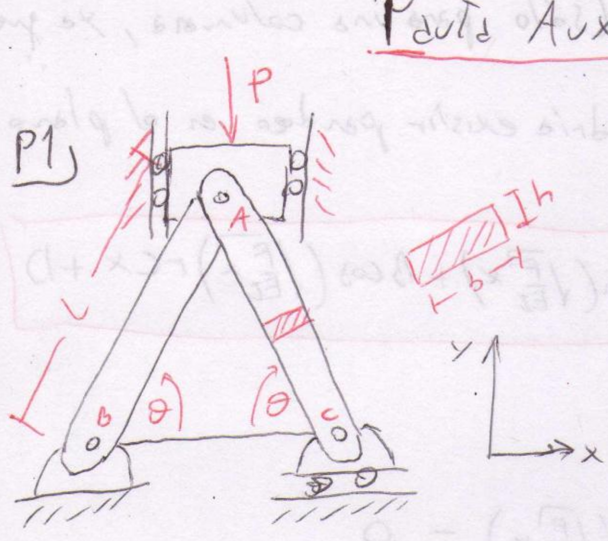
Pauta Aux. N°3 ME3202 2014

P1)



Necesitamos encontrar la Fza. que produce el pandeo en las columnas y la carga en el cable

* DCL pasador A

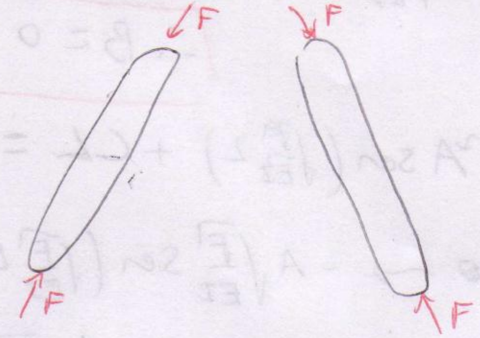
$\sum F_y = 0$

$2F\operatorname{sen}\theta - P = 0$

$$\boxed{\therefore F = \frac{P}{2\operatorname{sen}\theta}}$$

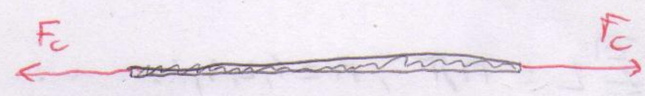
(Fza. que causa pandeo)

* DCL barras AC y AB



* DCL pasador B y C

* DCL cable



→ Desde pasador C

$\sum F_x = 0 \rightsquigarrow F\cdot\cos\theta - F_c = 0$

$$\boxed{\therefore F_c = F\cdot\cos\theta = \frac{P}{2\tan\theta}}$$

(Fza que causa falla en cable)

Evaluando en el cable

$\sigma_{adm} = \frac{\sigma_0}{F.S} \Rightarrow \boxed{F_{max} = \frac{\sigma_0 \cdot A_{cable}}{F.S}}$

$$F_{c_{max}} = \frac{200\cdot10^6 \frac{N}{m^2}\cdot\frac{\pi}{4}\cdot(1{,}5\cdot10^{-2})^2 m^2}{2}$$

$$\boxed{F_{c_{max}} \cong 17671{,}46\ [N]}$$

$\therefore P = F_c \cdot 2\tan\theta \rightarrow \boxed{P_{max} = 131901{,}56\ [N]}$

* Evaluemos la carga crítica de pandeo (sólo para una columna, ya que son iguales)

* Dada la configuración de las columnas, podría existir pandeo en el plano X-Y ó en el plano Y-Z

$\boxed{\text{* Plano } X-Y}$

- $\dfrac{d^4y}{dx^4} + \dfrac{F}{EI}\dfrac{d^2y}{dx^2} = 0 \rightsquigarrow \boxed{y(x) = A\,\mathrm{sen}\left(\sqrt{\tfrac{F}{EI}}\,x\right) + B\cos\left(\sqrt{\tfrac{F}{EI}}\,x\right) + Cx + D}$

- Cond. de Borde:

$\hookrightarrow y(0) = 0 \rightsquigarrow \boxed{B + D = 0}$

$\hookrightarrow \dfrac{d^2y}{dx^2}(0) = -A\dfrac{F}{EI}\,\mathrm{sen}\left(\sqrt{\tfrac{F}{EI}}\,x\right) - B\dfrac{F}{EI}\cos\left(\sqrt{\tfrac{F}{EI}}\,x\right) = 0$

$$\boxed{\therefore B = 0 \rightsquigarrow D = 0}$$

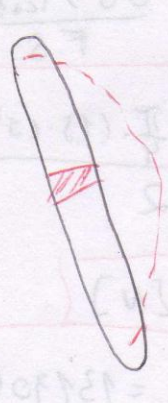
$\hookrightarrow y(L) = 0 = A\,\mathrm{sen}\left(\sqrt{\tfrac{F}{EI}}\,L\right) + CL = 0 \quad (*)$

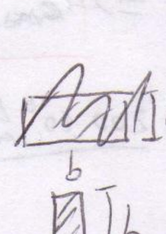
$\hookrightarrow \dfrac{d^2y}{dx^2}(L) = 0 \rightsquigarrow -A\dfrac{F}{EI}\,\mathrm{sen}\left(\sqrt{\tfrac{F}{EI}}\,L\right) = 0$

$$\therefore \mathrm{sen}\left(\sqrt{\tfrac{F}{EI}}\,L\right) = 0 \quad (C = 0 \text{ en } (*))$$

$\Rightarrow \sqrt{\tfrac{F}{EI}}\,L = n\pi,\ n = 1, 2, 3, \dots \quad \therefore F_{cr}^{(n)} = \dfrac{n^2\pi^2}{L^2}EI$

$$\boxed{F_{cr}^{(1)} = \frac{\pi^2}{L^2}EI} \quad (1^{\circ} \text{ modo})$$





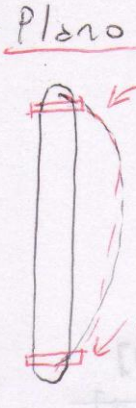
$\Rightarrow I_z = \dfrac{hb^3}{12} = \cancel{1{,}40625 \cdot 10^{-8}\,[m^4]}$
$1{,}5625 \cdot 10^{-7}\,[m^4]$

$\Rightarrow F_{cr}^{(1)} = \cancel{26\,370{,}35\,[N]} = 293003{,}881\,[N]$

$(DCL) \therefore \boxed{P_{cr} = 2F_{cr}^{(1)} \cdot \mathrm{sen}\,\theta = \cancel{50943{,}603\,[N]}}$

$$\boxed{= 566040{,}031\,[N]}$$

* Plano Y-Z



• C.B.

↳ $y(0) = 0 \rightsquigarrow B + D = 0 \rightsquigarrow \boxed{B = -D}$

↳ $\frac{dy}{dx}(0) = A\sqrt{\frac{F}{EI}}\cos\left(\sqrt{\frac{F}{EI}}x\right) - B\sqrt{\frac{F}{EI}}\operatorname{sen}\left(\sqrt{\frac{F}{EI}}x\right) + C = 0$

$\therefore A\sqrt{\frac{F}{EI}} + C = 0 \rightsquigarrow \boxed{C = -A\sqrt{\frac{F}{EI}}}$

↳ $y(L) = 0 \rightsquigarrow A\operatorname{sen}\left(\sqrt{\frac{F}{EI}}L\right) - D\cos\left(\sqrt{\frac{F}{EI}}L\right) - A\sqrt{\frac{F}{EI}}L + D = 0$

↳ $\frac{dy}{dx}(L) = 0 \rightsquigarrow A\sqrt{\frac{F}{EI}}\cos\left(\sqrt{\frac{F}{EI}}L\right) + D\sqrt{\frac{F}{EI}}\operatorname{sen}\left(\sqrt{\frac{F}{EI}}L\right) - A\sqrt{\frac{F}{EI}} = 0$

• Ordenando de forma matricial

$$\begin{pmatrix} \operatorname{sen}\left(\sqrt{\frac{F}{EI}}L\right) - \sqrt{\frac{F}{EI}}L & -\cos\left(\sqrt{\frac{F}{EI}}L\right) + 1 \\ \sqrt{\frac{F}{EI}}\cos\left(\sqrt{\frac{F}{EI}}L\right) - \sqrt{\frac{F}{EI}} & +\sqrt{\frac{F}{EI}}\operatorname{sen}\left(\sqrt{\frac{F}{EI}}L\right) \end{pmatrix} \begin{pmatrix} A \\ D \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

• Para que el sistema tenga solución → det(matriz) = 0. Haciendo $k = \sqrt{\frac{F}{EI}}$

$\cancel{-k\operatorname{sen}\left(\sqrt{\frac{F}{EI}}L\right)}$

$\cancel{-k\operatorname{sen}(kL)(\operatorname{sen}(kL) - kL) + \cos(kL)(k\cos(kL) - k) = 0}$

$-k\operatorname{sen}(kL)(\operatorname{sen}(kL) - kL) + (1 - \cos(kL))(k\cos(kL) - k) = 0$

$-k\operatorname{sen}^{\cancel{2}}(kL) + k^2L\operatorname{sen}(kL) - k\cos(kL) + k + k\cos^{\cancel{2}}(kL) - k\cos(kL) = 0$

$\Rightarrow k + k^2L\operatorname{sen}(kL) - 2k\cos(kL) + k = 0$

$\boxed{\therefore kL\operatorname{sen}(kL) - 2\cos(kL) + 2 = 0}$

f)

$\cdot$ Soln $\rightsquigarrow \cos(kL) = 1$ → Soln. trivial $KL = 0$

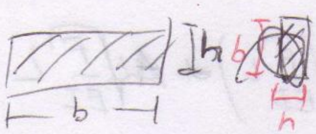
→ Soln. no trivial $KL = 2\pi n$

$$\sqrt{\frac{F_{cr}}{EI}}\, L = 2\pi n$$

$$\Rightarrow \boxed{F_{cr}^{(1)} = \frac{4\pi^2}{L^2} EI}$$

* En este caso

$$I = \frac{b \cdot h^3}{12} = \cancel{1,5625 \cdot 10^{-7}} [m^4]$$
$$= \cancel{1,5625 \cdot 10^{-7} [m^4]}$$
$$= 1,40625 \cdot 10^{-8} [m^4]$$



$$\Rightarrow F_{cr}^{(1)} = \cancel{1172015,523 [N]} \rightsquigarrow \boxed{P_{cr} = \cancel{2264160,125 [N]}}$$
$$105481,397 [N] \longrightarrow \boxed{= 203774,11 [N]}$$

* En resumen:

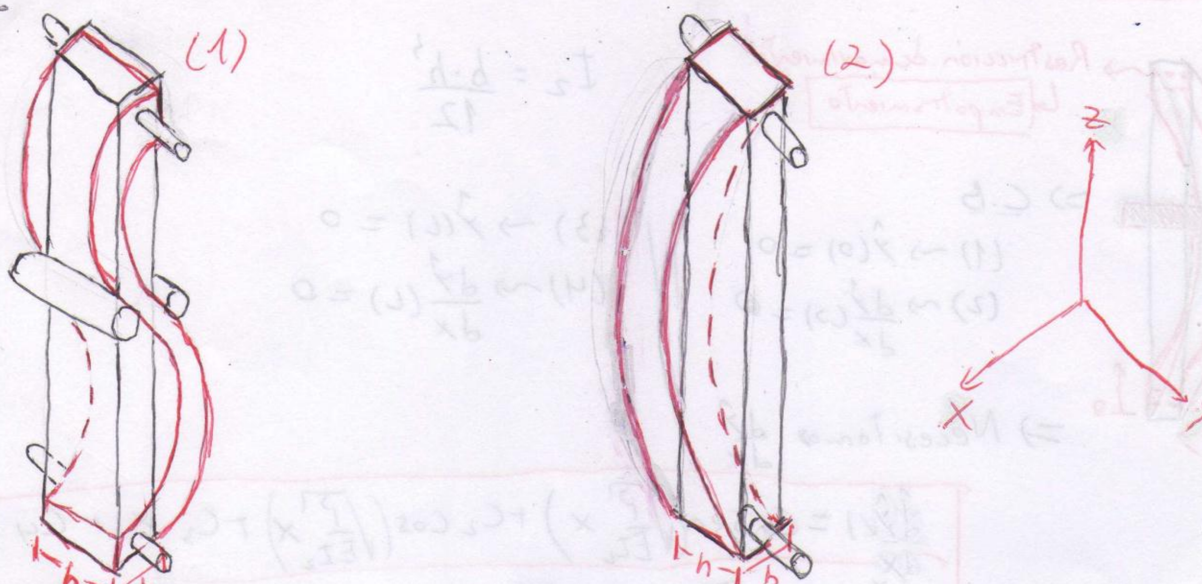
$$P_{max}(\text{Cable}) = 131901,56 [N]$$

$$P_{cr}^{(1)}(x\text{-}x) = \frac{566040,031}{F.S.} = 283020,016 [N]$$

$$P_{cr}^{(1)}(y\text{-}z) = \frac{203774,11}{F.S.} = 101887,055 [N]$$

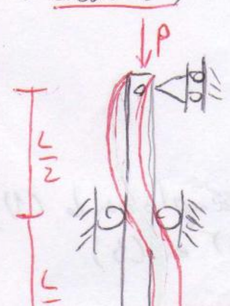
∴ La carga máxima para que ningún componente falle es de 101887,055 [N] (Pandeo en el plano y-z)

P2)



* Calculamos la carga crítica en ambos casos

→ Caso (1)



$$\frac{d^4 \hat{y}}{dx^4} + \frac{P}{EI_1}\frac{d^2\hat{y}}{dx^2} = 0 \quad \wedge \quad I_1 = \frac{hb^3}{12}$$

$$\Rightarrow \hat{y}(x) = C_1 \operatorname{Sen}\left(\sqrt{\frac{P}{EI_1}}\,x\right) + C_2 \cos\left(\sqrt{\frac{P}{EI_1}}\,x\right) + C_3 x + C_4$$

⇒ C.B.

$$(1) \rightsquigarrow \hat{y}(0) = 0 \qquad (3) \rightsquigarrow \hat{y}(L) = 0$$

$$(2) \rightsquigarrow \frac{d^2\hat{y}}{dx^2}(0) = 0 \qquad (4) \rightsquigarrow \frac{d^2\hat{y}}{dx^2}(L) = 0$$

⇒ Necesitamos $\frac{d^2\hat{y}}{dx^2}$:

$$\frac{d^2\hat{y}}{dx^2} = -C_1 \frac{P}{EI_1} \operatorname{Sen}\left(\sqrt{\frac{P}{EI_1}}\,x\right) - C_2 \frac{P}{EI_1}\cos\left(\sqrt{\frac{P}{EI_1}}\,x\right)$$

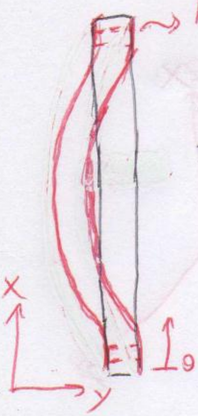
* Desde P1: $C_2 = -C_4 = C_3 = 0$

$$\wedge \quad \operatorname{Sen}\left(\sqrt{\frac{P}{EI_1}}\,L\right) = 0 \Longrightarrow \boxed{P_{cr}^{(n)} = \frac{EI_1 n^2 \pi^2}{L^2}}$$

* Dado que hay una restricción en $\frac{L}{2}$, el modo 1 de pandeo no puede darse. Por lo tanto, la viga se deflectará en el 2º modo $(n=2)$

$$\boxed{\therefore P_{cr}^{(2)} = \frac{4EI_1\pi^2}{L^2}}$$

→ Caso (2)



→ Restricción de movimiento

↳ $\boxed{\text{Empotramiento}}$

$$I_2 = \frac{b \cdot h^3}{12}$$

⇒ C.B

(1) → $\hat{y}(0) = 0$

(2) → $\frac{d\hat{y}}{dx}(0) = 0$

(3) → $\hat{y}(L) = 0$

(4) → $\frac{d\hat{y}}{dx}(L) = 0$

⇒ Necesitamos $\frac{d\hat{y}}{dx}$

$$\boxed{\begin{aligned} \hat{y}(x) &= C_1 \operatorname{Sen}\left(\sqrt{\frac{P}{EI_2}}\,x\right) + C_2 \cos\left(\sqrt{\frac{P}{EI_2}}\,x\right) + C_3\,x + C_4 \\ \frac{d\hat{y}}{dx} &= C_1\sqrt{\frac{P}{EI_2}} \cos\left(\sqrt{\frac{P}{EI_2}}\,x\right) - C_2\sqrt{\frac{P}{EI_2}} \operatorname{Sen}\left(\sqrt{\frac{P}{EI_2}}\,x\right) + C_3 \end{aligned}}$$

* (1) → $-C_2 = C_4$
* (2) → $-C_1\sqrt{\frac{P}{EI_2}} = C_3$
* (3) → $C_1 \operatorname{Sen}\left(\sqrt{\frac{P}{EI_2}}\,L\right) + C_2 \cos\left(\sqrt{\frac{P}{EI_2}}\,L\right) + C_3 L + C_4 = 0$ / Reemplazando (1) y (2) en (3)

$$\boxed{C_1\left(\operatorname{Sen}\left(\sqrt{\frac{P}{EI_2}}\,L\right) - \sqrt{\frac{P}{EI_2}}\,L\right) + C_2\left(\cos\left(\sqrt{\frac{P}{EI_2}}\,L\right) - 1\right) = 0} \quad (*)$$

* (4) → $C_1\sqrt{\frac{P}{EI_2}} \cos\left(\sqrt{\frac{P}{EI_2}}\,L\right) - C_2\sqrt{\frac{P}{EI_2}} \operatorname{Sen}\left(\sqrt{\frac{P}{EI_2}}\,L\right) + C_3 = 0$

$$\boxed{C_1\left(\cancel{\sqrt{\frac{P}{EI_2}}} \cos\left(\sqrt{\frac{P}{EI_2}}\,L\right) - \cancel{\sqrt{\frac{P}{EI_2}}}\right) - C_2\cancel{\sqrt{\frac{P}{EI_2}}} \operatorname{Sen}\left(\sqrt{\frac{P}{EI_2}}\,L\right) = 0} \quad (**)$$

→ Despejamos C_2 desde (**) y lo reemplazamos en (*)

Sea $k = \sqrt{\frac{P}{EI_2}}$ ⇒ $\boxed{C_2 = \frac{\cos(kL) - 1}{\operatorname{Sen}(kL)}}$

$$\Rightarrow C_1\left(\mathrm{Sen}(KL)-KL\right)+\frac{(\cos(KL)-1)}{\mathrm{Sen}(KL)}\left(\cos(KL)-1\right)C_1=0 \quad / \cdot \mathrm{Sen}(KL)$$

$$\underbrace{\mathrm{Sen}^2(KL)-KL\,\mathrm{Sen}(KL)+\cos^2(KL)}_{1}-2\cos(KL)+1=0$$

$$\Rightarrow \boxed{KL\,\mathrm{Sen}(KL)+2\cos(KL)=2}$$

$\to KL=0$ (Soln. trivial)

$\to \boxed{KL=2\pi n}$

$$\hookrightarrow \sqrt{\frac{P}{EI_2}}\,L=2\pi n \quad ; \; n=1,2,3,\ldots$$

$$\therefore \text{Para } n=1 \Rightarrow \boxed{P_{cr}^{(1)}=\frac{4EI_2\pi^2}{L^2}}$$

* Las cargas críticas en ambos casos deben ser iguales:

$$P_{cr}^{(2)}=P_{cr}^{(1)} \Rightarrow \frac{\cancel{4}E I_1\cancel{\pi^2}}{\cancel{L^2}}=\frac{\cancel{4}E I_2\cancel{\pi^2}}{\cancel{L^2}} \Rightarrow I_1=I_2$$

$$\Rightarrow \frac{hb^3}{12}=\frac{bh^3}{12} \longrightarrow \boxed{b=h}$$

* Existe otra forma de resolver (*) y (**) que es expresar ese sistema de ecs para C_1 y C_2 en forma matricial $(Ax=b)$:

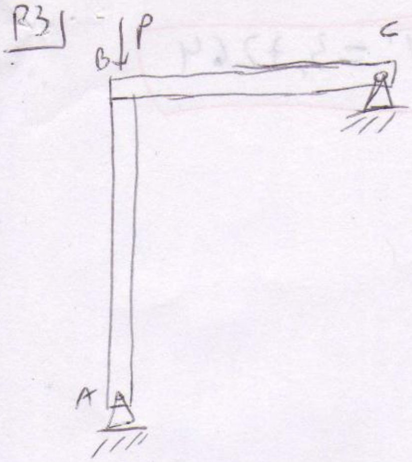
$$\begin{pmatrix} \mathrm{Sen}(KL)-KL & \cos(KL)-1 \\ \cos(KL)-1 & -\mathrm{Sen}(KL) \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

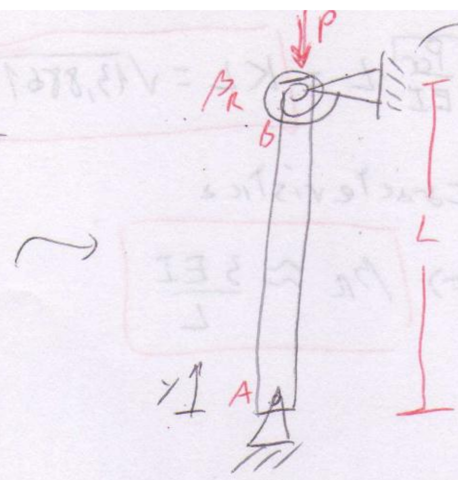
* Para que el sistema tenga solución $\Rightarrow \det(A)=0$:

$$(\mathrm{Sen}(KL)-KL)(-\mathrm{Sen}(KL))-(\cos(KL)-1)^2=0$$

$$\boxed{\therefore KL\,\mathrm{Sen}(KL)+2\cos(KL)=2}$$

P3)





$M_R = -\beta_R \cdot \theta_B$

$\boxed{K = \sqrt{\frac{P}{EI}}}$

a) $\frac{d^4y}{dx^4} + \frac{P}{EI}\frac{d^2y}{dx^2} = 0 \Rightarrow \boxed{y(x) = A\,sen(kx) + B\cos(kx) + Cx + D}$

$\boxed{\begin{array}{l} y'(x) = Ak\cos(kx) - Bk\,sen(kx) + C \\ y''(x) = -Ak^2 sen(kx) - Bk^2\cos(kx) \end{array}}$

*C.B:

- $\frac{d^2y}{dx^2}(0) = 0 \rightsquigarrow \boxed{B = 0}$; $\cdot\, y(0) = 0 \rightsquigarrow \boxed{D = 0}$
- $y(L) = 0 \Rightarrow A \cdot sen(kL) + CL = 0 \longrightarrow \boxed{C = -A\frac{sen(kL)}{L}}$ (*)
- $EI \cdot \frac{d^2y}{dx^2}(L) = -\beta_R \cdot \frac{dy}{dx}(L)$

$EI(-\cancel{A}k^2 sen(kL)) = -\beta_R(\cancel{A}k\cos(kL) + C) = -\beta_R\left(Ak\cos(kL) - \frac{A\,sen(kL)}{L}\right)$

$k^2 EI\,sen(kL) = k\beta_R\cos(kL) - \frac{sen(kL)}{L}$

$k^2L^2 = \frac{kL^2\beta_R}{EI}\frac{\cos(kL)}{sen(kL)} - \frac{L\beta_R}{EI}$

$\boxed{\therefore \frac{\beta_R L}{EI}(\cot(kL)\cdot kL - 1) = (kL)^2}$

b) $P_{cr} = 13,886\frac{EI}{L^2} \Rightarrow \sqrt{\frac{P}{EI}} \cdot L = \boxed{K \cdot L = \sqrt{13,886} \approx 3,7264}$

*Reemplazando en la ec. característica:

$\beta_R = \frac{(kL)^2}{(kL\cdot\cot(kL) - 1)} \cdot \frac{EI}{L} \Rightarrow \boxed{\beta_R \approx \frac{3EI}{L}}$

b) $P_{cr} = 13,8861 \frac{EI}{L^2} \rightsquigarrow \sqrt{\frac{P_{cr}}{EI}} \cdot L = \boxed{KL = \sqrt{13,8861} = 3,7264}$

* Reemplazando en la ec. característica

$$\beta_R = \frac{(KL)^2 \cdot EI}{(KL\cot(KL) - 1) \cdot L} \Rightarrow \boxed{\beta_R \approx \frac{3EI}{L}}$$