Pauta Ejercicio #6.

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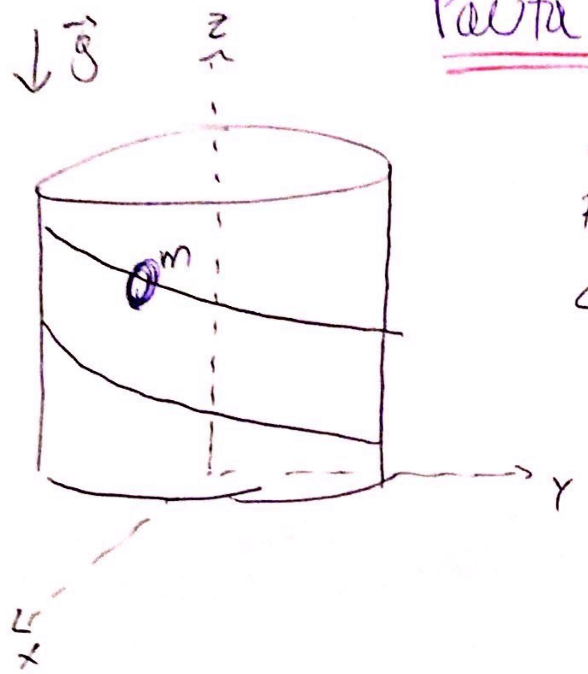


$Z = h - \phi R_1$

$F_{roce} = -c\vec{v}$

CI: $\phi(0) = 0$; $z(0) = h$, $\dot{\phi}(0) = 0$

a) Obtener $\hat{t}$ y el vector $\vec{N}$ general

En cilíndricas:

$$\vec{r} = \rho\hat{\rho} + z\hat{k} = R_0\hat{\rho} + (h - \phi R_1)\hat{k}$$

$$\vec{v} = \rho\dot{\phi}\hat{\phi} + \dot{z}\hat{k} = R_0\dot{\phi}\,\hat{\phi} - R_1\dot{\phi}\hat{k} = \dot{\phi}(R_0\hat{\phi} - R_1\hat{k})$$

$$\vec{a} = R_0\ddot{\phi}\hat{\phi} - R_0\dot{\phi}^2\hat{\rho} - R_1\ddot{\phi}\hat{k}$$

Sabemos que:

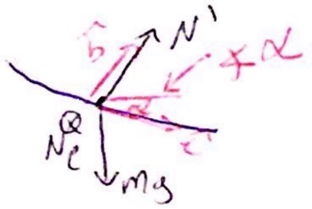
$$\hat{t} = \frac{\vec{v}}{\|\vec{v}\|} = \frac{(R_0\hat{\phi} - R_1\hat{k})\dot{\phi}}{|\dot{\phi}|\sqrt{R_0^2 + R_1^2}} = \frac{R_0\hat{\phi} - R_1\hat{k}}{\sqrt{R_0^2 + R_1^2}}$$ 0.5

Luego sea $\vec{N} = \begin{pmatrix} -N_\rho \\ N_\phi \\ N_k \end{pmatrix}$ el vector normal, sabemos que $\hat{t} \cdot \vec{N} = 0$

$$\Rightarrow R_0 N_\phi = R_1 N_k \Rightarrow N_\phi = \frac{R_1}{R_0} N_k$$

$$\Rightarrow \vec{N} = -N_\rho\hat{\rho} + \frac{R_1}{R_0} N_k\,\hat{\phi} + N_k\hat{k}$$ 0.5

b) DCL



Donde $N' = \sqrt{N_k^2+N_\phi^2} = N_k\sqrt{1+\frac{R_1^2}{R_0^2}} = \frac{N_k}{R_0}\sqrt{R_1^2+R_0^2}$

Podemos ahora escribir la aceleración en términos de $\hat{t}$ como: $\vec{v} = \|\vec{v}\|\hat{t} = \dot\phi\sqrt{R_1^2+R_0^2}\,\hat{t}$

$\vec{a} = R_0\dot\phi^2\hat{n} + \ddot\phi\sqrt{R_1^2+R_0^2}\,\hat{t}$

$\Rightarrow \hat{n})\quad mR_0\dot\phi^2 = N_e$

Además: $\sin\alpha = \frac{R_1}{\sqrt{R_1^2+R_0^2}}$ $\quad\cos\alpha = \frac{R_0}{\sqrt{R_1^2+R_0^2}}$

$\hat{t})$ ~~$m\ddot\phi\sqrt{R_1^2+R_0^2}\,\hat{t} = mg\sin\alpha\,\hat{t} - c\dot\phi(R_0\hat\phi + R_1\hat{k})$~~

Perdón

$m\ddot\phi\sqrt{R_1^2+R_0^2} = mg\frac{R_1}{\sqrt{R_1^2+R_0^2}} - c\dot\phi\sqrt{R_1^2+R_0^2}$

$\hat{b})\quad mg\frac{R_0}{\sqrt{R_1^2+R_0^2}} = N_2$.

De $\hat{t}$ tenemos que:

$$\ddot\phi + \frac{c}{m}\dot\phi = \frac{gR_1}{R_1^2+R_0^2}\ \Big/\cdot e^{\frac{c}{m}t} \Rightarrow \ddot\phi e^{\frac{c}{m}t} + \frac{c}{m}e^{\frac{c}{m}t}\dot\phi = \frac{gR_1}{R_0^2+R_1^2}e^{c/m\,t}\ \Big/\int dt$$

$$\dot\phi e^{\frac{c}{m}t} = \frac{gR_1m}{c(R_1^2+R_0^2)}e^{c/m\,t} + C. \quad \underline{\underline{CI}}\ \dot\phi(0)=0 \Rightarrow C = -\frac{gR_1m}{c(R_1^2+R_0^2)}$$

$$\Rightarrow \boxed{\dot\phi = \frac{gR_1m}{c(R_1^2+R_0^2)}\left(1-e^{-\frac{c}{m}t}\right)}$$

2.0

Así integrando $\dot\phi$ se tiene: $\boxed{\phi = \frac{gR_1m^2}{c^2(R_1^2+R_0^2)}\left(e^{-\frac{c}{m}t}+\frac{c}{m}t-1\right)}$

1.0

Luego reemplazando $\dot\phi$ en $\hat{n}$ y sabiendo que $\frac{mgR_0}{\sqrt{R_1^2+R_0^2}} = \sqrt{N_k^2+N_\phi^2}$

$$\Rightarrow N_e = mR_0\cdot\left(\frac{gR_1m}{c(R_1^2+R_0^2)}\right)^2\left(1-e^{-\frac{c}{m}t}\right)^2$$

$N_k = \frac{R_0^2mg}{R_0^2+R_1^2}$; $N_\phi = \frac{R_1R_0mg}{R_0^2+R_1^2}$

$\vec{N} = -N_e\hat{e} + N_\phi\hat\phi + N_k\hat{k}$

No es necesario reemplazar.

1.0