# AUX #4

P1 y P2 PAUTA AUX #3

P3 $$\mathbb{E}\left(\frac{X_1+\dots+X_k}{X_1+\dots+X_n}\right)=\sum_{i=1}^{k}\mathbb{E}\left(\frac{X_i}{X_1+\dots+X_n}\right) \quad \star$$

AHORA NOTEMOS QUE

$$1=\mathbb{E}\left(\frac{X_1+\dots+X_n}{X_1+\dots+X_n}\right)=\sum_{i=1}^{n}\mathbb{E}\left(\frac{X_i}{X_1+\dots+X_n}\right)$$
$$= n\,\mathbb{E}\left(\frac{X_1}{X_1+\dots+X_n}\right)$$
$$\Longrightarrow \quad \mathbb{E}\left(\frac{X_1}{X_1+\dots+X_n}\right)=\frac{1}{n}$$

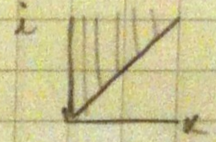
POR LO TANTO

$$\star=\frac{k}{n}.$$

P4 PRIMERO NOTEMOS QUE

$$\sum_{k=1}^{\infty}\mathbb{P}(N\geq k)=\sum_{k=0}^{\infty}\mathbb{P}(N>k)$$

POR LO TANTO PROBAREMOS LA IDENTIDAD PEDIDA SOLO CON UNA DE ELLAS




$$\sum_{k=1}^{\infty}\mathbb{P}(N\geq k)=\sum_{k=1}^{\infty}\sum_{i=k}^{\infty}\mathbb{P}(N=i)$$
$$=\sum_{i=1}^{\infty}\sum_{k=1}^{i}\mathbb{P}(N=i)$$
$$=\sum_{i=1}^{\infty}\mathbb{P}(N=i)\sum_{k=1}^{i}1=\sum_{i=1}^{\infty}i\,\mathbb{P}(N=i)=\mathbb{E}(N)$$

EL INTERCAMBIO DE LA SUMATORIA SE JUSTIFICA CON TONELLI

P5

a) PARA

$k=1 \quad \mathbb{P}(X \geq 1) = 1$

$k=2 \quad \mathbb{P}(X \geq 2) = 1$

$k=3 \quad \mathbb{P}(X \geq 3) = 1 - \underbrace{\mathbb{P}(X=2)}_{1/n} - \underbrace{\mathbb{P}(X=1)}_{0}$

$= 1 - 1/n$

$k=4 \quad \mathbb{P}(X \geq 4) = \mathbb{P}(X \geq 3, X \neq 3)$

$= \mathbb{P}(X \neq 3 \mid X \geq 3)\, \mathbb{P}(X \geq 3)$

$= \left(1 - \frac{2}{n}\right)\left(1 - \frac{1}{n}\right)$

POR INDUCCIÓN

$k < N \quad \mathbb{P}(X \geq k) = \mathbb{P}(X \geq k-1) \cdot \left(1 - \frac{k-2}{n}\right)$

$k = n+2 \quad \mathbb{P}(X \geq n+2) = 0$

b) $\mathbb{E}(X) = \sum_{k=1}^{\infty} \mathbb{P}(X \geq k)$

$= \sum_{k=1}^{n+1} \mathbb{P}(X \geq k)$

$= 1 + 1 + \left(1 - \frac{1}{n}\right) + \left(1 - \frac{1}{n}\right)\left(1 - \frac{2}{n}\right) + \dots + \left(1 - \frac{1}{n}\right) \cdots \left(1 - \frac{n-1}{n}\right)$

P6 a) $\mathrm{VAR}(X) = \mathbb{E}\left((X - \mathbb{E}(X))^2\right)$

$= \mathbb{E}\left(X^2 - 2X\mathbb{E}(X) + \mathbb{E}(X)^2\right)$

$= \mathbb{E}(X^2) - 2\mathbb{E}(X)\mathbb{E}(X) + \mathbb{E}(X)^2$

$= \mathbb{E}(X^2) - \mathbb{E}(X)^2$

b) $\mathrm{VAR}(aX+b) = \mathbb{E}\left((aX+b)^2\right) - \mathbb{E}(aX+b)^2$

$= \mathbb{E}(a^2X^2 + 2aXb + b^2) - [a\mathbb{E}(X) + b]^2$

$= a^2\mathbb{E}(X^2) + \cancel{2ab\mathbb{E}(X)} + \cancel{b^2} - [a^2\mathbb{E}(X)^2 - \cancel{2a\mathbb{E}(X)b} + \cancel{b^2}]$

$= a^2(\mathbb{E}(X^2) - \mathbb{E}(X)^2)$

$= a^2\, \mathrm{VAR}(X)$

c) $\mathbb{E}(Y) = \mathbb{E}\left(\frac{X-\mathbb{E}(X)}{\sqrt{VAR(X)}}\right) = \frac{1}{\sqrt{VAR(X)}}\,\mathbb{E}(X-\mathbb{E}(X))$

$= \frac{1}{\sqrt{VAR(X)}}(\mathbb{E}(X)-\mathbb{E}(X))$

$= 0$

$VAR(Y) = VAR\left(\frac{X-\mathbb{E}(X)}{\sqrt{VAR(X)}}\right) = \frac{1}{VAR(X)}\,VAR(X-\mathbb{E}(X))$

$= \frac{VAR(X)}{VAR(X)}$

**P7** $\mathbb{E}(X) = \sum_{k=0}^{\infty} k \cdot e^{-\lambda}\frac{\lambda^k}{k!} = \sum_{k=1}^{\infty} k e^{-\lambda}\frac{\lambda^k}{k!}$

$= \sum_{k=1}^{\infty} \lambda \cdot e^{-\lambda}\frac{\lambda^{k-1}}{(k-1)!} = \lambda \underbrace{\sum_{j=0}^{\infty} e^{-\lambda}\frac{\lambda^j}{j!}}_{1} = \lambda$

$\mathbb{E}(X^2) = \sum_{k=0}^{\infty} k^2 e^{-\lambda}\frac{\lambda^k}{k!} = \lambda\sum_{j=0}^{\infty}(j+1)e^{-\lambda}\frac{\lambda^j}{j!}$

$= \lambda\left[\sum_{j=0}^{\infty} j e^{-\lambda}\frac{\lambda^j}{j!} + \underbrace{\sum_{j=0}^{\infty} e^{-\lambda}\frac{\lambda^j}{j!}}_{1}\right]$

$= \lambda\left[\lambda\underbrace{\sum_{j=0}^{\infty} e^{-\lambda}\frac{\lambda^j}{j!}}_{1} + 1\right]$

$= \lambda(\lambda+1)$

$VAR(X) = \mathbb{E}(X^2) - \mathbb{E}(X)^2$

$= \lambda(\lambda+1) - \lambda^2$

$= \lambda.$

**P8** $\mathbb{P}\left(\sum_{i=1}^{n} X_i = k, M=n\right) = \mathbb{P}\left(\sum_{i=1}^{n} X_i = k \mid M=n\right)\mathbb{P}(M=n)$

$\overset{\perp}{=} \mathbb{P}\left(\sum_{i=1}^{n} X_i = k\right) e^{-\lambda}\frac{\lambda^n}{n!} = \binom{n}{k} p^k (1-p)^{n-k} \frac{e^{-\lambda}\lambda^n}{n!}$

$$\sum_{n=k}^{\infty} \mathbb{P}\left(\sum_{i=0}^{\infty} X_i = k, M=n\right) = \sum_{n=0}^{\infty} \binom{n}{k} p^k (1-p)^{n-k} \cdot e^{-\lambda} \frac{\lambda^n}{n!}$$

$$= \sum_{n=k}^{\infty} \frac{n!}{(n-k)!\,k!} \cdot p^k (1-p)^{n-k} e^{-\lambda} \frac{\lambda^n}{n!}$$

$$= \sum_{n=k}^{\infty} \frac{(\lambda p)^k e^{-\lambda}}{k!} \frac{((1-p)\lambda)^{n-k}}{(n-k)!}$$

$$= \frac{(\lambda p)^k e^{-\lambda}}{k!} \sum_{n=k}^{\infty} \frac{((1-p)\lambda)^{n-k}}{(n-k)!}.$$

$$= \frac{(\lambda p)^k e^{-\lambda}}{k!} \underbrace{\sum_{i=0}^{\infty} \frac{((1-p)\lambda)^i}{i!}}_{e^{-(1-p)\lambda}}$$

$$= \frac{(\lambda p)^k e^{-\lambda}}{k!}$$

$$= \frac{(\lambda p)^k}{k!} e^{-p\lambda}$$

P9 a) $1 = \sum_{k=0}^{\infty} p_X(k) = \eta \sum_{k=0}^{\infty} \left(\frac{1}{3}\right)^k = \eta \frac{1}{1-1/3} = \eta \cdot \frac{3}{2}$

$\Rightarrow \eta = \frac{2}{3}$

b) $\mathbb{P}(W \leq k)$

$= \mathbb{P}(\min(Z,X) \leq k)$

$= 1 - \mathbb{P}(\min(Z,X) > k)$

$= 1 - \mathbb{P}(Z > k, X > k)$

$= 1 - \mathbb{P}(Z > k)\,\mathbb{P}(X > k)$

$= 1 - \left(\sum_{i=k+1}^{n} \binom{n}{i} q^i (1-q)^{n-i}\right)\left(\sum_{j=k+1}^{\infty} \frac{2}{3}\left(\frac{1}{3}\right)^k\right)$

$= 1 - \left(\sum_{i=k+1}^{n} \binom{n}{i} q^i (1-q)^{n-i}\right) \cdot (1/3)^k$

NOTEMOS QUE $\mathbb{P}(W \leq n) = 1$. YA QUE EN EL PEOR CASO LA BINOMIAL ES $n$.

P101

$$\mathbb{P}(X+Y=k) = \sum_{i=0}^{k} \mathbb{P}(X+Y=k \mid Y=i)\,\mathbb{P}(Y=i)$$

$$= \sum_{i=0}^{k} \mathbb{P}(X=k-i)\,\mathbb{P}(Y=i)$$

$$= \sum_{i=0}^{k} e^{-\lambda_1}\frac{(\lambda_1)^{k-i}}{(k-i)!}\, e^{-\lambda_2}\frac{(\lambda_2)^{i}}{i!}$$

$$= \frac{e^{-(\lambda_1+\lambda_2)}}{k!} \sum_{i=0}^{k} \frac{k!}{i!\,(k-i)!} (\lambda_1)^{k-i}(\lambda_2)^{i}$$

$$= \frac{e^{-(\lambda_1+\lambda_2)}}{k!} \sum_{i=0}^{k} \binom{k}{i} (\lambda_1)^{k-i}(\lambda_2)^{i}$$

$$= \frac{e^{-(\lambda_1+\lambda_2)}}{k!} (\lambda_1+\lambda_2)^{k}.$$

$\Rightarrow X+Y \sim \text{POISSON}(\lambda_1+\lambda_2)$. $\blacksquare$

# AUX #5

**P1|** VER **P3|** AUX #3

**P2| P3| P4| P5|** VER **P7|, P8|, P9| P10|** AUX #4

**P6|** $\mathbb{P}(F_X^{-1}(U) \leq t)$
$\mathbb{P}(\inf\{x : F_X(x) \geq U\} \leq t)$

DEMOSTREMOS QUE

$$A := \{F_X^{-1}(U) \leq t\} = \{U \leq F_X(t)\} =: B$$

$\subseteq|$ SI $\omega \in A \Rightarrow \inf\{x : F_X(x) \geq U(\omega)\} \leq t$

- SI $\inf\{x : F(x) \geq U(\omega)\} = t$
  $\Rightarrow \exists\, x_n \searrow t$ TQ $F(x_n) \geq U(\omega)$
  PERO $F$ CONTINUA POR LA DERECHA $\Rightarrow F(t) \geq U(\omega)$

- SI $\inf\{x : F(x) \geq U(\omega)\} < t$
  $\exists\, x_n \searrow$ $\exists N$ TQ $F_X(x_n) \geq U(\omega)$ Y $t > x_n$ $\forall n > N$
  $\rightarrow$ COMO $F_X$ CONT POR LA DERECHA
  $F_X(t) \geq U(\omega)$

$\supseteq|$ SI $\omega \in B \Rightarrow U(\omega) \leq F_X(t)$
$\Rightarrow \inf\{x : F_X(x) \geq U(\omega)\} \leq t$
$\Rightarrow \omega \in A.$

CON ESTO $A = B$.

$$\mathbb{P}(F_X^{-1}(U) \leq t) = \mathbb{P}(U \leq F_X(t)) = \frac{F_X(t)}{1} = F_X(t)$$

$$\Rightarrow F_X^{-1}(U) \sim X$$

P5.

# AUX #6

P1) SEA X V.A

a) $\mathbb{P}(\Omega) = 1 = \int_0^\infty K e^{bx} = \frac{K}{b} \cdot e^{bx} \Big|_0^\infty = \frac{K}{b} e^{b\infty} - 1$

si $b > 0$ $e^{b\infty} = \infty$

POR LO QUE $b \leq 0$

LUEGO $e^{b\infty} = 0$

$\Rightarrow \quad 1 = -\frac{K}{b} \quad \Rightarrow \quad K = -b$

CON ESTO

$$f_X(x) = \begin{cases} K e^{-Kx} & x > 0 \quad \text{CON } K > 0. \\ 0 & \sim \end{cases}$$

$$\mathbb{P}(X \in I) = 1 - \mathbb{P}(X \in \mathbb{Q}) = 1 - \underbrace{\sum_{i \in \mathbb{Q}} \underbrace{\mathbb{P}(X = i)}_{0}}_{\text{SUMA NUM.}} = 1$$

b) $\mathbb{P}(X > x) = \int_x^\infty K e^{-Kt} dt = \left( e^{-Kt} \Big|_x^\infty \right) = e^{-Kx}$

$$\mathbb{P}(X > s+t \mid X > s) = \frac{\mathbb{P}(X > s+t, X > s)}{\mathbb{P}(X > s)}$$

$$= \frac{\mathbb{P}(X > s+t)}{\mathbb{P}(X > s)} = \frac{e^{-K(s+t)}}{e^{-Ks}} = e^{-Kt}$$

$$= \mathbb{P}(X > t)$$

P2) ~~$\mathbb{P}(Z < s) = \mathbb{P}(X + Y < s) = \int_0^{s \wedge 1/2} \mathbb{P}(X < s - t) f_Y(t) dt$~~

$X_i$ = V.A DEL TROZO MAS CHICO DE LA i-ESIMA BARRA

$Y_i$ = VA DEL TROZO MAS GRANDE DE LA i-ESIMA BARRA

SI $x \le L/2$

$$\mathbb{P}(Y_1 \le x) = \mathbb{P}(X_1 \le x) = \int_0^x f_c(t)\,dt + \int_{L-x}^{L} f_c(t)\,dt$$

$$= \frac{2x}{L}$$

PARA $t > L/2$.

$$\mathbb{P}(Y_2 > t) = \mathbb{P}(X_2 \le L-t) = 2\frac{(L-t)}{L}$$

$$\Rightarrow \mathbb{P}(Y_2 \le t) = \frac{L-2L+2t}{L} = \frac{2t-L}{L}$$

$$\Rightarrow f_{X_1}(t) = \frac{2}{L} \qquad f_{Y_2}(t) = \frac{2}{L}$$

PARA $z \in [0, 3L/2]$

$$\mathbb{P}(Z \le z) = \mathbb{P}(X_1+Y_2 \le z) = \int_{L/2}^{L} \mathbb{P}(X_1 \le z-t)\,\frac{2}{L}\,dt$$

$$= \int_{L/2}^{L} \left[ \frac{2(z-t)}{L} \cdot \underbrace{\mathbb{1}_{\{z-t<L/2\}}}_{z-L/2<t} + \mathbb{1}_{\{z-t>L/2\}} \right] \frac{2}{L}\,dt$$

$$= \int_{z-L/2}^{L} \frac{4(z-t)}{L^2}\,dt + \int_{L/2}^{z-L/2} \frac{2}{L}\,dt$$

$$= \left. -\frac{2(z-t)^2}{L^2} \right|_{z-L/2}^{L} + \frac{2(z-L)}{L}$$

$$= -\frac{2(z-L)^2}{L^2} + \frac{2(L/2)^2}{L^2} + \frac{2(z-L)}{L}$$

$$= \frac{-2z^2+4zL-2L^2+L^2/2+2zL-2L^2}{L^2}$$

$$= \frac{-2z^2+6zL-\frac{7}{2}L^2}{L^2}$$

$=$

$=$

b) $\mathbb{E}(Z) = \mathbb{E}(X_1 + Y_2) = \mathbb{E}(X_1) + \mathbb{E}(Y_2)$
$= L/4 + 3L/4$
$= L.$

**P3** a) $\mathbb{P}(Z_1 \leq t) = \mathbb{P}(\max_{i \in [n]}\{X_i\} \leq t)$

$= \mathbb{P}(X_1 \leq t, \ldots, X_n \leq t)$
$= \mathbb{P}(X_1 \leq t) \cdots \mathbb{P}(X_n \leq t)$

$= \mathbb{P}(X_1 \leq t)^n$
$= F_X(t)^n$

$\rightarrow f_{Z_1}(t) = n F_X(t)^{n-1} \cdot f_X(t)$

$\mathbb{P}(Z_2 \leq t) = \mathbb{P}(\min_{i \in [n]} X_i \leq t)$

$= -\mathbb{P}(\min_{i \in [n]} X_i > t) + 1$

$= 1 - \mathbb{P}(X_1 > t, \ldots, X_n > t)$
$= 1 - \mathbb{P}(X_1 > t) \cdots \mathbb{P}(X_n > t)$
$= 1 - (1 - \mathbb{P}(X_1 \leq t))^n$

$f_{Z_2}(t) = n(1 - F_{X_1}(t))^{n-1} f_{X_1}(t)$

b) $f_{Z_2} = \left(1 - \prod_{i=1}^{n} (1 - \mathbb{P}(X_i \leq t))\right)'$

$= \left(1 - \prod_{i=1}^{n} 1 - (1 - e^{-\alpha_i t})\right)'$

$= \left(1 - e^{-\sum \alpha_i t}\right)'$

$= \sum \alpha_i \, e^{-\sum \alpha_i t}$

$\rightarrow Z_2 \sim \mathrm{EXP}(\sum \alpha_i)$

c) $\mathbb{P}(X_k = Z_2) = \mathbb{P}(\underset{i\in[n]}{\text{ARG MIN}}\, X_i = X_k)$

$$= \mathbb{P}\left( X_k \leq \min_{i\in[n]\setminus k} X_i \right)$$

$$= \int_0^\infty \mathbb{P}\left(\min_{i\in[n]\setminus k} X \geq t\right) f_{X_k}(t)\, dt$$

$$= \int_0^\infty \left( e^{-\sum_{i=1\setminus k}^{n} \alpha_i t} \right) \cdot \alpha_k e^{-\alpha_k t}\, dt$$

$$= \frac{\alpha_k}{\sum \alpha_i} \int_0^\infty \sum_{i=1}^{n} \alpha_i e^{-\sum \alpha_k t}\, dt$$

$$= \frac{\alpha_k}{\sum \alpha_i}$$

$\boxed{P4}$ i) $\mathbb{E}(X_1) = \int_1^\infty x \cdot 4x^{-5}\, dx$

$$= \int_1^\infty 4x^{-4}\, dx = -\frac{4}{3} x^{-3} \Big|_1^\infty = \frac{4}{3}$$

ii) $\text{VAR}(X_1) = \int_1^\infty x^2\, 4x^{-5}\, dx = -\frac{4}{2} x^{-2} \Big|_1^\infty = 2$

iii) $\mathbb{E}(X_1 + \dots + X_n) \stackrel{\leq}{=} \sum_{i=1}^{n} \mathbb{E}(X_i) = n\mathbb{E}(X_1) = \frac{4n}{3}$

$$\text{VAR}(X_1 + \dots + X_n) = \sum_{i=1}^{n} \text{VAR}(X_i) = n\,\text{VAR}(X_1) = 2n$$

iv) SEA $\varepsilon > 0$ ENTONCES

$$\mathbb{P}\left( \frac{|X_1 + \dots + X_n - \mathbb{E}(X_1 + \dots + X_n)|}{n} \geq \varepsilon \right)$$

$$= \mathbb{P}\left( (X_1 + \dots + X_n - \mathbb{E}(X_1 + \dots + X_n))^2 \geq \varepsilon^2 n^2 \right) \leq \frac{\mathbb{E}\left((\sum X_i - \mathbb{E}(\sum X_i))^2\right)}{\varepsilon^2 n^2}$$

$$= \frac{\text{VAR}(\sum X_i)}{\varepsilon^2 n^2}$$

$$= \frac{2n}{\varepsilon^2 n^2} = \frac{2}{\varepsilon^2 n} \xrightarrow{n \to 0} 0$$

PS a) $F_X(1) = 1 = \int_0^{1/2} Ax + \int_{1/2}^{1} B(1-x)$

$$= \frac{A}{2}\left(\frac{1}{2}\right)^2 + \frac{B}{2}\left[-(1-x)^2\right]_{1/2}^{1}$$

$$= \frac{A}{8} + \frac{B}{2}\cdot\frac{1}{4} = \frac{A+B}{8}$$

$A+B = 8$

b) si $x<0$ $F_X(x) = 0$

si $0 < x \leq 1/2$

$$F_X(x) = \int_0^x Ax = \frac{A}{2}x^2$$

si $1/2 < x \leq 1$

$$F_X(x) = \frac{A}{8} + \int_{1/2}^{x} B(1-\cancel{t})\,d\cancel{t}$$

$$= \frac{A}{8} + \frac{B}{2}\left.-(1-\cancel{t})^2\right|_{1/2}^{x}$$

$$= \frac{A}{8} + \frac{B}{2}\left[-(1-x)^2 + \frac{1}{4}\right]$$

$$= \frac{A}{8} + \frac{B}{8} - \frac{B(1-x)^2}{2}$$

si $x>1$ $F_X(x) = 1.$

c) $\mathbb{P}(X > 1/2) = 1 - \mathbb{P}(X \leq 1/2) = \frac{B}{8}$

$\mathbb{P}(X < 1/2) = \mathbb{P}(X \leq 1/2) = A/8$

$\mathbb{P}(1/4 \leq X \leq 3/4) = \mathbb{P}(X \leq 3/4) - \mathbb{P}(X \leq 1/4)$

$$= \frac{A+B}{8} - \frac{B}{2}\frac{1}{16} - \frac{A}{2}\cdot\frac{1}{16} = \frac{A+B}{8}\left(1 - \frac{1}{4}\right)$$

$= 3/4$

IV) $\mathbb{P}(X<0.2 \mid X\leq 1/2) = \dfrac{\mathbb{P}(X\leq 0.2)}{\mathbb{P}(X\leq 1/2)} = \dfrac{\frac{A}{2}\frac{4}{100}}{\frac{A}{8}} = \dfrac{4}{25}$

$\mathbb{P}(X\geq 1/2 \mid 1/4 < X < 3/4) =$
$\dfrac{\mathbb{P}(1/2\leq X\leq 3/4)}{\mathbb{P}(1/4<X\leq 3/4)} = \dfrac{\frac{B}{8}-\frac{B}{2}\frac{1}{16}}{3/4} = \dfrac{\frac{3B}{16\cdot 2}}{3/4}\cdot\dfrac{B}{8}$

V) $\mathbb{E}(X) = \int_0^{1/2} Ax^2 + \int_{1/2}^{1} tB(1-t)\,dt = \dfrac{A}{24}+\dfrac{B}{12} = \dfrac{1}{3}+\dfrac{B}{24}$

$\mathbb{E}(X^2) = \int_0^{1/2} x^2\cdot A\,x\,dx + \int_{1/2}^{1} x^2 B(1-x)\,dx$

$A\,\dfrac{x^4}{4}\Big|_0^{1/2}$

$= \dfrac{A}{4}\cdot\dfrac{1}{16} + B\,\dfrac{x^3}{3}\Big|_{1/2}^{1} - \dfrac{Bx^4}{4}\Big|_{1/2}^{1}$

$= \dfrac{A}{16\cdot 4} + \dfrac{B}{3}\left(1-\dfrac{1}{8}\right) - \dfrac{B}{4}\left(1-\dfrac{1}{16}\right)$

$= \dfrac{3A + 8\cdot 7B - 15\cdot 3B}{16\cdot 4\cdot 3}$

$= \dfrac{3A+56B-45B}{16\cdot 4\cdot 3}$

$= \dfrac{3\cdot 8 + 8B}{16\cdot 4\cdot 3} = \dfrac{1}{8}+\dfrac{B}{24}$

$\mathbb{V}\mathrm{AR}(X) = \dfrac{1}{8}+\dfrac{B}{24} - \left(\dfrac{1}{3}+\dfrac{B}{24}\right)^2$

$= \dfrac{1}{8}+\dfrac{B}{24} - \dfrac{1}{9} + 2\cdot\dfrac{1}{3}\cdot\dfrac{B}{24} + \dfrac{B^2}{24^2}$ //

i) $\mathbb{P}(X > \mu + a) = \int_{\mu+a}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dt$

$\cancel{x = \mu}$

$x - \mu = (v - \mu)$

$a = -v + \mu$

$a = \mu - a$

$= -\int_{\mu-a}^{-\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(v-\mu)^2}{2\sigma^2}} dv$

$= \int_{\infty}^{\mu-a} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(v-\mu)^2}{2\sigma^2}} dv$

$= \mathbb{P}(X < \mu - a)$.

ii) $\mathbb{P}(Z \leq t) = \mathbb{P}\left(\frac{X-\mu}{\sigma} \leq t\right) = \mathbb{P}(X \leq t\sigma + \mu)$

$= \int_{-\infty}^{t\sigma+\mu} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$ $\quad v = \left(\frac{x-\mu}{\sigma}\right)$

$\int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}\cancel{\sigma}} e^{-\frac{v^2}{2}} dv \cdot \cancel{\sigma}$

$\Rightarrow Z \sim N(0,1)$ pues $f_Z(t) = f_{N(0,1)}(t)$

iii) $\mathbb{P}(Y \leq t) = \mathbb{P}(e^Z \leq t)$ $\quad$ sop$(Y) \in [0, +\infty)$

$= \mathbb{P}(Z \leq \log(t))$

$\Rightarrow f_Y(t) = (\mathbb{P}(Z \leq \log(t)))'$

$= f_Z(\log(t)) \cdot \frac{1}{t}$

$= \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(\log(t))^2}{2}} \cdot \frac{1}{t}$

iv) $\mathbb{E}(Y) = \mathbb{E}(e^Z) = \int_{-\infty}^{\infty} e^x \cdot \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x)^2}{2}} dx$

$= \int_{-\infty}^{\infty} e^{-\frac{x^2 - 2x}{2}} \frac{1}{\sqrt{2\pi}\sigma} dx$

$= \int_{-\infty}^{\infty} e^{-\frac{(x-1)^2}{2}} \frac{1}{\sqrt{2\pi}\sigma} + \int_{-\infty}^{\infty} e^{1/2} \cdot \frac{1}{\sqrt{2\pi}\sigma} dx$

$$\mathbb{E}(e^Z) = \int_{-\infty}^{\infty} e^x \cdot c \cdot e^{-\frac{x^2}{2}} dx$$

$$= c\int_{-\infty}^{\infty} e^{1/2} e^{-\frac{x^2+2x-1}{2}} dx$$

$$= c\int_{-\infty}^{\infty} e^{1/2} \cdot e^{-\frac{(x-1)^2}{2}} dx \qquad v = x-1$$

$$= c\int_{-\infty}^{\infty} e^{1/2} \cdot e^{-\frac{v^2}{2}} dv$$

$$= e^{1/2}$$

$$\mathbb{E}((e^Z)^2) = c\int_{-\infty}^{\infty} e^{2x} \cdot \cancel{c} \cdot e^{-\frac{x^2}{2}} dx$$

$$= c\int_{-\infty}^{\infty} e^{-\frac{x^2+4x-4}{2}} e^{2} dx$$

$$= c\int_{-\infty}^{\infty} e^{-\frac{(x-2)^2}{2}} e^2 dx \qquad v = x-2$$

$$= c\int_{-\infty}^{\infty} e^{-v} \cdot e^2 dv$$

$$= e^2$$

$$\mathbb{V}\mathrm{AR}(Y) = e^2 - (e^{1/2})^2$$

$$= (e^1 - 1) e^1$$

$$\mathrm{COV}(X, Y) = \mathbb{E}(X,$$