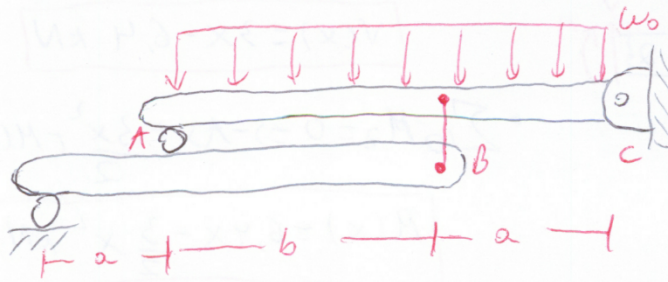


P1)



$$w_0 = 3 \frac{\text{kN}}{\text{m}}$$

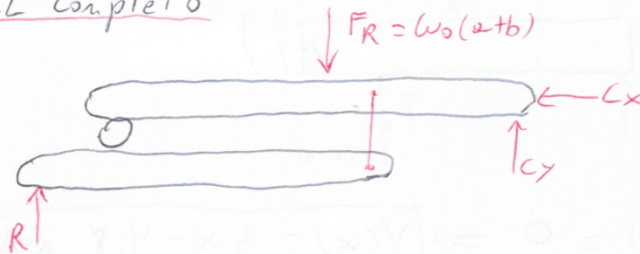
$$a = 1 \text{ m}$$

$$b = 3 \text{ m}$$



• DCL's

i) DCL completo



$$\sum F_x = 0 \Rightarrow C_x = 0$$

$$\sum F_y = 0 \Rightarrow R + C_y - F_R = 0$$

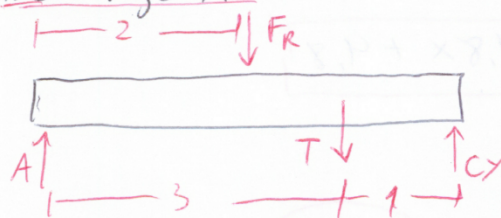
$$C_y = F_R - R \quad (i)$$

$$\sum M_z = 0 \Rightarrow 2F_R - 5R = 0$$

$$\therefore R = \frac{2}{5}F_R = 4,8 \text{ kN}$$

$$\text{en (i)} \quad C_y = 7,2 \text{ kN}$$

ii) DCL viga AC



$$\sum F_y = 0 \Rightarrow A - F_R - T + C_y = 0$$

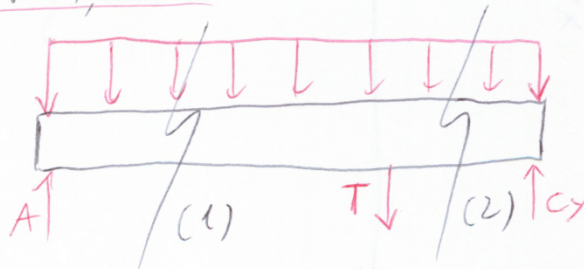
$$A = F_R + T - C_y \quad (ii)$$

$$\sum M_z = 0 \Rightarrow -2F_R - 3T + 4C_y = 0$$

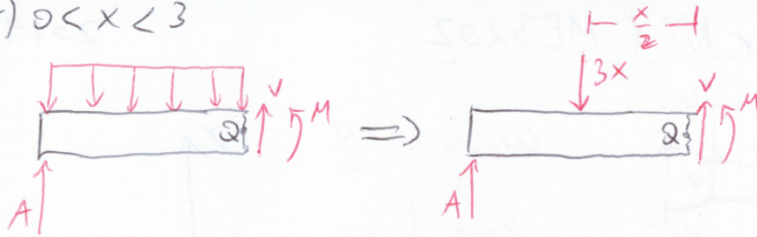
$$\therefore T = \frac{1}{3}(4C_y - 2F_R) = 1,6 \text{ kN}$$

$$\text{en ii)} \quad A = 6,4 \text{ kN}$$

• Cálculo de $V(x)$, $M(x)$



1) $0 < x < 3$



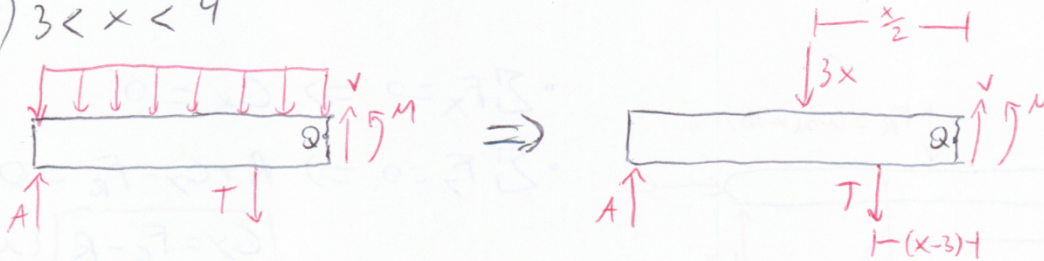
$$\sum F_y = 0 \Rightarrow A - 3x + V(x) = 0$$

$$V(x) = 3x - 6,4 \text{ kN}$$

$$\sum M_z = 0 \Rightarrow -Ax + \frac{3x^2}{2} + M(x) = 0$$

$$M(x) = 6,4x - \frac{3}{2}x^2 \text{ kN}\cdot\text{m}$$

2) $3 < x < 4$

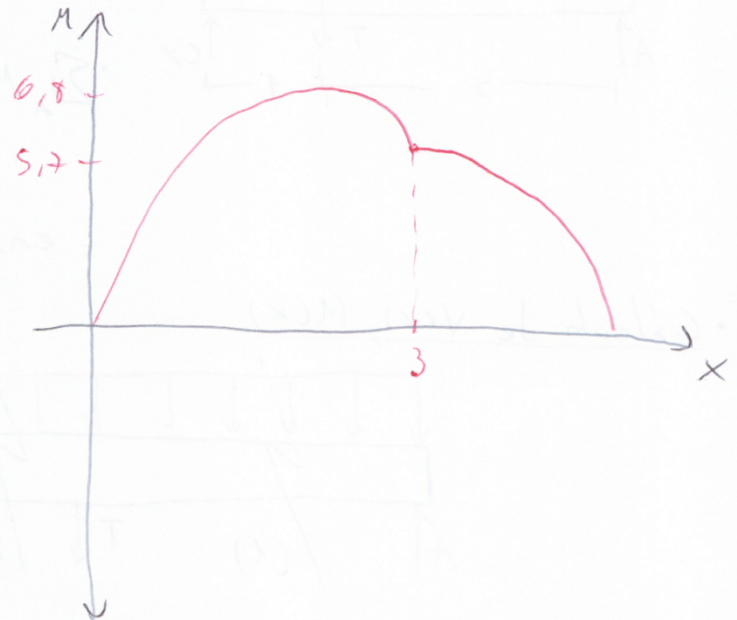
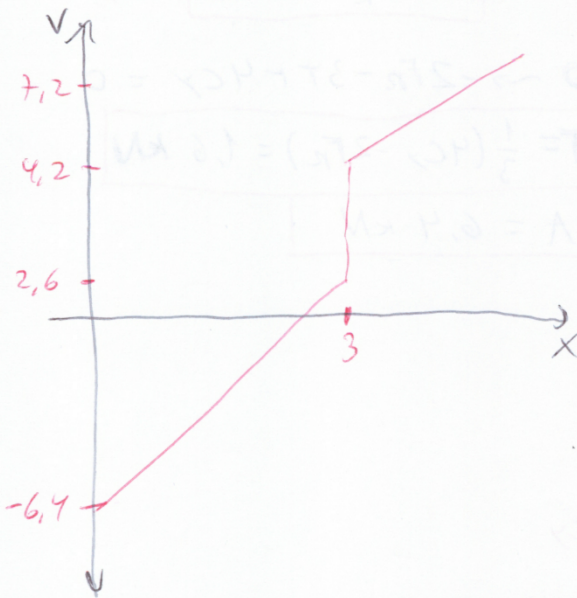


$$\sum F_y = 0 \Rightarrow A - 3x - T + V(x) = 0 \Rightarrow V(x) = 3x - 4,8 \text{ kN}$$

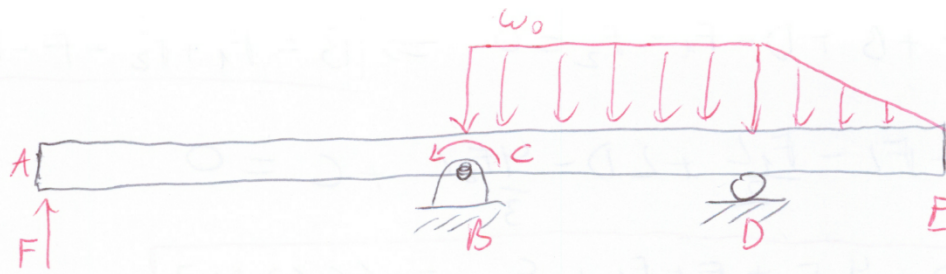
$$\sum M_z = 0 \Rightarrow -Ax + \frac{3}{2}x^2 + T(x-3) + M(x) = 0$$

$$\frac{3}{2}x^2 + x(T-A) - 3T + M(x) = 0$$

$$\therefore M(x) = -\frac{3}{2}x^2 + 4,8x + 4,8$$



P2)



$$F = 1000 \text{ N}$$

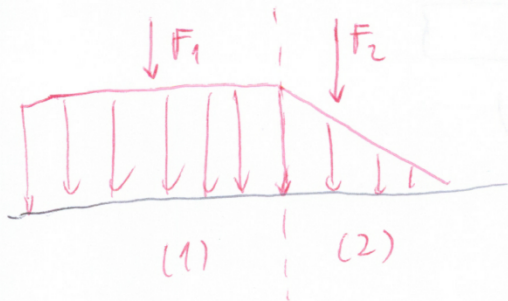
$$C = 200 \text{ Nm}$$

$$L = 2 \text{ m}$$

$$w_0 = 500 \text{ N/m}$$

• DCL's: ¿Cómo determinar f.e.s. equivalentes?

$$F_{eq} = \int w(x) dx \quad ; \quad x = \frac{\int x w(x) dx}{\int w(x) dx}$$



→ Podemos separar la f.e.s. distribuida en dos, una constante y una lineal

$$\hookrightarrow (1): w(x) = w_0 \Rightarrow F_1 = \int_0^{L/2} w_0 dx = w_0 L = 1000 \text{ N}$$

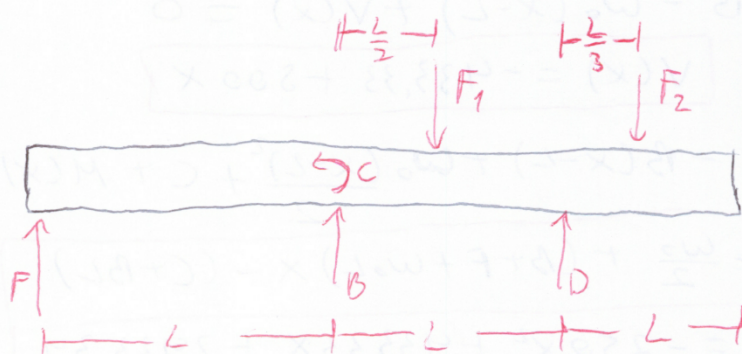
$$x_1 = \frac{\int_0^{L/2} x w_0 dx}{w_0 L} = \frac{L}{2} = 1 \text{ m}$$

$$\hookrightarrow (2): w(x) = \frac{w_0}{L} (L-x) \Rightarrow F_2 = \int_0^{L/2} \frac{w_0}{L} (L-x) dx = \frac{w_0 L}{2} = 500 \text{ N}$$

$$x_1 = \frac{\int_0^{L/2} \frac{w_0}{L} (L-x) x dx}{\frac{w_0 L}{2}} = \frac{L}{3} = \frac{2}{3} \text{ m}$$

(Detalle de cálculos en apéndice A.1)

DCL



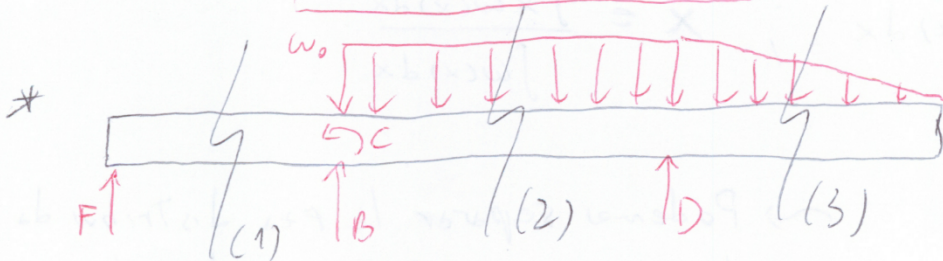
$$\bullet \sum F_y = 0 \Rightarrow F + B + D - F_1 - F_2 = 0 \Rightarrow \boxed{B = F_1 + F_2 - F - D} \quad (i)$$

$$\bullet \sum_b M_z = 0 \Rightarrow -FL - \frac{F_1 L}{2} + LD - \frac{4F_2 L}{3} + C = 0$$

$$\boxed{D = \frac{4}{3}F_2 + F + \frac{F_1}{2} + \frac{C}{L} = 2066,67 \text{ [N]}}$$

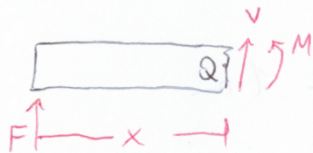
en (i)

$$\boxed{B = -1566,67 \text{ [N]}}$$



• Cálculo de cargas internas:

$$1) 0 < x < L$$

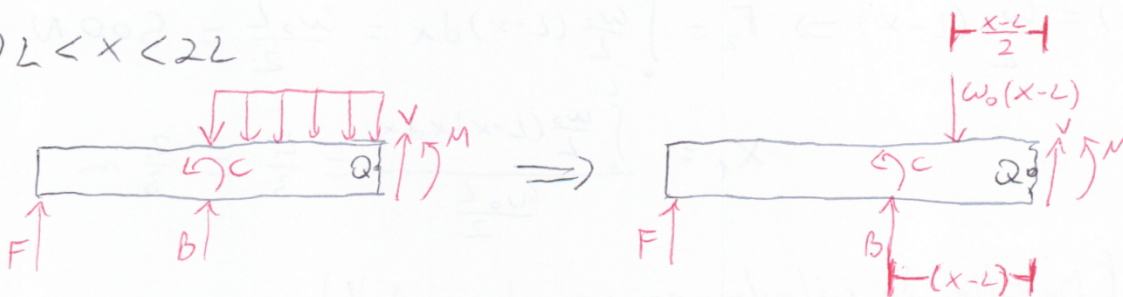


$$\bullet \sum F_y = 0 \leadsto \boxed{V = -F = -1000 \text{ N}}$$

$$\bullet \sum_a M_z = 0 \leadsto -Fx + M = 0 \Rightarrow M(x) = Fx$$

$$\boxed{M(x) = 1000x \text{ Nm}}$$

$$2) L < x < 2L$$



$$\bullet \sum F_y = 0 \leadsto F + B - w_0(x-L) + V(x) = 0$$

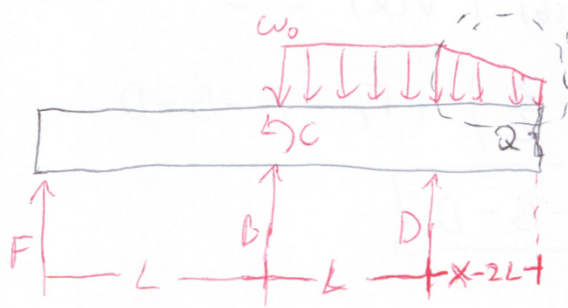
$$\boxed{V(x) = -433,33 + 500x}$$

$$\bullet \sum_a M_z = 0 \leadsto -Fx - B(x-L) + w_0 \frac{(x-L)^2}{2} + C + M(x) = 0$$

$$\boxed{M(x) = -\frac{w_0}{2} + (B+F+w_0L)x - (C+BL)}$$

$$\boxed{M(x) = -250x^2 + 433,33x + 2933,34}$$

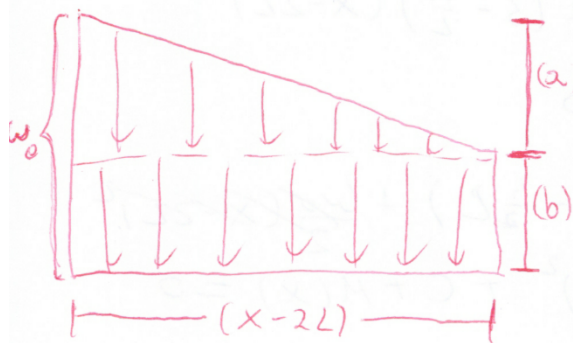
$$2L < x < 3L$$



¿Cómo calcular este segmento de fza distribuida?

→ Separarlo en dos secciones, una constante y una lineal, obteniendo dos fzas. equivalentes (A continuación)

→ Usar la función $w(x)$ y reintegrar (Apéndice A.2)



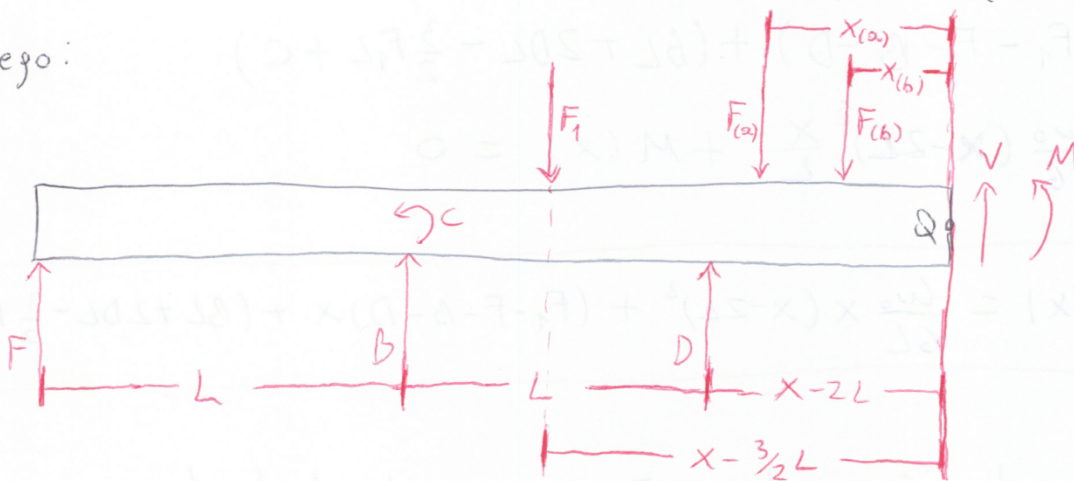
$$* w(x) = \frac{w_0}{L} (L - x)$$

$$\hookrightarrow (b) = \frac{w_0}{L} (L - (x - 2L)) = w_0 \left(3 - \frac{x}{L}\right)$$

(Fza. lineal evaluada en $x = (x - 2L)$
nos da el valor de $w(x)$ en ese punto)

$$\hookrightarrow (a) = w_0 - w_0 \left(3 - \frac{x}{L}\right) = w_0 \left(\frac{x}{L} - 2\right)$$

Luego:



Con:

$$F_{(a)} = w_0 \left(\frac{x}{L} - 2\right) (x - 2L) \quad \wedge \quad X_{(a)} = \frac{x - 2L}{2}$$

$$F_{(b)} = \frac{w_0}{2} \left(3 - \frac{x}{L}\right) (x - 2L) \quad \wedge \quad X_{(b)} = \frac{2}{3} (x - 2L)$$

$$\Rightarrow \sum F_x = 0 \leadsto F + B + D - F_1 - F_{(a)} - F_{(b)} + V(x) = 0$$

$$\therefore V(x) = \omega_0(x-2L) \left(\frac{x}{L} - 2 + 3 - \frac{x}{L} \right) + F_1 - F - B - D$$

$$V(x) = \omega_0(x-2L) + F_1 - F - B - D$$

$$\Rightarrow \sum M_2 = 0 \leadsto -Fx - B(x-L) - D(x-2L) + F_1 \left(x - \frac{3}{2}L \right) + \frac{\omega_0}{2} \left(\frac{x}{L} - 2 \right) (x-2L)^2 + \frac{\omega_0}{3} \left(3 - \frac{x}{L} \right) (x-2L)^2$$

$$+ C + M(x) = 0$$

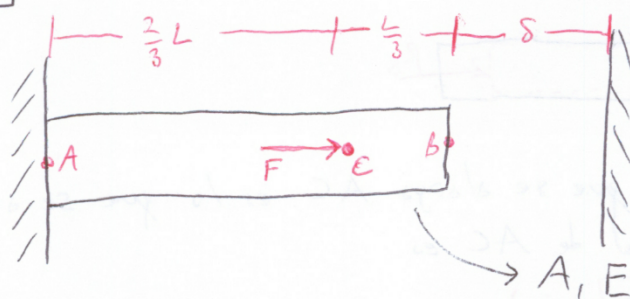
$$\leadsto -Fx - B(x-L) - D(x-2L) + F_1 \left(x - \frac{3}{2}L \right) + \frac{\omega_0}{2} (x-2L)^2 + \omega_0 \left(\frac{1}{2} \frac{x}{L} - 1 + 1 - \frac{1}{3} \frac{x}{L} \right) (x-2L)^2 + C + M(x) = 0$$

$$\leadsto x(F_1 - F - B - D) + (BL + 2DL - \frac{3}{2}F_1L + C) + \frac{\omega_0}{6} (x-2L)^2 \frac{x}{L} + M(x) = 0$$

$$\therefore M(x) = \frac{\omega_0}{6L} x(x-2L)^2 + (F_1 - F - B - D)x + (BL + 2DL - \frac{3}{2}F_1L + C)$$

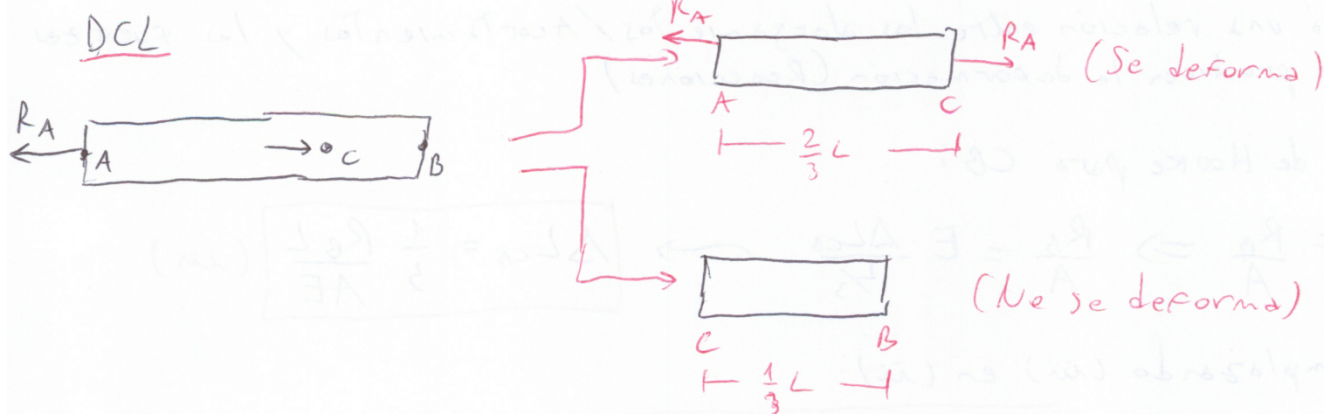
*C! > si hiciésemos los cortes comenzando desde el lado derecho? (Ver apéndice A.3)

P3)



* Para resolver problemas hiperestáticos con deformación lineal (donde haya que usar Ley de Hooke), conviene siempre encontrar una relación entre los alargamientos (o acortamientos) de las distintas elementos que componen el sistema

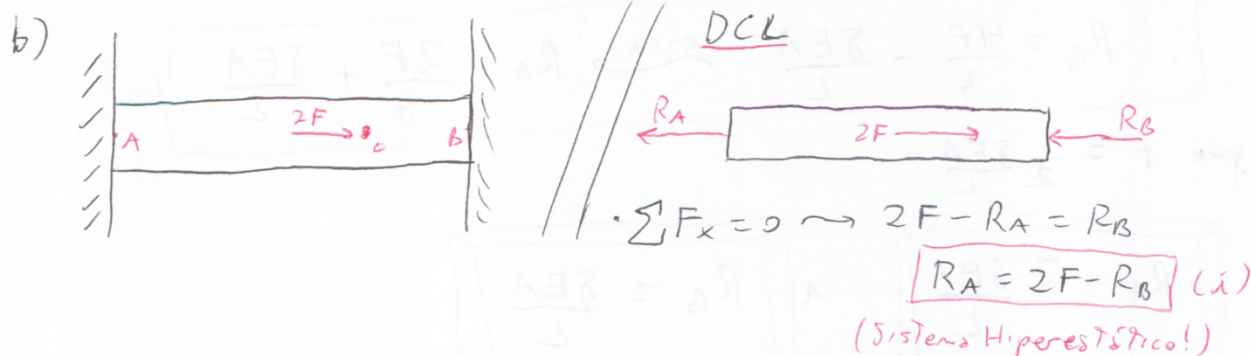
a) Analicemos qué parte de la barra se estira realmente. Corte justo antes y después de C:



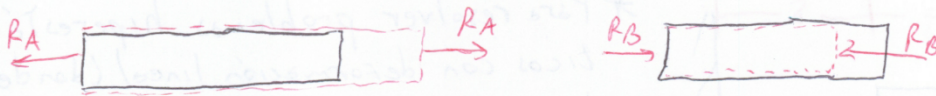
Luego, en la deformación, el largo inicial es $\frac{2}{3}L$ > NO L !!

$$\Rightarrow \left. \begin{aligned} \sigma &= \frac{F}{A} \\ \sigma &= E \cdot \varepsilon \\ \varepsilon &= \frac{\Delta L}{L_0} \end{aligned} \right\} \frac{F}{A} = E \cdot \frac{\Delta L}{L_0} \quad // \text{ Para que haya contacto con la pared, } \Delta L = \delta. \text{ Despejando } F$$

$$F = \frac{AES}{\frac{2}{3}L} = \frac{3AES}{2L}$$



* Haciendo un corte antes y después de C:



→ Una vez en contacto con la pared, lo que se alarga AC es lo que se acorta CB. Luego, el alargamiento total de AC es:

$$\Delta L_{AC} = \Delta L_{CB} + \delta \quad (iii)$$

→ Una vez encontrada la relación de alargamiento, ahora se debe usar la ec. constitutiva (Ley de Hooke) para cada elemento del sistema, lo cual nos dará una relación entre los alargamientos/acortamientos y las fuerzas que producen la deformación (Reacciones)

• Ley de Hooke para CB:

$$\sigma = \frac{R_B}{A} \Rightarrow \frac{R_B}{A} = E \cdot \frac{\Delta L_{CB}}{\frac{2}{3}L} \rightarrow \Delta L_{CB} = \frac{1}{3} \frac{R_B L}{AE} \quad (iv)$$

• Reemplazando (iv) en (iii):

$$\Delta L_{AC} = \frac{1}{3} \frac{R_B L}{AE} + \delta \quad (v)$$

• Ley de Hooke para AC:

$$\sigma = \frac{R_A}{A} \Rightarrow \frac{R_A}{A} = E \cdot \frac{\Delta L_{AC}}{\frac{1}{3}L} \quad // \text{ Reemplazando (i) y (v) en la expresión anterior:}$$

$$\Rightarrow \frac{(2F - R_B)}{A} = \frac{3E}{2L} \left(\delta + \frac{1}{3} \frac{R_B L}{AE} \right) \rightarrow 2F - R_B = \frac{3EA\delta}{2L} + \frac{R_B}{2}$$

$$\therefore R_B = \frac{4F}{3} - \frac{\delta EA}{L} \xrightarrow{\text{en (i)}} R_A = \frac{2F}{3} + \frac{\delta EA}{L}$$

• Dado que $F = \frac{3}{2} \frac{\delta EA}{L}$

$$R_A = 2 \frac{\delta EA}{L}$$

$$R_B = \frac{\delta EA}{L}$$