



a) por geometría y condición de no resbalar:

$$Z_c = R \cos \theta + R \theta \sin \theta \quad (1)$$

$$\Rightarrow V = MgR (\cos \theta + \theta \sin \theta) \quad (2)$$

b)

$$K = K_{cm} + K_{rel} \quad (3)$$

$$K_{cm} = \frac{1}{2} M (\dot{X}_c^2 + \dot{Z}_c^2) \quad (4)$$

$$\text{geometría} \rightarrow X_c = R (\sin \theta - \theta \cos \theta) \quad (5)$$

$$(1) \Rightarrow \dot{Z}_c = R \theta \dot{\theta} \cos \theta$$

$$(5) \Rightarrow \dot{X}_c = R \theta \dot{\theta} \sin \theta$$

$$\text{en (4)} \Rightarrow K_{cm} = \frac{1}{2} M (R \theta)^2 \dot{\theta}^2$$

$$K_{rel} = \frac{1}{2} I \dot{\theta}^2 \quad \text{dado} \quad I = \frac{1}{12} M (2R)^2 \Rightarrow K_{rel} = \frac{1}{6} M R^2 \dot{\theta}^2$$

$$\text{en (3)} \Rightarrow K = \frac{1}{2} M R^2 \left\{ \theta^2 + \frac{1}{3} \right\} \dot{\theta}^2$$

c)

$$V = mgR (\cos \theta + \theta \sin \theta)$$

$$\begin{aligned} V' &= mgR (-\sin \theta + \sin \theta + \theta \cos \theta) \\ &= mgR \theta \cos \theta \rightarrow \text{equilibrium } \theta = 0 \end{aligned}$$

$$V'' = mgR (\cos \theta - \theta \sin \theta)$$

$$V''(0) = mgR$$

$$\Rightarrow \omega_0^2 = \frac{mgR}{\frac{mR^2}{3}} = 3 \frac{g}{R}$$

d)

$$R\theta_i = R \Rightarrow \theta_i = 1 \text{ radian}$$

$$\left. \begin{array}{l} V_i = mgR (\cos 1 + \sin 1) \\ K_i = 0 \end{array} \right\} E_0 = mgR (\cos 1 + \sin 1)$$

$$V + K = E_0 \quad \forall t$$

$$\Rightarrow mgR (\cos \theta + \theta \sin \theta) + \frac{1}{2} mR^2 \left(\dot{\theta}^2 + \frac{1}{3} \right) \dot{\theta}^2 = E_0$$

$$m\dot{\theta} = 0 \Rightarrow mgR + \frac{1}{2} mR^2 \cdot \frac{1}{3} \dot{\theta}_0^2 = mgR (\cos 1 + \sin 1)$$

$$\Rightarrow \dot{\theta}_0^2 = \frac{6g}{R} (\cos 1 + \sin 1) \underset{-1}{\approx} 2.29 \frac{g}{R}$$