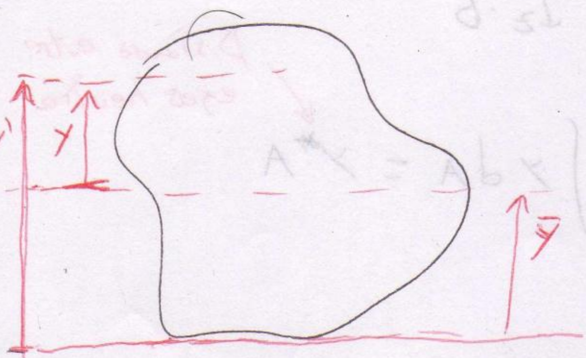
Pauta Auxiliar N°4 ME3202 2014.

* Cálculo de propiedades de Área (I_z, Q)

↳ 2° momento de área (I_z)



- $\bar{y}$ (Eje neutro) $= \dfrac{\int y' \, dA}{A}$

- $I_z = \int y^2 \, dA = \int (y' - \bar{y}) \, dA$

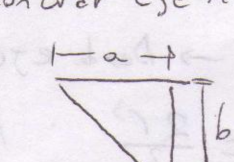
- Si hay varias fig. geométricas

$$\bar{y}_T = \frac{\sum_i \bar{y}_i \cdot A_i}{\sum A_i}$$

- Traslado de eje de rotación (Teorema de los ejes paralelos)

$$\bar{I}_z = I_z + \delta^2 A$$

* Encontrar eje neutro y ~~...~~ I_z de:



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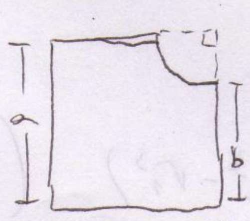
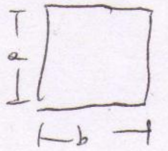


* Secciones conocidas:

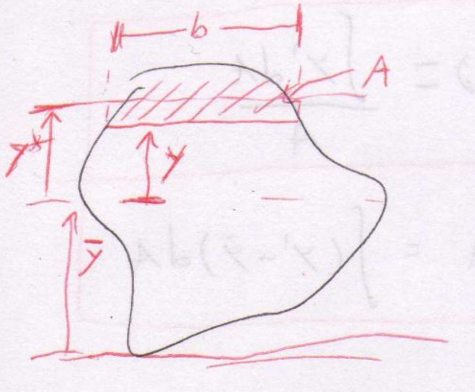


$I_z = \dfrac{b a^3}{12}$,

$I_z = \dfrac{\pi d^4}{64}$

↳ 1° Momento de Área (Q):

* Para encontrar el esf. de corte en una sección arbitraria, se requiere calcular Q:



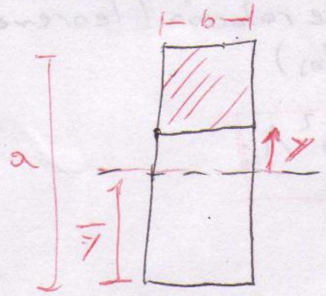
$$\tau = \frac{V(x) \cdot Q}{I_z \cdot b}$$

donde

$$Q = \int y \, dA = y^{*} A$$

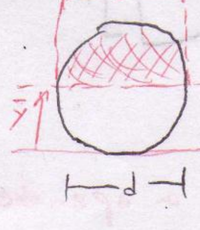
Distancia entre ejes neutros

* Secciones conocidas:



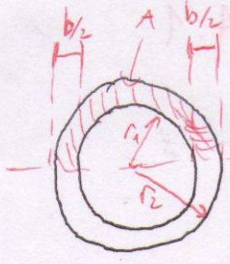
$$Q = \frac{b}{2}\left(\frac{h^2}{4} - y^2\right) \Rightarrow \tau = \frac{V}{2 I_z}\left(\frac{h^2}{4} - y^2\right)$$

* Se puede observar que el esf. es máximo cuando $y = 0$ (o sea, en el eje neutro!!)



(Esf. máximo en sec. circular → Desde eje neutro)

$$Q = A \cdot \bar{y} = \left(\frac{\pi r^2}{2}\right)\left(\frac{4r}{3\pi}\right) = \frac{2 r^3}{3} \quad ; \quad b = 2r$$



$$Q = \frac{2}{3}(r_2^3 - r_1^3)$$

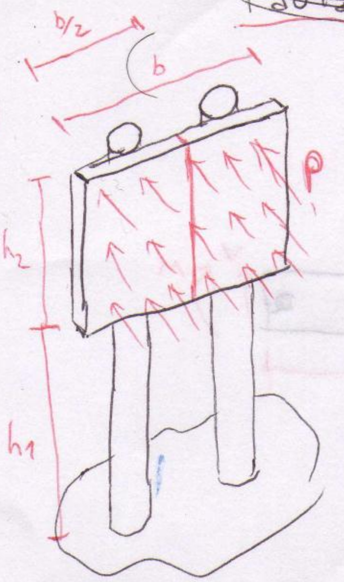
(Los momentos de área se pueden superponer!!)

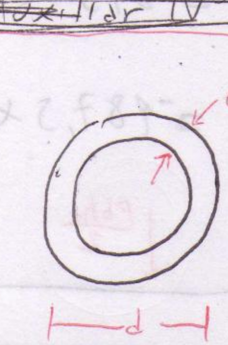
$$I_z = \frac{\pi}{4}(r_2^4 - r_1^4) = \frac{\pi}{64}(d_2^4 - d_1^4)$$

$$b = 2(r_2 - r_1)$$

~~Pauta Auxiliar N°4 ME3202~~ ~~2019~~

P1





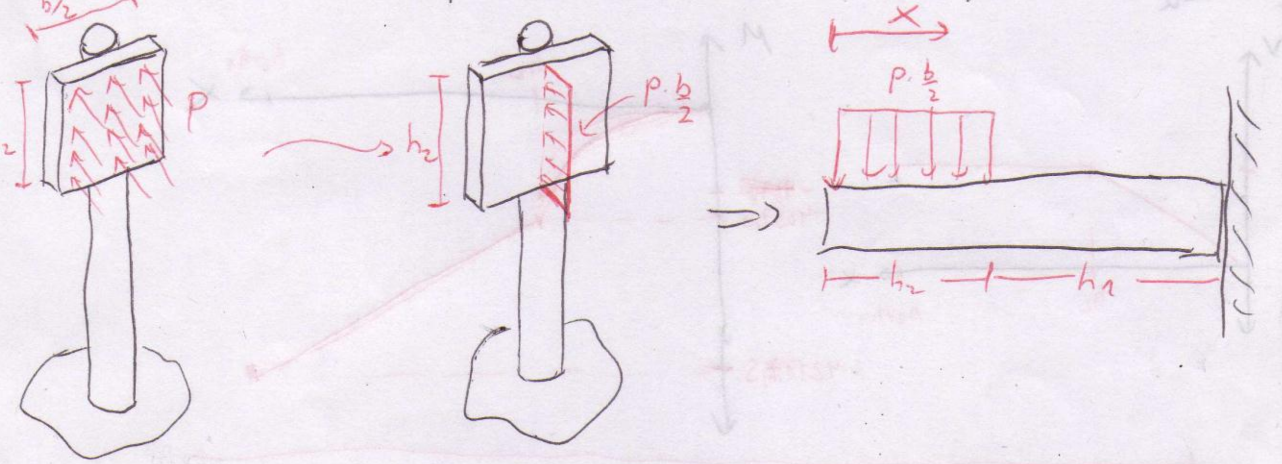
$P = 75\ lb/ft$

$h_1 = 20\ ft$

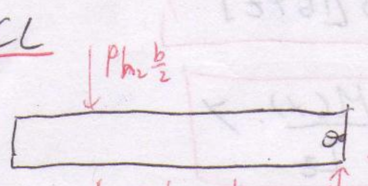
$h_2 = 5\ ft$

$b = 10\ ft$

• Veamos como afecta la presión del viento a cada poste



• DCL



• $\sum F_y = 0 \Rightarrow \boxed{R = \frac{P h_2 b}{2} = 1875\ lb}$

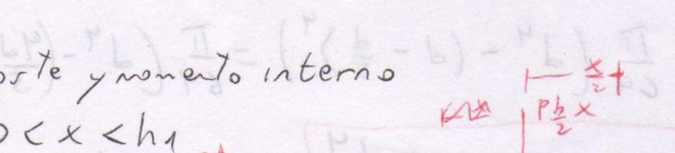
• $\sum_a M_z = 0 \Rightarrow -M_R + P h_2 \frac{b}{2}\left(h_1 + \frac{h_2}{2}\right) = 0$

$\boxed{\therefore M_R = \cancel{3837,5}\ [lb \cdot ft]}$

$\boxed{42187,5\ [lb \cdot ft]}$

• Corte y momento interno

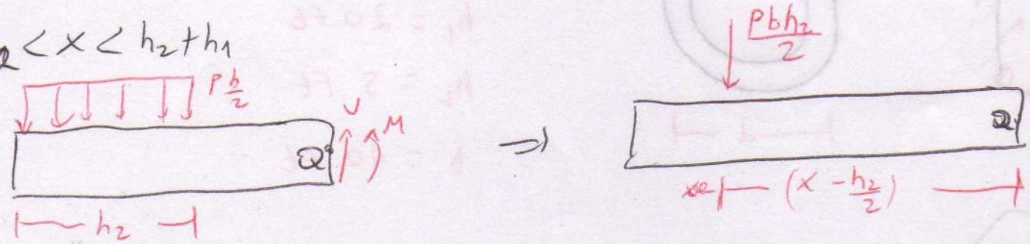
* $0 < x < h_1$



$$\cdot \sum F_y = 0 \Rightarrow V(x) = \frac{Pb}{2}x = 375x$$

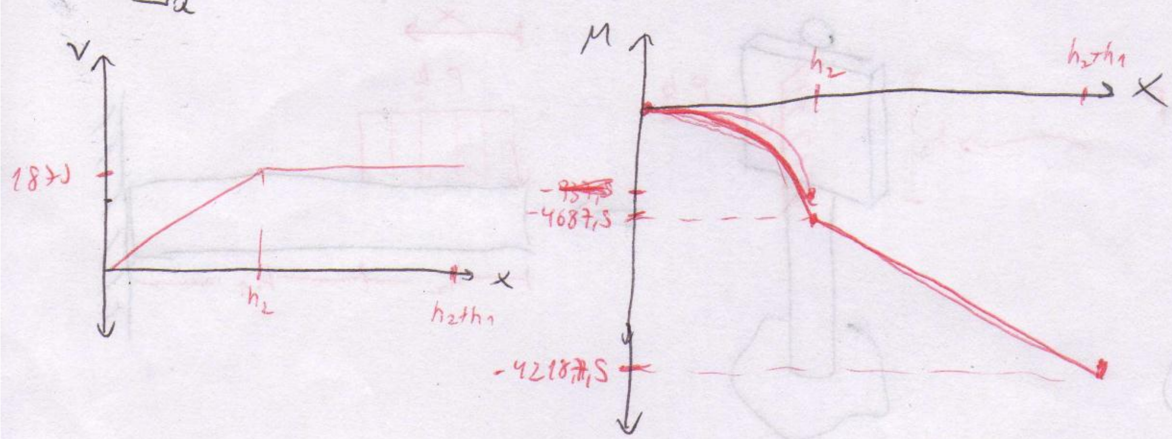
$$\cdot \sum_a M_z = 0 \Rightarrow M(x) = -\frac{Pbx^2}{4} = -187{,}5x^2$$

• $h_2 < x < h_2 + h_1$



$$\cdot \sum F_y = 0 \Rightarrow V(x) = \frac{Pbh_2}{2} = 1875$$

$$\cdot \sum_a M_z = 0 \Rightarrow M(x) = -\frac{Pbh_2}{2}\left(x - \frac{h_2}{2}\right) = -1875x + 4687{,}5$$



$$\boxed{\therefore V_{max} = 1875\ lb \quad y \quad |M_{max}| = 42187{,}5\ [lb\ ft]}$$

a) Esfuerzo normal debido a Flexión: $\boxed{\sigma_x = -\frac{M(x)}{I_z} \cdot y}$

$$\rightarrow \sigma_{max} = -\frac{M_{max} \cdot y_{max}}{I_z} \quad / \quad M_{max} = -42187{,}5\ [lb \cdot ft], \quad y_{max} = d/2$$

$$* I_z = \frac{\pi}{64}\left(d^4 - (d-2t)^4\right) = \frac{\pi}{64}\left(d^4 - \left(d - \frac{d}{5}\right)^4\right) = \frac{\pi}{64}\left(d^4 - \left(\frac{4d}{5}\right)^4\right)$$

$$= \frac{\pi}{64} \cdot \frac{369}{625} d^4 \Rightarrow \boxed{I_z = 0{,}02898\, d^4}$$

$$\therefore \sigma_{max} = \frac{42187{,}5\,[lb\cdot ft]\cdot \frac{d}{2}}{0{,}02898\,d^4} = 7500\,\left[\frac{lb}{ft^2}\right]$$

$$\Rightarrow \frac{727872{,}671}{d^3} = 7500 \quad \boxed{\therefore d = 4{,}595\,[ft]}$$

b) Esfuerzo de corte debido a flexión : $\tau_{max} = \frac{V_{max} Q}{I \cdot b}$

$V_{max} = 1875\,[lb]$; $b = 2(d-(d-2t)) = 2(d-(d-\frac{d}{5})) = 2(d-\frac{4d}{5})$

$$\boxed{\therefore b = \frac{2}{5}d}$$

$$\cancel{I_z = \frac{\pi}{64}(d^4-(d-2t)^4)} \quad \boxed{I_z = 0{,}02898\,d^4}$$

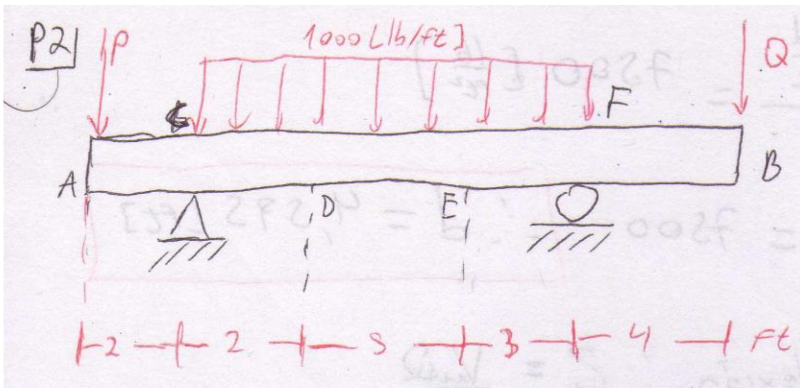
$$Q = \frac{2}{3}(r_2^3 - r_1^3) = \frac{2}{3}\cdot\frac{1}{8}\left(d^3-(d-2t)^3\right) = \frac{1}{12}\left(d^3-\left(\frac{4d}{5}\right)^3\right)$$

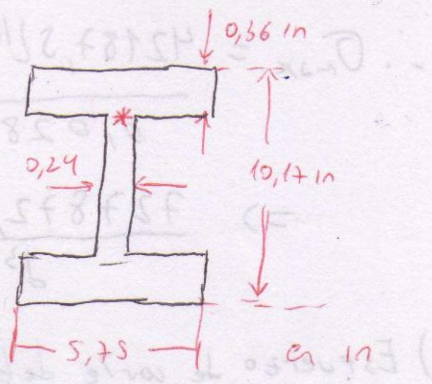
$$= \frac{1}{12}\cdot\frac{1}{125}\,d^3 = 6{,}667\cdot 10^{-4}\,d^3$$

$$\Rightarrow \tau_{max} = \frac{1875\cdot 6{,}667\cdot 10^{-4}\,d^3}{0{,}02898\,d^4\cdot\frac{2}{5}d} = \frac{107{,}83}{d^2} = 2000\ psi$$

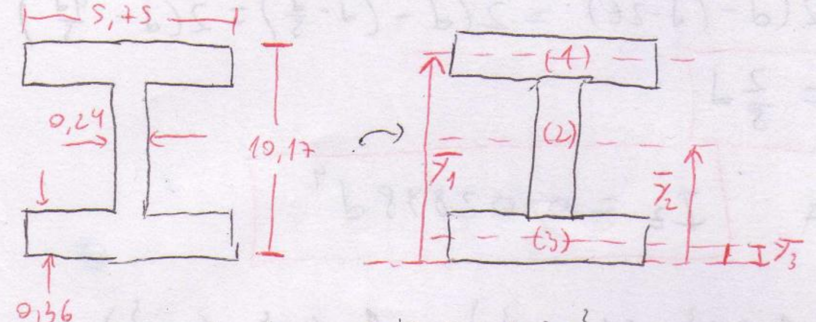
$$\boxed{\therefore d = 0{,}232\,[ft]}$$

P2





• Encontremos el $\bar{I}_z$ de la sección completa



$$\bar{Y}_T = \frac{10,17}{2} = 5,085 \text{ in}$$

$$I_1 = \frac{b\cdot h^3}{12} = \frac{5,75\cdot 0,36^3}{12} = 0,0223 \text{ in}^4$$

$$I_3 = 0,0223 \text{ in}^4$$

$$I_2 = \frac{0,24\cdot(10,17-2\cdot 0,36)^3}{12} = 16,878 \text{ in}^4$$

* Trasladamos las inercias al eje neutro ($\bar{Y}_T = \bar{Y}_2$) de la figura completa

• $\bar{I}_1 = I_1 + 5,75\cdot 0,36\,(\bar{Y}_1-\bar{Y}_2)^2 = I_1 + 5,75\cdot 0,36\left(\left(10,17-\frac{0,36}{2}\right)-\frac{10,17}{2}\right)^2$

$$\boxed{\therefore \bar{I}_1 = 49,824 \text{ in}^4}$$

$$\boxed{\cdot\ \bar{I}_2 = I_2}$$

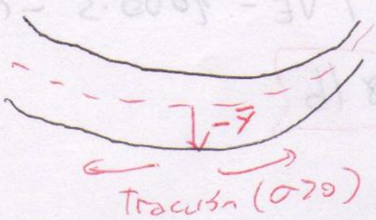
• $\bar{I}_3 = I_3 + 5,75\cdot 0,36\,(\bar{Y}_2-\bar{Y}_3)^2 = I_3 + 5,75\cdot 0,36\left(\frac{10,17}{2}-\frac{0,36}{2}\right)^2$

$$\boxed{\therefore \bar{I}_3 = 49,824 \text{ in}^4}$$

* Finalmente

$$\boxed{\bar{I}_z = \bar{I}_1+\bar{I}_2+\bar{I}_3 = 116,526\,[\text{in}^4] = 5,611\cdot 10^{-3}\,[\text{ft}^4]}$$

* El problema posee 4 incógnitas (P, Q y las reacciones), por lo que se necesita información adicional, que está dada por los esfuerzos en D y E



$$\sigma = -\frac{M}{I_z} y = \frac{M\bar{y}}{I_z}$$

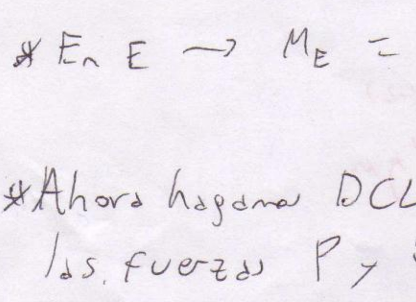
$$\boxed{\therefore M = \frac{\sigma I_z}{\bar{y}}}$$

* En D $\rightsquigarrow$ $M_D = \dfrac{2{,}07\,[ksi] \cdot 5{,}611 \cdot 10^{-3}\,[ft^4]}{5{,}085\,[in] \cdot 0{,}0833\,\left[\frac{ft}{in}\right]} = 27{,}42\,[lb\cdot ft]$

* En E $\rightarrow$ $M_E = \dfrac{0{,}776\,[ksi] \cdot 5{,}611 \cdot 10^{-3}\,[ft^4]}{5{,}085\,[in] \cdot 0{,}0833\,\frac{ft}{in}} = 10{,}28\,[lb\cdot ft]$

* Ahora hagamos DCL por tramos para encontrar las reacciones y las fuerzas P y Q

$\rightsquigarrow$ DCL tramo DE



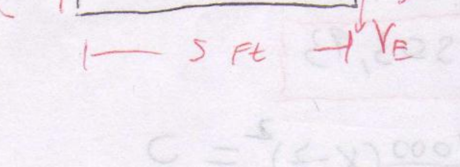
$\bullet \sum_E M_z = 0 \Rightarrow 5V_D + M_D - M_E + 1000 \cdot 5 \cdot 2{,}5 = 0$

$$\boxed{\therefore V_D = 2503{,}43\ lb}$$

$\bullet \sum F_y = 0 \Rightarrow V_D - V_E - 1000 \cdot 5 = 0$

$$\boxed{V_E = -2496{,}57\ lb}$$

$\rightsquigarrow$ DCL tramo ACD



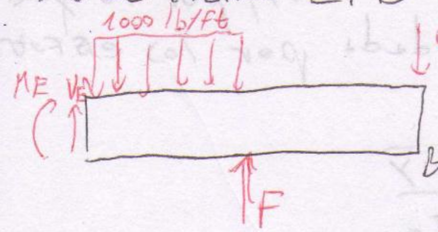
$\bullet \sum_A M_z = 0 \Rightarrow 2C - 4V_D + M_D - 1000 \cdot 2 \cdot 3 = 0$

$$\boxed{\therefore C = 7993{,}15\ lb}$$

$\bullet \sum F_y = 0 \Rightarrow -P + C - V_D - 1000 \cdot 2 = 0$

$$\boxed{\therefore P = 3489{,}72\ lb}$$

$\rightsquigarrow$ DCL tramo EFB



$$\sum_B M_z = 0 \Rightarrow 4F + 7V_E - 1000\cdot 3\cdot(4+1{,}5) = 0$$

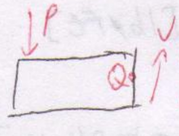
$$\boxed{\therefore F = 8493{,}998\ lb}$$

$$\sum F_y = 0 \Rightarrow -Q + F + V_E - 1000\cdot 3 = 0$$

$$\boxed{Q = 2997{,}428\ lb}$$

• Corte interno y momento interno

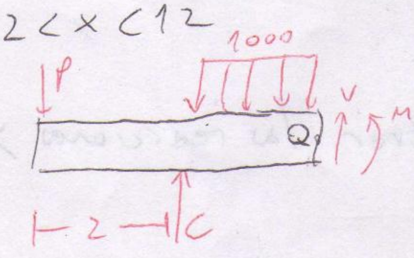
$\rightarrow 0 < x < 2$

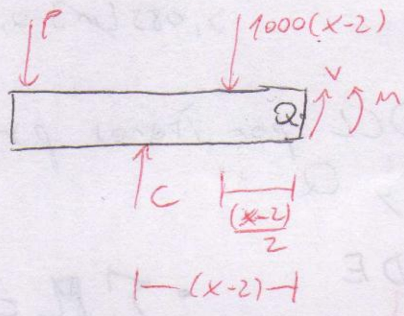


$$\sum F_y = 0 \Rightarrow V = P = 3489{,}22\ lb$$

$$\sum M = 0 \Rightarrow M(x) = -Px = -3489{,}22x\ lb\cdot ft$$

$\rightarrow 2 < x < 12$





$$\sum F_y = 0 \Rightarrow V(x) - P + C - 1000(x-2) = 0$$

$$V(x) = 1000x + 2\cdot 1000 + P - C$$

$$\boxed{V(x) = 1000x \cancel{+} 2503{,}93}$$

$$\sum_Q M_z = 0 \Rightarrow M(x) + Px - C(x-2) + \frac{1000}{2}(x-2)^2 = 0$$

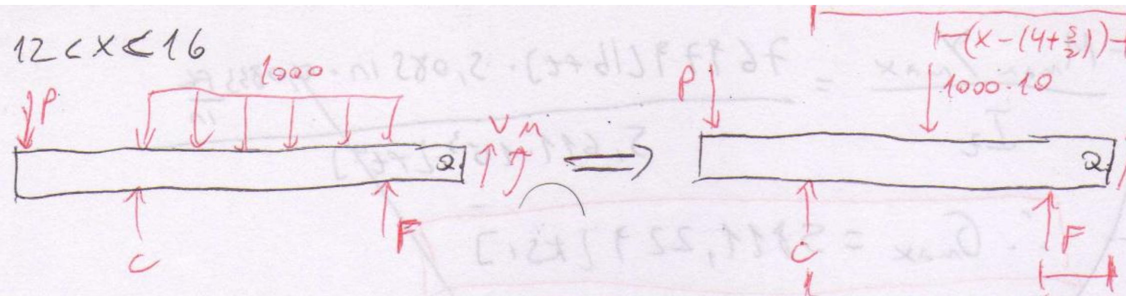
$$M(x) = -500(x^2 - 2x + 4) - Px + Cx - 2C$$

$$= -500x^2 + (1000 + P + C)x + (500\cdot 4 + 2C)$$

$$\cancel{M(x) = -500x^2 + 3503{,}43x + 13986{,}3}$$

$$\boxed{M(x) = -500x^2 + 5503{,}43x - 15986{,}3}$$

$\rightarrow 12 < x \leq 16$



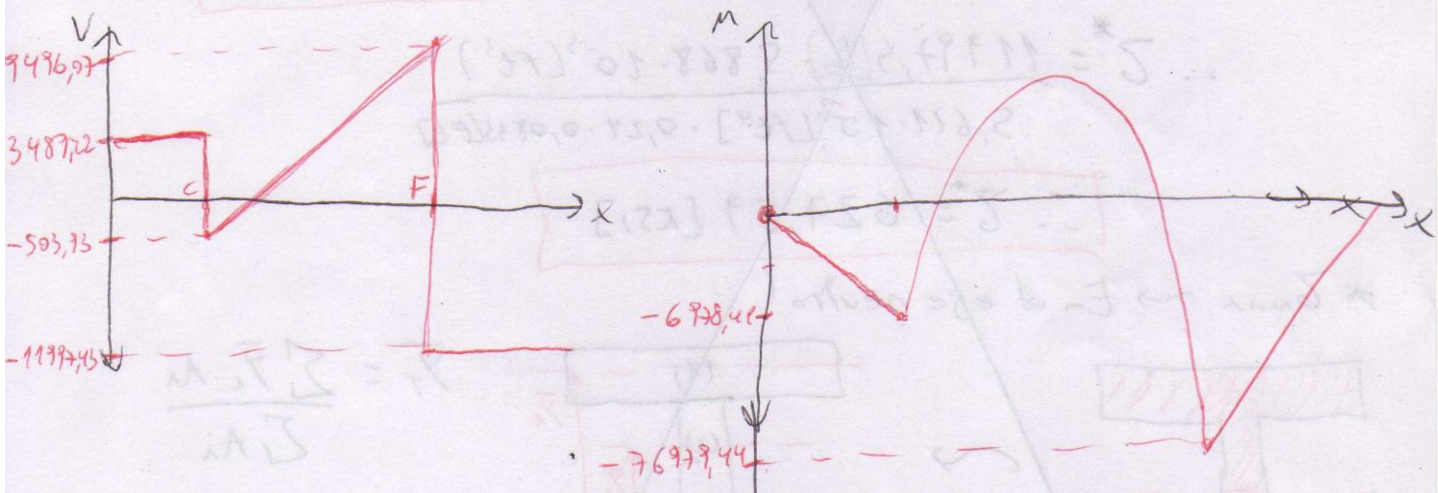
$\cdot \sum F_y = 0 \Rightarrow -P + C - 10000 + F + V = 0$

$$\boxed{V = -11997,43 \text{ lb}}$$

$\cdot \sum_{a} M_z = 0 \Rightarrow Px - C(x-2) + 10000(x-6,5) - F(x-12) + M(x) = 0$

$$M(x) = (-P + C - 10000 + F)x + (\cancel{-2C} + 10000 \cdot 6,5 - 12F)$$

$$\boxed{M(x) = 2997,428x - 52914,276}$$

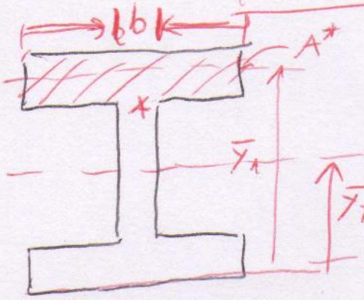


$$\boxed{\begin{aligned} V_{max} &= -11997,43 \, [\text{lb}] \\ M_{max} &= -76979,44 \, [\text{lb} \cdot \text{ft}] \end{aligned}}$$

b) $\sigma_{max} = -\dfrac{M_{max}\, Y_{max}}{I_z} = \dfrac{76947\,[lb\,ft] \cdot 5,085\,[in] \cdot 0,0833\,\frac{ft}{in}}{5,611 \cdot 10^{-3}\,[ft^4]}$

$$\boxed{\sigma_{max} = 5811,229\,[KSI]}$$

c)



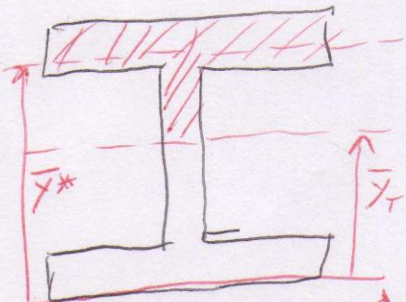
*Utilicemos Vmax para el cálculo

$$\tau^* = \frac{V_{max}\, Q}{I_z \cdot b}$$

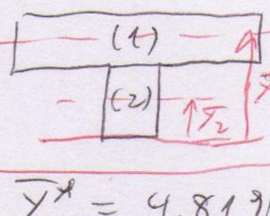
$\rightsquigarrow Q = A^*(\bar{y}_1 - \bar{y}_T) = 5,868 \cdot 10^{-3}\,[ft^3]$

$$\therefore \boxed{\tau^* = 627,59\,[KSI]}$$

$\tau_{max} \rightsquigarrow$ En el eje neutro!

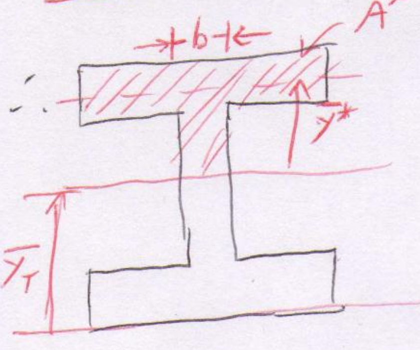


$\longrightarrow$



$$\frac{\sum \bar{y}_i \cdot A_i}{\sum A_i} = \bar{y}^*$$

$$\boxed{\bar{y}^* = 4,819\,[in] = 0,401\,ft}$$



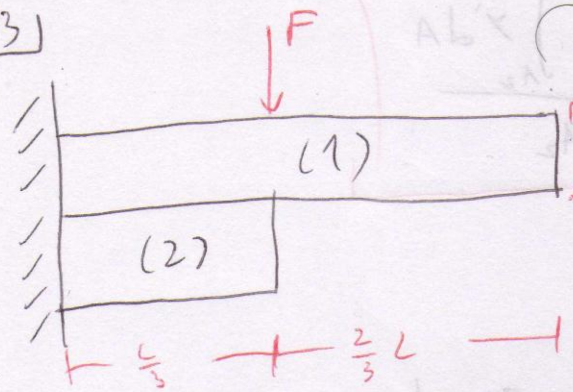
- $A^* = 3,204\ in^2 = 0,0222\ ft^2$

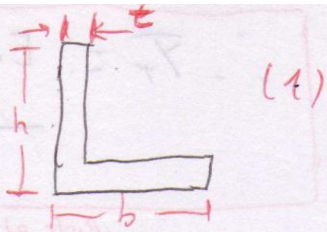
$\therefore \boxed{Q = A^* \cdot \bar{y}^* = 8,918 \cdot 10^{-3}\,[ft^3]}$

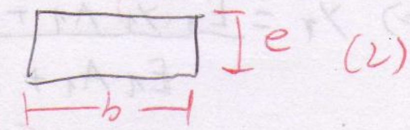
- Finalmente:

$$\boxed{\tau = 954,435\,[KSI]}$$

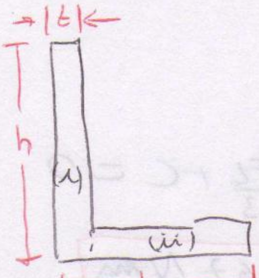
P3]







a) Viga (1)



$$\bar{y}_1 = \frac{\sum_i y_i \cdot A_i}{\sum_i A_i} = \frac{\bar{y}_{(i)} A_{(i)} + \bar{y}_{(ii)} \cdot A_{(ii)}}{A_{(i)} + A_{(ii)}}$$

$$= \frac{\frac{h}{2} \cdot h \cdot t + \frac{t}{2} \cdot t(b-t)}{ht + t(b-t)}$$

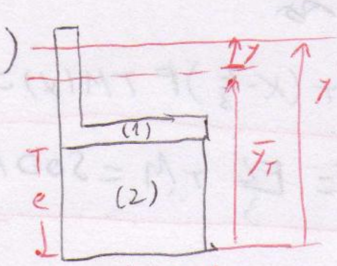
$$\Rightarrow \boxed{\bar{y}_1 = 3{,}038\ cm}$$

$$\rightarrow \bar{I}_{(i)} = I_{(i)} + \delta^2 A_{(i)} = \frac{t h^3}{12} + \left(\bar{y} - \frac{h}{2}\right) \cdot t \cdot h = 249{,}249\ cm^4$$

$$I_{(ii)} = I_{(ii)} + \delta^2 A_{(ii)} = \frac{(b-t) \cdot t^3}{12} + \left(\bar{y} - \frac{t}{2}\right)^2 (b-t) \cdot t = 91{,}38\ cm^4$$

$$\boxed{\therefore I_1 = 3.406 \cdot 10^{-6}\ m^4}$$

b)



- Para calcular el eje neutro de una viga compuesta se usa:

$$E_1 \int_{A_1} y\, dA + E_2 \int_{A_2} y\, dA = 0$$

- Haciendo $y = y' - \bar{y}_T$

$$\Rightarrow E_1 \int_{A_1} (y' - \bar{y}_T) dA + E_2 \int_{A_2} (y' - \bar{y}_T) dA = 0$$

$$\Rightarrow \left[E_1 \int_{A_1} y' dA + E_2 \int_{A_1} y' dA\right] + \left[E_2 \bar{y}_T A_1 + E_2 \bar{y}_T A_2\right] = 0$$

$$\boxed{\therefore \bar{Y}_T = \frac{E_1\int_{A_1} y'\,dA + E_2\int_{A_2} y'\,dA}{E_1 \cdot A_1 + E_2 A_2}}$$

Parte al $\quad \frac{e}{2}$

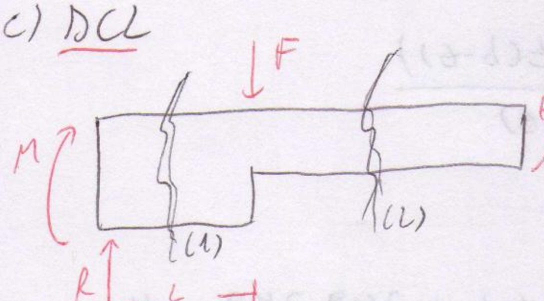
$$\Rightarrow \bar{Y}_T = \frac{E_1 \bar{y}_1 A_1 + E_2 \bar{y}_2 A_2}{E_1 A_1 + E_2 A_2}$$

Desde la base!

$$= \frac{E_1(\bar{Y}_1 + e)(ht + t(b-t)) + E_2 \cdot \frac{e}{2} \cdot eb}{E_1(ht + t(b-t)) + E_2 eb}$$

$$\boxed{\therefore \bar{Y}_T = 4{,}6967 \text{ cm}}$$

c) DCL



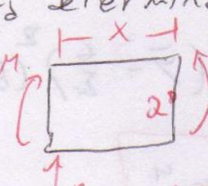
$\cdot \sum M_z = 0 \rightsquigarrow M - \frac{FL}{3} + C = 0$

$\boxed{\therefore M = 166{,}667 \text{ N·m}}$

$\cdot \sum F_y = 0 \rightsquigarrow \boxed{R = F = 1000 \text{ N}}$

• Hacemos cortes para determinar la distribución de momento interno

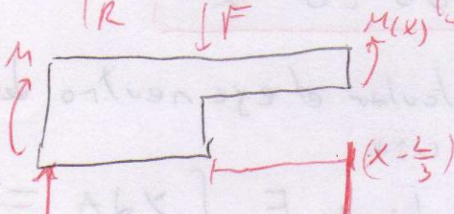
(1) $0 < x < \frac{L}{3} \Rightarrow$



$\cdot \sum M_z = 0 \rightsquigarrow -M - Rx + M(x) = 0$

$\boxed{\therefore M(x) = 166{,}667 + 1000x}$

(2) $\frac{L}{3} < x < L \Rightarrow$



$\cdot \sum M_z = 0 \rightsquigarrow$

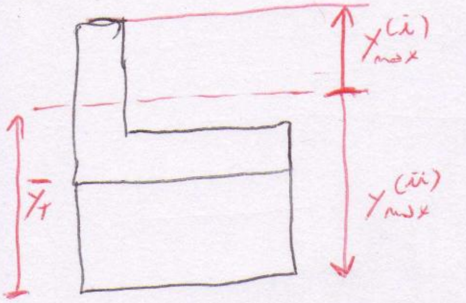
$\Rightarrow M - Rx + (x - \frac{L}{3})F + M(x) = 0$

$\boxed{\therefore M(x) = \frac{FL}{3} + M = 500 \text{ N·m}}$

• El momento máximo se da para cada intervalo (1): $M_{max} = M(\frac{L}{2}) = 166{,}667 + 1000 \cdot \frac{L}{3}$

$\boxed{M_{max} = 500 \text{ N·m}}$

• Para determinar Y_{max}, se hace para cada material



$\Rightarrow Y_{max}^{(i)} = h+e-\bar{Y}_T = 0{,}143$ cm (superior)

$Y_{max}^{(ii)} = -\bar{Y}_T = 4{,}6967$ cm (inferior)

• Usando las fórmulas para Esf. máximo en vigas compuestas:

$$\boxed{\sigma_{(i),max} = \frac{-E_1}{E_1 I_1 + E_2 I_2} \cdot M_{max} \cdot Y_{max}^{(i)} = -4{,}152\,[MPa]\ \text{(Compresión)}}$$

$$\boxed{\sigma_{(ii),max} = \frac{-E_2}{E_1 I_1 + E_2 I_2}\, M_{max} \cdot Y_{max}^{(ii)} = 1{,}507\,[MPa]\ \text{(Tracción)}}$$

•(2): $M_{max} = 500$ Nm (cte)

$Y_{max} = 12 - \bar{Y}_1 = 8{,}962$ [cm] (Parte superior)

$$\Rightarrow \boxed{\sigma_{max} = -\frac{M_{max} \cdot Y_{max}}{I_1} = 13{,}454\,[MPa]}$$