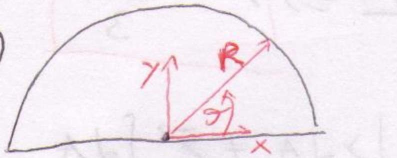
Apéndice Aux N° 4

↳ Cálculo de ejes neutros y momentos de inercia de secciones particulares

1-)

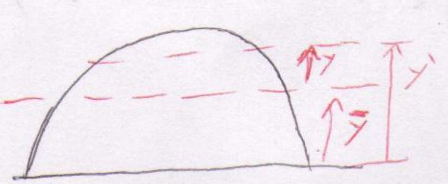


En coordenadas polares $\rightarrow dA = r\,d\theta\,dr$

$y = r\,\text{sen}\theta$

$$\Rightarrow \bar{y} = \frac{\int y\,dA}{A} = \frac{\int_0^{\pi}\int_0^{R} (r\,\text{sen}\theta)\,r\,d\theta\,dr}{A} = \frac{\int_0^{\pi}\int_0^{R} r^2\,\text{sen}\theta\,d\theta\,dr}{\frac{1}{2}\pi R^2}$$

$$= \underbrace{-\cos\theta\Big|_0^{\pi}}_{2} \cdot \frac{R^3}{3} \cdot \frac{1}{\frac{1}{2}\pi R^2} \Rightarrow \boxed{\bar{y} = \frac{4R}{3\pi}}$$



$$I_z = \int y^2\,dA = \int (y' - \bar{y})^2\,dA$$

$$= \int y'^2\,dA - 2\bar{y}\int y'\,dA + \bar{y}^2\int dA$$

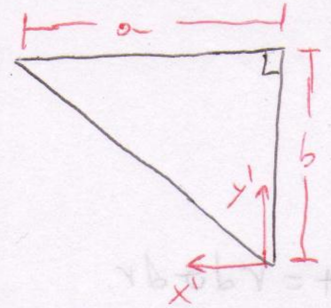
$$\rightarrow I_z = \int_0^{\pi}\int_0^{R} r^2\,\text{sen}^2\theta\, r\,d\theta\,dr - \cancel{2\frac{4R}{3\pi}\left(\frac{4R}{3\pi}\right)} + \frac{16R^2}{9\pi^2}\int_0^{\pi}\int_0^{R} r\,d\theta\,dr$$

$$2\cdot\frac{4R}{3\pi}\int_0^{\pi}\int_0^{R} r\,\text{sen}\theta\, r\,d\theta\,dr$$

$$= \frac{R^4}{4}\underbrace{\int_0^{\pi}\text{sen}^2\theta\,d\theta}_{\frac{1}{2}(\theta - \text{sen}\theta\cos\theta)\big|_0^{\pi}} - \frac{8R^4}{9\pi}\int_0^{\pi}\text{sen}\theta\,d\theta + \frac{8R^4}{9\pi^2}\cdot\pi$$

$$= \frac{\pi R^4}{8} - \frac{16R^4}{9\pi} + \frac{8R^4}{9\pi} \Longrightarrow \boxed{I_z = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)R^4}$$

2-)



$$x' = \frac{a}{b}y' \Rightarrow \bar{y} = \frac{\int y' x' dy'}{\frac{ab}{2}}$$

$$\rightarrow \bar{y} = \frac{\int_0^b y'^2 \frac{a}{b} dy}{\frac{ab}{2}} = \frac{\frac{b^{\cancel{3}}}{3} \cdot \frac{a}{\cancel{b}}}{\frac{\cancel{a}b}{2}} \Rightarrow \boxed{\bar{y} = \frac{2b}{3}}$$

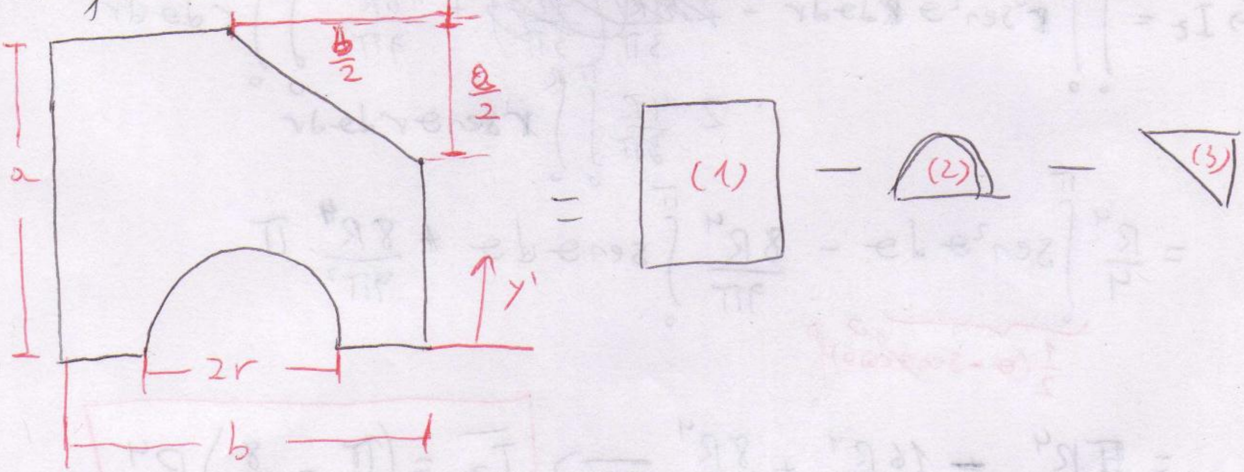
$$\bar{I}_z = \int y^2 dA = \int (y' - \bar{y})^2 dA = \int y'^2 dA - 2\bar{y}\int y\, dA + \bar{y}^2 \int dA$$

$$= \int_0^b y'^3 \frac{a}{b} dy' - \frac{4b}{3}\int_0^b y'^2 \frac{a}{b} dy' + \frac{4b^2}{3}\int_0^b y' \frac{a}{b} dy'$$

$$= \frac{a}{b}\left(\frac{b^4}{4} - \frac{4b}{3}\cdot\frac{b^3}{3} + \frac{4b^2}{3}\cdot\frac{b^2}{2}\right) = \frac{a}{b}\left(\frac{b^4}{4} - \frac{4}{9}b^4 + \frac{2b^4}{3}\right)$$

$$\therefore \boxed{\bar{I}_z = \frac{ab^3}{36}}$$

3-) ¿Y qué hacemos con esta figura?



$$\therefore \boxed{\bar{y}_T = \frac{\bar{y}_1 \cdot A_1 - \bar{y}_2 \cdot A_2 - \bar{y}_3 \cdot A_3}{A_1 - A_2 - A_3}}$$

* $\bar{Y}_1 = \frac{a}{2} \wedge \bar{I}_1 = \frac{b \cdot a^3}{12} \wedge A_1 = ab$

Desde ~~la~~ base!

$$\cancel{\Rightarrow \bar{Y}_T = \frac{a}{2}ab - \left[\left(\frac{b}{2}\cdot\left(\frac{a}{2}\right)^3\cdot\frac{1}{36}\right) + \frac{a}{2}\right]\cdot\frac{ab}{8} - \frac{4r}{3\pi}}$$

Desde la base!

$$\bar{Y}_T = \frac{\frac{a}{2}\cdot ab - \left[\frac{2b}{2}\cdot\frac{1}{3} + \frac{a}{2}\right]\frac{ab}{8} - \frac{4r}{3\pi}\cdot\frac{1}{2}\pi r^2}{ab - \frac{ab}{8} - \frac{1}{2}\pi r^2}$$

$$= \frac{\frac{a^2b}{2} - \left(\frac{b}{3} + \frac{a}{2}\right)\frac{ab}{8} - \frac{4r}{3\pi}\cdot\frac{1}{2}\pi r^2}{\frac{7ab}{8} - \frac{1}{2}\pi r^2}$$

$$= \frac{\frac{a^2b}{2} - \frac{ab^2}{24} - \frac{a^2b}{16} - \frac{2}{3}r^3}{\frac{7ab}{8} - \frac{1}{2}\pi r^2}$$

$$\boxed{\therefore \bar{Y}_T = \frac{\frac{ab}{8}\left(\frac{7a}{2} - \frac{b}{3}\right) - \frac{2}{3}r^3}{\frac{7ab}{8} - \frac{1}{2}\pi r^2}}$$

* Los momentos de inercia se pueden sumar o restar siempre y cuando estén todos con respecto al eje neutro de la sección completa. Entonces, se calculan las inercias con respecto a $\bar{Y}_T$ usando el teorema de los ejes paralelos y luego se resta (No haremos eso aquí, pero RECUÉRDENLO!)