \frac{(L_{AC} - l)}{G_{AC} \cdot J_3} \right)^{-1}}$$

* $\frac{(L_{Al} - l)}{G_{Al} \cdot J_1} = \frac{(70 - 20) \cdot 10^{-2}\ [m]}{27 \cdot 10^{9}\left[\frac{N}{m^2}\right] \cdot 7{,}4603 \cdot 10^{-6}\ [m^4]} = 2{,}4823 \cdot 10^{-6}\left[\frac{1}{N \cdot m}\right]$

* $A = \frac{75 \cdot 10^{9}\left[\frac{N}{m^2}\right](7 \cdot 10^{-2})^4\ [m^4]}{64} + \frac{27 \cdot 10^{9}\left[\frac{N}{m^2}\right]}{64}\left(10^4 - 7^4\right)\left(10^{-2}\right)^4\ [m^4]$

$\rightsquigarrow \cancel{A = 32058{,}28\ [N \cdot m^2]}$ $\Rightarrow \frac{A \cdot l}{2\pi} = 5{,}2879 \cdot 10^{-7}\left[\frac{1}{N \cdot m}\right]$

$A = 1{,}6613 \cdot 10^{-5}\left[\frac{1}{N \cdot m^2}\right]$

* $\frac{(L_{AC} - l)}{G_{AC} \cdot J_3} = \frac{(70 \cdot 10^{-2} - 20 \cdot 10^{-2})\ [m]}{75 \cdot 10^{9}\left[\frac{N}{m^2}\right] \cdot 2{,}357 \cdot 10^{-6}\ [m^4]} = 3{,}8822 \cdot 10^{-6}\left[\frac{1}{N \cdot m}\right]$

$$\boxed{\therefore\ T = 40574{,}6\ [N\,mm]}$$ //

b) ~~El esf. má~~

$$\tau = \frac{T\cdot r}{J} = \frac{G\theta r}{L} \rightsquigarrow \text{Esf. máximo} \sim r = \frac{d}{2}$$

$\Rightarrow$ En el tubo hay 2 zonas. Calculemos el máximo en ambas:

* Zona 1 (Tubo de aluminio)

$$\tau = \frac{T\cdot \frac{d_{Al}}{2}}{J_1} \Rightarrow \boxed{T = \frac{2\tau J_1}{d_{Al}} = 4476\ N\cdot m}$$

* Zona 2 (Eje compuesto) $\rightarrow$ No existe un J explícito

$$\tau = G r\left(\frac{\theta}{L}\right) \wedge T = \frac{2\pi\theta}{L}\left(\%\right) \Rightarrow \frac{\theta}{L} = \frac{T}{2\pi}\cdot A$$

$$\therefore \tau = G\cdot\frac{d_{Al}}{2}\cdot\frac{T}{2\pi}\cdot A \rightsquigarrow \boxed{T = \frac{4\tau\pi}{G_{Al} d_{Al}\cdot A} = 8404{,}8\ N\cdot m}$$

(Máximo!)

c) Dos zonas:

$$\text{* Zona 2} \rightsquigarrow \boxed{T = \frac{4\tau\pi}{G_{AC}\cdot d_{AC}\cdot A} = 8644{,}9\ [N\cdot m]}\ \text{(Máximo!)}$$

$$\text{* Zona 3} \rightsquigarrow T = \frac{\tau\cdot J_3}{\frac{d_{AC}}{2}} = 4040{,}87\ [N\cdot m]$$ //