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Multimarket contact and collusive behavior

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Traditional analyses of industrial behavior typically link the exercise of market power in an industry to internal features such as demand conditions, concentration, and barriers-to-entry. Nevertheless, some economists have remained concerned that external factors, such as contact across markets, may also play a significant role in determining the level of competitiveness in any particular industry. In this article, we examine the effect of multimarket contact on the degree of cooperation that firms can sustain in settings of repeated competition. We isolate conditions under which multimarket contact facilitates collusion and show that these collusive gains are achieved through modes of behavior that have been identified in previous empirical studies of multimarket firms.

1. Introduction

■ Traditional analyses of industrial behavior typically link the exercise of market power in an industry to internal features such as demand conditions, concentration, and barriers-to-entry. Nevertheless, some economists have remained concerned that external factors may also play a significant role in determining the level of competitiveness in any particular industry. One aspect of this concern relates to the potential effects of multimarket contact between firms. The possibility that such contact could foster anticompetitive outcomes was first raised in 1955 by Corwin Edwards, who said

> When one large conglomerate enterprise competes with another, the two are likely to encounter each other in a considerable number of markets. The multiplicity of their contact may blunt the edge of their competition.[1]

This potential for "mutual forebearance" is not limited to conglomerates but exists for any multiproduct firms, including "single-product" firms that operate in a number of distinct geographic markets.

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[1] Corwin Edwards, as quoted in Scherer (1980, p. 340).

Despite the obvious prevalence of multimarket contact among firms, however, relatively little research has analyzed its effect on economic performance. Although a number of authors have recently attempted to study this issue empirically, the existing literature contains virtually no formal theoretical analyses.[2]

One recent exception is Bulow, Geanakoplos, and Klemperer (1985). These authors investigate the effects of cost- and demand-based linkages across markets in the context of static oligopolistic models. While these conditions give rise to linkages in strategic interaction across markets, their analysis does not address the issue that multimarket contact may affect firms' abilities to sustain noncompetitive ("collusive") outcomes. The object of this article is to provide such an analysis. Given this aim, we focus on settings of repeated interactions between firms. Moreover, to highlight the strategic linkages between markets, we assume away the demand- and cost-based linkages that motivated their analysis.

In what follows, we examine the effect of multimarket contact on the degree of cooperation that firms can sustain in settings of repeated competition. In particular, we contrast the most collusive equilibrium outcomes that can be sustained in the presence of multimarket contact with those attainable when all products are produced by single-product firms. This exercise requires us to adopt a concept of strategic equilibrium. The most widely accepted concept is that of subgame perfection. The set of subgame perfect equilibria may also be viewed as the set of credible nonbinding agreements available to firms, since any element of this set specifies actions that are in each firm's individual self-interest at all times. Thus, following Abreu (1986, 1988), we investigate the effect of multimarket contact by contrasting the most collusive subgame perfect equilibria (those yielding Pareto-undominated payoffs for the firms) for these two settings.[3]

We begin Section 2 by discussing some general aspects of the link between multimarket contact and collusive behavior. There, we point out that multimarket contact relaxes the incentive constraints governing the implicit agreements between firms, and that this has the potential to improve firms' abilities to sustain collusive outcomes.

To assess the effects of multimarket contact more fully, we then turn to an analysis of price competition with homogeneous products in each market. We begin Section 3 by proving an irrelevance result: when markets are identical, firms are identical, and technology exhibits constant returns to scale, then multimarket contact does not enhance firms' abilities to sustain collusive prices. Nevertheless, certain natural conditions do give rise to collusive gains from multimarket contact. In Sections 4 through 6 we investigate these conditions by successively relaxing each of the three assumptions which generate the irrelevance result, allowing in turn for differing markets, differing firms, and scale economies. Of particular interest is the fact that, in each of these cases, the gains from multimarket contact are achieved by using modes of behavior that have been identified in previous studies of multimarket firms. For instance, when firms differ in their costs of production across markets or when scale economies are present, multimarket contact allows the development of "spheres of influence," which enable firms to sustain higher levels of profits and prices. In addition, geographically-based, reciprocal trades of output—a common practice in many industries in which transportation costs are high—may, in such circumstances, facilitate the maintenance of collusive prices. When markets are subject to imperfectly correlated random shocks, even risk-neutral firms will wish to diversify their multimarket holdings. For similar reasons,

[2] Existing empirical work includes Mueller (1977), Heggestad and Rhoades (1978), Whitehead and Luytjes (1983), Whitehead (1978), Scott (1982), Rhoades and Heggestad (1985), Mester (1985), and Gelfand and Spiller (1986). We discuss this work in Section 8.

[3] One might want to impose further restrictions on the set of equilibrium outcomes. In fact, as we discuss later, we focus on stationary outcomes in the text (for stationary models) because these require relatively little coordination between the firms. An alternative that we do not pursue here is to consider group incentive constraints, i.e., "collective dynamic consistency" or "renegotiation-proofness," as in Bernheim and Ray (1989), Farrell and Maskin (1989), or more general coalitional incentive constraints, as in Bernheim, Peleg, and Whinston (1987).

multimarket firms should prefer to operate simultaneously in both mature and rapidly growing industries.

In Section 7 we extend our analysis to the case of heterogeneous products. This allows us to highlight several interesting issues that do not arise when products are homogeneous. Finally, Section 8 summarizes our central conclusions and discusses their relations to the existing empirical literature.

2. General aspects of market contact

■ When markets are not inherently linked, it is easy to see that multimarket contact cannot reduce firms' abilities to collude. Since firms can always treat each market in isolation, the set of subgame perfect equilibria cannot be reduced by the introduction of multimarket contact. It is somewhat more difficult to understand the mechanism through which multimarket contact can increase collusion. Edwards' view is the most commonly held:

> [Firms which compete against each other in many markets] may hesitate to fight local wars vigorously because the prospects of local gain are not worth the risk of general warfare . . . A prospect of advantage from vigorous competition in one market may be weighed against the danger of retaliatory forays by the competitor in other markets.[4]

Edwards' appealing assertion is that collusive outcomes are easier to sustain with multimarket contact because there is more scope for punishing deviations in any one market. The problem with this argument is that once a firm knows that it will be punished in every market, if it decides to cheat, it will do so in every market. This observation raises the possibility that increasing the number of markets over which firms have contact may simply proportionately raise the costs and benefits of an optimal deviation.

In fact, multimarket contact does generally alter the strategic environment in a substantive way. To see this, consider two markets, A and B, and two firms, 1 and 2, which operate in both markets. Let the strategy set of firm i in market k be S_{ik}. Firm i's static payoff function in market k is given by $\pi_{ik}(s_{ik}, s_{jk})$, where $s_{ik} \in S_{ik}$ and $s_{jk} \in S_{jk}$. Suppose that the optimal punishment of firm i in market k yields a discounted payoff to firm i of $\underline{v}_{ik}$, and, to keep things simple, consider only stationary equilibrium paths. If the firms treat the markets separately (act like single-product firms in each market), then strategies (s_{1k}, s_{2k}) are supportable as a perfect equilibrium outcome path in market k if and only if

$$\pi_{ik}(\hat{s}_{ik}(s_{jk}), s_{jk}) + \delta\underline{v}_{ik} \leq \left(\frac{1}{1-\delta}\right)\pi_{ik}(s_{ik}, s_{jk}) \tag{1}$$

for $i = 1, 2$, where $\hat{s}_{ik}(s_{jk})$ is firm i's static best response to s_{jk} and δ is the discount factor used by both firms. (See Abreu (1988).) In contrast, in any optimal multimarket collusive equilibrium, firms recognize that any deviation will be met with punishment in *both* markets (Abreu, 1988). As a result, if a firm decides to deviate, it will do so in *both* markets. Consequently, strategies $[(s_{1A}, s_{2A}), (s_{1B}, s_{2B})]$ are supportable as a perfect equilibrium outcome path if

$$\sum_{k=A,B} \{\pi_{ik}(\hat{s}_{ik}(s_{jk}), s_{jk}) + \delta\underline{v}_{ik}\} \leq \left(\frac{1}{1-\delta}\right) \sum_{k=A,B} \pi_{ik}(s_{ik}, s_{jk}).^{5}$$

Thus, multimarket contact serves to *pool* the incentive constraints of the two markets.[6] This pooling can potentially relax binding incentive constraints, thereby increasing collusive

[4] Corwin Edwards, as quoted in Scherer (1980, p. 340).

[5] For expositional simplicity, we are ignoring the effect that multimarket contact may have on punishments. We consider this issue in Section 7.

[6] Telser (1980) briefly considers this aspect of multiproduct operation toward the end of his discussion of self-enforcing agreements.

profits. In order to gain a better understanding of the circumstances in which such gains are possible and of how firms act to take advantage of these gains, we next turn to an analysis of more structured models of oligopolistic pricing. In Sections 3 through 6 we consider price-setting (Bertrand) models of repeated interactions with homogeneous products; in Section 7 we investigate the heterogeneous product case.

For each of these models, we characterize and compare the most collusive equilibria with and without multimarket contact. Three points should be noted about this analysis. First, though our analysis follows that of Abreu (1988) in utilizing optimal punishments, our basic points would also apply for a variety of other punishments (e.g., reversion to the static equilibrium for some finite number of periods, T). Second, in analyzing situations with symmetrically positioned firms, we focus on equilibria which yield identical payoffs ("symmetric-payoff equilibria") to these firms; with asymmetrically positioned firms, we take a more agnostic position and examine the full Pareto frontier of equilibria. Third, for stationary models, we focus on stationary equilibria. We find these equilibria more plausible in such settings because they require less coordination between the firms. In many cases this additional restriction is without consequence, since the most collusive symmetric-payoff equilibrium is actually stationary. When this restriction is consequential, however, it does not drive our basic points regarding the gain in collusive ability that comes from multimarket contact.

3. A simple model of multimarket contact: an irrelevance result

■ We begin by introducing a simple model of multimarket contact with repeated (Bertrand) price competition. For expositional purposes, both here and in the following sections, we limit our discussion to a consideration of the gains from contact over two markets, which we label A and B. Trading in both markets occurs at the same set of points in discrete time, $\{t\}_{t=0}^{\infty}$. Demand in market k in each period is given by a decreasing continuous function, $Q(\cdot)$, of the price in market k, p_k. This demand relationship is identical for both markets.

Again, for expositional purposes, we shall suppose that two firms (labelled $i = 1, 2$) operate in both markets.[7] At every point in time, t, each firm i announces its current prices for the two markets, $\{p_{ik}(t)\}_{k=A,B}$. Consumers observe all announced prices and purchase each good from the firm with the lowest price. When different firms announce identical prices in the same market, consumers are indifferent between the suppliers, and we may resolve this indifference to achieve any desired division of demand.[8] We assume that each

[7] The result of this section is unaffected by the number of firms assumed. Note, however, that we do implicitly assume that entry is blocked. This could, for example, be due to patents. Alternatively, this may be due to the existence of sunk costs associated with entry that make entry unprofitable even when it is followed by collusion. In addition, potential entrants may fear that their entry into the industry will upset the collusive nature of industry pricing.

[8] We shall make use of this freedom to divide the market when prices are equal. One useful way to think about this is to imagine a market in which products are almost perfectly homogeneous. In particular, firm i's demand function is given by

$$q_i(p_i, p_j) = 0 \quad \text{if} \quad p_i > p_j + e,$$

by

$$Q(p_i) \quad \text{if} \quad p_i < p_j - e,$$

and by

$$q(p_j - p_i) \quad \text{if} \quad p_i \in [p_j - e, p_j + e],$$

where $q(p_j - p_i)$ is an increasing continuous function mapping to $[0, Q(p_i)]$. Then, for sufficiently small e, the firms can achieve any split of market demand they desire with almost no effect on profits.

firm must meet all of the demand for its output at its announced price. Both firms produce output at some constant marginal (and average) cost, c, which is identical in both markets. For simplicity, we assume that industry profits, $(p - c)Q(p)$, are concave in price and denote the joint monopoly price by p^m. Finally, both firms have discount factor δ.

Abreu (1988) has shown that one can obtain all subgame perfect equilibrium paths in discounted, infinitely repeated games by considering strategies with a very simple structure. These strategies entail the use of optimal punishments that are applied whenever players deviate from the equilibrium path. The optimal punishment for each player is the perfect equilibrium which provides him with the lowest payoff that he receives in any perfect equilibrium. Since, in our model, firms always have the option to shut down, optimal punishments cannot be negative. Furthermore, there is a simple perfect equilibrium which yields discounted profits of zero to both players; this consists of the repeated static Bertrand solution, in which the price is set equal to the marginal cost by both firms. Consequently, we may describe an equilibrium as a path of prices and associated profits, $\{p_{ik}(t), \pi_{ik}(t)\}_{t=0}^{\infty}$, $i = 1, 2$, $k = A, B$, where it is understood that we punish any deviations from this path by retreating to the Bertrand solution forever.

For this simple model of multimarket contact, an irrelevance result holds.

Proposition 1. When identical firms with identical constant-returns-to-scale technologies meet in identical markets, multimarket contact does not aid in sustaining collusive outcomes.

This irrelevance result holds not only for the case of stationary symmetric-payoff equilibria (which we discuss here), but also if we consider the entire set of subgame perfect equilibria. For completeness, we present the argument for the unrestricted case (including some extensions discussed below) in Appendix A.[9] We now turn to the case of stationary symmetric-payoff equilibria

Proof. Consider first the single-market outcome. An outcome with stationary price $p \in [c, p^m]$ and equal market shares is sustainable if and only if, for $i = 1, 2$,

$$(p - c)Q(p) \leq \left(\frac{1}{1-\delta}\right)(1/2)(p - c)Q(p). \tag{2}$$

The left-hand side of (2) is firm i's discounted profit if it deviates (it slightly undercuts the price, p, and sells to the whole market; reversion to the Bertrand equilibrium then follows), while the right-hand side is firm i's discounted profit from abiding by the agreement. Simplifying (2) reveals that if $\delta < 1/2$, no price above c is sustainable, while if $\delta \geq 1/2$, there is a symmetric equilibrium that sustains the monopoly price.

Now consider multimarket equilibria. In searching for the most profitable stationary symmetric-payoff equilibrium, we can first restrict our attention to outcomes in which both firms name the same price in every period. To see this point, suppose that an equilibrium prescribes a lower price for firm 1 than for firm 2 in market k. If we adjust firm 2's prescribed price so that it equals firm 1's price and assign a zero market share to firm 2, both firms' profits are unchanged. Moreover, this change lowers firm 1's gain from deviation and leaves firm 2's unchanged. The revised strategies therefore constitute an equilibrium. Accordingly, we let p_k denote the price charged by the firms in market k and $(\lambda_{1k}, \lambda_{2k})$ denote the two firms' market shares in that market.

Next, note that we can also restrict our attention to those outcomes in which $p_k \in [c, p^m]$ for $k = A, B$: if either $p_k > p^m$ or $p_k < c$, then there exists a $\hat{p} \in [c, p^m]$ that results in equal or greater profits for both firms (keeping market shares fixed) and lower

[9] It is also worth noting that once one restricts attention to symmetric-payoff equilibria, stationarity is optimal. This is shown in Proposition B1 in Appendix B.

gains from deviating. Thus, if both firms name price $\hat{p}$, it would be sustainable as an equilibrium, and the firms would both earn (weakly) more than if they named p_k.[10]

Prices $(p_A, p_B) \in [c, p^m]^2$ and market shares $\{\lambda_{1k}, \lambda_{2k}\}_{k=A,B}$ are sustainable if and only if, for $i = 1, 2$,

$$\sum_{k=A,B} \left\{ \left[\left(\frac{1}{1-\delta} \right) \lambda_i (p_k - c) Q(p_k) \right] - [(p_k - c) Q(p_k)] \right\} \geq 0. \tag{3}$$

Summing (3) over $i = 1, 2$, we see that this requires that

$$\sum_{k=A,B} (p_k - c) Q(p_k) [\delta - 1/2] \geq 0. \tag{4}$$

Thus, if $\delta < 1/2$, it is again impossible to sustain any prices above c, so multimarket contact replicates the single-market outcome in *both* markets. If $\delta \geq 1/2$, on the other hand, then a completely monopolistic outcome is possible even without multimarket contact. *Q.E.D.*

Thus, in this simple model, multimarket contact does not facilitate collusive behavior. As a prelude to the next several sections, it is worth emphasizing our three central assumptions: (i) markets are identical; (ii) firms are identical; and (iii) technology is constant returns to scale. The irrelevance of multimarket contact does not depend on all aspects of these assumptions. For example, if either demand or the level of (constant) marginal costs differs across markets, the same line of argument establishes irrelevance (just replace $Q(p)$ by $Q_k(p)$ and c by c_k above).[11] Nevertheless, certain aspects of these assumptions are critical. In the next three sections we consider several cases of special interest.

4. Differences between markets: conglomeration and the transfer of market power

■ The first central assumption in Section 3 is that the markets are identical. In order to identify specific differences between markets that give rise to gains from multimarket contact, it is helpful to begin by thinking about stationary equilibria in a single market, k. By the same logic as in the previous section (parallel to condition (2)), if there are N identical firms, collusion is sustainable in a stationary symmetric-payoff equilibrium if and only if

$$N \leq \frac{1}{1-\delta}.^{12} \tag{5}$$

When this condition is satisfied strictly, firms have slack enforcement power in market k. If these firms also participate in a market in which this condition is violated, they may be able to put this slack enforcement power to use.

Recall that when markets are identical, differ in demand, or differ in marginal cost, then multimarket contact generates no gain in collusive ability. In essence, the pooling of

[10] These two arguments also apply for the models considered in Sections 4 and 6, and so we shall make use of these restrictions in our analysis there without further comment. In Section 5, where firms differ in their costs, we can, without loss of generality, restrict our attention to equilibria in which firms name identical prices, which lie above the lowest-cost level and below the high-cost monopoly price.

[11] The irrelevance result (for the unrestricted equilibrium set) also holds if we allow certain forms of nonstationarity. In particular, we can let demand in market k be a function of time and can let (constant) marginal costs vary with time as well, as long as there exists a constant, Θ, such that $\Pi_A^m(t)/\Pi_B^m(t) = \Theta$, where $\Pi_k^m(t)$ is the monopoly profit level in market k in period t. This is established formally in Appendix A in the course of proving Proposition A1 for the unrestricted case.

[12] Proposition B2 and Corollary B1 in Appendix B establish that when $N < (1-\delta)^{-1}$, all subgame perfect equilibria yield a discounted payoff of zero to every firm. Thus, by demonstrating that multimarket contact can sustain stationary collusive outcomes when this inequality holds, we also establish that this contact yields gains even when we allow for nonstationary outcomes.

incentive constraints does not help in such cases because either the incentive constraints in *both* markets can be satisfied individually at the monopoly price or *neither* can be satisfied individually at any price above cost. This is not true, however, when markets differ in terms of N or δ, and in such cases, multimarket contact may facilitate collusion. It is perhaps unnatural to assume that a firm uses different discount factors to value the net income streams associated with different activities. However, the importance of δ suggests a more general point: potential gains from multimarket contact may arise when firms attach more weight to future outcomes in some markets than in others. In the next two subsections, we explore these factors in greater detail.

□ **Number of firms.** For purposes of illustration, we consider a situation in which market A is a duopoly and market B consists of $N > 2$ competitors. To focus on the case of interest, we make the following three assumptions.

Assumption 1. $2(1 - \delta) < 1$.

Assumption 2. $N(1 - \delta) > 1$.

Assumption 3. $(N - 2)(1 - \delta) < 1$.

Assumption 1 implies that complete collusion can be sustained (strictly) in the duopolistic market, A. Assumption 2 implies that, in the absence of multimarket contact, the only outcome in the N-firm market, B, involves pricing at cost.[13] Finally, Assumption 3 implies that if market B had only $(N - 2)$ firms, then complete collusion would be sustainable.

Suppose now that each of the market A duopolists owns a market B firm. We again examine the set of optimal stationary symmetric-payoff equilibria. (These yield identical payoffs to identical firms, i.e., one payoff to each of the two conglomerates and another to each of the $N - 2$ market B firms.) It is not difficult to show that an optimal equilibrium within this class involves identical market shares in each market for the two conglomerates, and we impose this condition in the discussion that follows.[14]

Suppose then that the market A price is $p_A > c$, which yields aggregate profits in market A of $\Pi_A = (p_A - c)Q(p_A)$. By Assumption 1, the incentive constraint for each conglomerate in market A is nonbinding. In particular, the net gains of deviating for each conglomerate (given the worst possible punishments) are

$$\Pi_A\left[1 - \frac{1}{2}\left(\frac{1}{1 - \delta}\right)\right] < 0. \tag{6}$$

The conglomerates can potentially use this slack enforcement power to induce a partially or completely collusive outcome in market B.

This outcome occurs as follows. Each conglomerate sets output so that the market share of its market B subsidiary is less than $(1/N)$. This leaves a greater share of market B for the other $N - 2$ firms. A single-market firm, i, with market share λ_i will not undercut a price $p_B \in (c, p^m]$ if and only if

$$(p_B - c)Q(p_B) \leq \left(\frac{1}{1 - \delta}\right)\lambda_i(p_B - c)Q(p_B), \tag{7}$$

[13] The reader may wonder about the consistency of Assumption 2 with our implicit assumption that entry is blockaded; that is, why did these N firms spend money to enter an industry in which they would earn nothing? This can be justified in several ways. For example, the N firms may be those that, *ex post,* were successful in stochastic research and development programs. Alternatively, the firms may have originally expected demand to grow rapidly (we argue in Section 4 that this would make supracompetitive prices possible), but, *ex post,* demand has been stationary.

[14] The argument parallels that of the second step of the proof of Proposition B1 in Appendix B.

or

$$\lambda_i \geq (1 - \delta). \tag{8}$$

Thus, if the market share of each of these firms is at least $(1 - \delta)$, they will not undercut a collusive arrangement.

Of course, this strategy violates the market B incentive constraint for each conglomerate firm. Specifically, if the price in market B is p_B, then the net gains from deviating in market B (considered in isolation) for the two conglomerates, if they each have a market share of λ_c, are

$$\Pi_B\left[1 - \lambda_c\left(\frac{1}{1-\delta}\right)\right], \tag{9}$$

where $\Pi_B = (p_B - c)Q(p_B)$ is the aggregate profit level in market B. The preceding discussion implies that $\lambda_c \leq [1 - (N-2)(1-\delta)]$, so (9) is strictly positive. However, as long as the sum of the expressions in (6) and (9) is nonpositive, neither conglomerate firm will deviate. Multimarket contact allows these firms to transfer the ability to collude from market A to market B by pooling their incentive constraints across markets.

The optimal collusive equilibrium is easily derived. Since both profits and the degree of surplus enforcement power rise with Π_A (recall condition (6)), the price in market A is set at its monopolistic level, p^m. Given the resulting slack enforcement power in market A and the conglomerate market shares in market B of $\lambda_c \leq [1 - (N-2)(1-\delta)]/2$, the highest sustainable level of aggregate profit in market B, $\Pi_B^*(\lambda_c)$, satisfies

$$(p^m - c)Q(p^m)\left[1 - \frac{1}{2}\left(\frac{1}{1-\delta}\right)\right] + \Pi_B^*(\lambda_c)\left[1 - \lambda_c\left(\frac{1}{1-\delta}\right)\right] = 0. \tag{10}$$

Note that as λ_c increases from zero to $[1 - (N-2)(1-\delta)]/2$, $\Pi_B^*(\lambda_c)$ increases. This raises conglomerate profits and, under Assumption 1, also increases the profits of the $(N-2)$ market B firms, $(1 - 2\lambda_c)\Pi_B^*(\lambda_c)$. Thus, as long as the monopoly price cannot be sustained in market B for any λ_c, the optimal collusive outcome involves setting $\lambda_c = [1 - (N-2)(1-\delta)]/2$. (When the monopoly price can be sustained for some λ_c, there is a Pareto frontier of equilibria that correspond to different levels of λ_c, all of which sustain $p_B = p_B^m$.)

Several aspects of this result deserve highlighting. First, contrary to conventional wisdom, the purchase of market B firms by "powerful" market A firms would lead to a decline in these firms' market shares—indeed, the conglomerate firms achieve a collusive outcome precisely through the contraction of their shares. Second, under Assumptions 1 through 3, multimarket contact *always* yields a potential gain, since $\Pi_B^*(\lambda_c)$ is always positive; that is, we can always sustain a price above cost in market B. Third, note that exactly the same points hold if we let either the demand or the level of (constant) marginal cost vary by market. Moreover, since the potential gains associated with multimarket contact depend upon the level of monopoly profits in market A (recall condition (10)), if the demand in market A is sufficiently large or the cost sufficiently low, multimarket contact leads to the complete monopolization of market B; that is, the firms can sustain $p_B = p_B^m$.

□ **Growth rates, response lags, and fluctuations.** In this subsection, we discuss three factors that may cause firms to attach more weight to future outcomes in some markets than in others. Since the analytics for all of the factors are similar, we present the first two informally, developing only the third in detail.

First, demand may grow more rapidly in one industry than in another. When one considers a single market in isolation, the addition of a geometric growth rate alters nothing of substance. Indeed, for analytical purposes, one can interpret δ as the product of a discount factor (ρ) and a growth factor (γ_k). Thus, it is easier to cartelize a rapidly growing market than one in which demand is stagnant. Intuitively, rapid growth makes the consequences of punishment (which occurs in the future) more important relative to the gain from deviating (which is immediate). This observation suggests that multimarket contact may serve as a device for shifting punishment power from rapidly to slowly growing markets.[15] Unfortunately, when one considers interactions over several markets, it is no longer valid to interpret δ as $\rho\gamma_k$, since differential growth causes the relative size of the two markets to change from period to period. One can nevertheless show that if N (the number of firms in both industries) is fixed, gains from multimarket contact are always available whenever $\rho\gamma_A > \left(1 - \frac{1}{N}\right) > \rho\gamma_B$ (i.e., when collusion is sustainable in one market but not in the other).[16]

The tendency for established firms in mature industries to acquire subsidiaries in rapidly developing industries has often been attributed to the fact that established firms typically have high earnings but relatively poor internal investment opportunities, while rapidly growing firms have insufficient internal funds to finance all profitable projects. The present analysis suggests that the same tendency could arise in part from the desire to spread market power from one industry to another.

Second, firms may be able to respond more quickly to deviations from collusive agreements in some markets than in others. Actions may be directly observable and immediately punishable in some markets, while in others, defections may be detected and punishment initiated only with a lag or some statistical uncertainty. (See, for example, Green and Porter (1984).) As with growth rates, when one considers a single market in isolation, adding an explicit response lag (the amount of time required to initiate punishments subsequent to deviation) changes nothing of substance. Once again, we may interpret δ as a function of the discount factor and a market-specific response lag. Although this interpretation no longer holds when one considers interactions over several markets simultaneously (unless response lags are identical, the implied length of a single period differs between the two markets), one can nevertheless show that multimarket contact can create potential gains by allowing firms to shift enforcement power from a market in which responses are rapid to one in which they are sluggish.[17]

Finally, demand may fluctuate from period to period within each market. Rotemberg and Saloner (1986) have previously argued that when demand fluctuates in a single market, collusion should be countercyclical: the future seems more important relative to the present when demand is low than when it is high. Indeed, firms may have slack enforcement power in periods of low demand but may be unable to sustain collusion in periods of high demand. While firms cannot shift enforcement power across periods, they can shift it across markets. Thus, one would suspect that parallel mergers across industries would yield gains as long as the random shocks experienced by each market are not perfectly correlated. The tendency for conglomerate firms to diversify over markets which experience poorly correlated shocks (see Marshall, Yawitz, and Greenberg (1984)) has previously been attributed to risk aversion, taxes, and/or bankruptcy costs. Our analysis suggests that the ability to collude more effectively could also play a role.

[15] Harrington (1986) establishes a similar result. He considers two finite-horizon industries, one of which terminates before the other. Due to the existence of multiple static equilibria, one can enforce collusion in a single market until some critical period prior to termination. If multimarket firms operate in both industries, they may be able to maintain collusive outcomes in the short-horizon industry through its terminal period by shifting enforcement power from the long-horizon industry.

[16] In fact, under this condition, it is always possible to eventually sustain complete collusion in both industries; since market A becomes extremely large relative to market B, slack enforcement power in market A must eventually exceed the net gains from deviating from a monopolistic outcome in market B.

[17] Tirole (1988) provides a simple example that illustrates this basic idea, in which in one market, firms choose prices every period, while in the other, they choose prices every other period.

We now formally illustrate this final point. As in Section 3, we envision two firms, 1 and 2, operating simultaneously in two markets, A and B. We maintain all of our previous assumptions concerning demand and production costs, except that we now distinguish between two demand states for each market, signified by h (high) and l (low). We use $Q^s(\cdot)$ to denote demand (for either market) in state $s = h, l$, and assume that $Q^h(p) > Q^l(p)$ for all $p \geq 0$. As in Rotemberg and Saloner (1986), the realizations of these states are independent across periods. For illustrative purposes, we assume here that there is perfect negative correlation between the demand shocks in these two markets. Thus, with probability .5, market A is in state h and B is in state l, while with probability .5, the reverse is true. The general case is considered in Bernheim and Whinston (1987) and discussed briefly below.

In this model, optimal punishments consist of reverting to the static Bertrand solution in every period in every state—as before, this equilibrium yields net discounted profits of zero. A stationary equilibrium path specifies prices and market shares for each market for each state of nature. Once again, in looking for an optimal stationary symmetric-payoff equilibrium, we can restrict our focus to those equilibria that entail equal market shares within each market in all states. (This is also true of the single market case.) Furthermore, it is not difficult to show that we can also restrict ourselves to outcomes that treat the markets symmetrically. Consequently, an equilibrium is completely characterized by two prices, p_h and p_l. Both firms set prices equal to p_l in the low demand market and equal to p_h in the high demand market. Let π_s denote the corresponding profits for each firm in the market for which the realization is s. In the multimarket setting, by undercutting its opponent, either firm can temporarily capture all the business in both markets, earning profits that are arbitrarily close to $2(\pi_l + \pi_h)$. Thus, each firm's incentive constraint is

$$\frac{\delta}{1-\delta}[\pi_h + \pi_l] \geq \pi_h + \pi_l,$$

or $\delta \geq 1/2$. As long as this condition is satisfied, the firms can jointly achieve monopoly profits in both markets. When $\delta < 1/2$, no price above cost is feasible.

To gauge the gains from multimarket contact, we consider next the opportunities for cooperation in a single market, assuming that there are no conglomerate firms. This is essentially the problem treated by Rotemberg and Saloner (1986). In this case, stationary, symmetric equilibrium paths supported by Bertrand punishments are characterized by two prices, p_h and p_l, where p_s denotes the price quoted by both firms in state s. Again letting π_s be the associated level of profits for each firm, incentive compatibility requires that

$$\frac{\delta}{1-\delta}\left[\frac{\pi_h}{2} + \frac{\pi_l}{2}\right] \geq \max\{\pi_h, \pi_l\}.$$

For $\delta < 1/2$, the only nonnegative solution to this inequality is $\pi_h = \pi_l = 0$. For $\delta \geq 1/2$, the most collusive outcome yields

$$\pi_l = \pi_l^m$$

and

$$\pi_h = \min\left\{\left[\frac{1-\delta}{2-3\delta}\right]\pi_l^m, \pi_h^m\right\},$$

where $2\pi_s^m$ is the aggregate monopoly profit in state s. Thus, in the single-market setting, firms can sustain full cooperation in both states only when $\delta \geq \delta^*$, where

$$\delta^* = \left[\frac{2\pi_h^m - \pi_l^m}{3\pi_h^m - \pi_l^m}\right] > 1/2.$$

For $1/2 \leq \delta < \delta^*$, multimarket contact increases the ability to sustain collusive outcomes.

In Bernheim and Whinston (1987), we relax the assumption of perfect negative correlation. There we show that potential gains to multimarket contact exist as long as the coefficient of correlation between the demand shocks is less than unity. These gains rise monotonically as the correlation falls. Thus, firms should prefer to establish multimarket contact across markets for which the correlation of shocks is as low as possible.[18]

5. Differing firms: spheres of influence and reciprocal exchanges

■ The second central assumption in Section 3 is that firms are identical. In practice, of course, firms may have different production costs. Such differences may arise not only because of differing levels of technological knowledge and capability, but also, in markets where transportation costs are significant, due to differing plant locations.

In this section, we demonstrate that multimarket contact may facilitate collusion in the presence of such cost differences. Furthermore, we show that the firms' optimal behavior in such circumstances corresponds to patterns that have, in fact, been previously noted, i.e., the development of "spheres of influence" and the use of reciprocal trades of output.

The development of spheres of influence for multimarket firms was originally discussed by Edwards, who argued, "Each conglomerate competitor . . . may informally recognize the other's primacy of interest in markets important to the other, in the expectation that its own important interests will be similarly respected."[19] Thus, when firms compete simultaneously in several different markets, each may come to specialize in some subset of these markets, and such specialization may help firms maintain high prices.

To illustrate these points, it is useful to distinguish between two cases of differing costs. First, firms may be in a situation of "symmetric advantage": each may be more efficient in some markets but less efficient in others. The most obvious example occurs when transportation costs are important and firms' plants are geographically separated, but symmetric advantage can clearly arise in other situations as well. The other case is that of "absolute advantage," in which one firm is more efficient in all markets. We now consider each of these cases in turn.

□ **Symmetric advantage.** Once again, consider a model with two markets (A and B), two firms (1 and 2), and homogeneous products within each market. Let firm 1's constant marginal cost of production in market A be $\underline{c}$, while firm 2's marginal cost is $\bar{c}$, where $\bar{c} > \underline{c}$. In market B, production costs are reversed, with firm 2 being more efficient than firm 1. In each market k, demand in every period is described by the function $Q(p_k)$. For expositional purposes, we again assume that $(p - c)Q(p)$ is concave in p for any level of constant marginal costs, $c \in [\underline{c}, \bar{c}]$, and we denote the monopoly price for cost level c by $p^m(c)$.

As a point of comparison, we begin by examining the optimal single-market outcome. (That is, we assume that the firms compete only in market A.) In this model, optimal punishments yield discounted payoffs of zero for both firms—the punishment for firm i entails both firms naming a price of $\underline{c}$ in every future period and firm 1 making all of the sales.[20]

[18] Evidence indicating that conglomerate firms tend to diversify over poorly correlated markets does not, of course, differentiate between our explanation of this phenomenon and the alternatives mentioned earlier. However, it may be possible to test between these two competing explanations by examining other collateral implications. For example, our model implies that the variability of total profits in each market is higher when multimarket contact is present. (Contact does not affect profits in state l but raises them in state h.) We doubt that this prediction also follows from any of the alternatives.

[19] Corwin Edwards, as quoted in Scherer (1980, p. 340).

[20] While this punishment is subgame perfect, it does have the unattractive feature that firm 2 plays a weakly dominated strategy. It is not difficult, however, to construct other punishment paths that yield the deviator a discounted payoff of zero and that do not involve weakly dominated strategies. (These have a "stick and carrot" structure as in Abreu (1986).) In any case, our basic points regarding the effect of multimarket contact do not rely in any way on the use of optimal punishments.

Consider, first, the single-market stationary equilibria. Firm i will not cheat in market k when the price is $p_k \geq \bar{c}$ and its market share is λ_{ik} if and only if

$$\phi(p_k | c_{ik}) \leq \left(\frac{1}{1-\delta}\right)\lambda_{ik}(p_k - c_{ik})Q(p_k), \tag{11}$$

where c_{ik} is firm i's (constant) marginal cost in market k and $\phi(p | c) \equiv \max_{s \leq p} (s - c)Q(s)$. Noting that $\phi(p_k | c_{ik}) \geq (p_k - c_{ik})Q(p_k)$, we can sum (11) over $i = 1, 2$ and conclude that if $\delta < 1/2$, then no price above $\bar{c}$ is sustainable. When $\delta \geq 1/2$, firms can sustain collusive prices above $\bar{c}$. The crucial point for our purposes, however, is that some fraction of sales must be allocated to the inefficient firm to keep it from undercutting the collusive price. In particular, the inefficient firm must have a market share of at least $(1 - \delta)$.[21] If

$$\delta[p^m(\underline{c}) - \underline{c}] \geq (\bar{c} - \underline{c})Q(\bar{c}),$$

then a collusive outcome Pareto dominates (from the firms' perspectives) the static equilibrium in which both firms name a price of $\bar{c}$ and the efficient firm makes all of the sales; abstracting from coordination difficulties, we would therefore expect to observe collusive behavior with a price $p \in [p^m(\underline{c}), p^m(\bar{c})]$ and a market share for the inefficient firm of at least $(1 - \delta)$. If this inequality is not satisfied, then the efficient firm prefers the static ("noncollusive") outcome, while the inefficient firm prefers the reverse.[22] In neither case, however, do the firms achieve the joint profit-maximizing outcome, since this involves a price of $p^m(\underline{c})$ and requires that the efficient firm make all of the sales.

Now, consider the most collusive outcome in the two market setting. For expositional purposes, we focus here on stationary equilibria that involve symmetric outcomes: a price p is charged in both markets by both firms, and the efficient firm receives a market share, $\underline{\lambda}$, in each market. We argue below that the equilibria we derive are the optimal stationary symmetric-payoff equilibria, so this focus is unrestrictive.

As before, if a firm deviates, it receives a continuation payoff of zero. Thus, the optimal collusive outcome solves

$$\max_{\underline{\lambda}, p \geq \bar{c}} \underline{\lambda}(p - \underline{c})Q(p) + (1 - \underline{\lambda})(p - \bar{c})Q(p)$$

subject to

$$\phi(p | \underline{c}) + \phi(p | \bar{c}) \leq \left(\frac{1}{1-\delta}\right)\{\underline{\lambda}(p - \underline{c})Q(p) + (1 - \underline{\lambda})(p - \bar{c})Q(p)\}. \tag{12}$$

It is easy to see that any solution to this problem must involve $\underline{\lambda} = 1$. That is, the less efficient firm *completely* withdraws from each market.[23,24] It is then clear that the solution

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[21] Proposition B2 in Appendix B shows that when $\delta < 1/2$, sales never occur at a price above $\bar{c}$ in any period of a subgame perfect equilibrium. Our focus on stationary equilibria is therefore unrestrictive for such discount factors. When $\delta > 1/2$, however, this focus may be restrictive since, as Schmalensee (1987) has shown, the set of payoffs achievable by market sharing is nonconvex. (So, profits may be increased by allowing the firms to alternate production between them.) Even so, for any $\delta < 1$, some production must be allocated to the inefficient firm. As a result, the firms still cannot achieve the joint profit-maximizing solution. In contrast, multimarket contact (as we show in the text) can allow them to achieve this outcome.

[22] Note, however, that if punishments consisted of reversion to the static outcome, then a collusive price could *only* arise if $\delta[p^m(\underline{c}) - \underline{c}] \geq (\bar{c} - \underline{c})Q(\bar{c})$.

[23] The reader may perceive a tension between the assumption that entry is blockaded (see footnote 7), and the supposition that each incumbent firm can freely enter the other's market even after complete withdrawal. Implicitly, we assume that each incumbent has an advantage over potential entrants in both markets and that it retains at least a portion of this advantage even if it terminates operations in some market. For example, the advantage could arise from a patent, knowledge, or other sunk assets that are useful in both markets. Alternatively,

never involves a price in excess of $p^m(\underline{c})$, the monopoly price level for a firm with cost level $\underline{c}$, since a price of $p^m(\underline{c})$ generates larger profits than any higher price and involves identical deviation profits. Now, when $\underline{\lambda} = 1$, a price $p \leq p^m(\underline{c})$ is sustainable if and only if

$$(p - \bar{c}) \leq \left(\frac{\delta}{1 - \delta}\right)(p - \underline{c}). \tag{13}$$

Two conclusions follow immediately. First, if $\delta \geq 1/2$, complete monopolization is possible—the efficient firm sets price equal to $p^m(\underline{c})$ in each market. Second, for all $\delta \in (0, 1/2)$ at least *some* collusion is possible. The optimum involves a price such that (13) binds, unless $p^m(\underline{c})$ is sustainable, which occurs at some critical discount factor strictly less than 1/2.[25]

The fact that the optimal collusive outcome here involves the development of spheres of influence is not terribly surprising, since such a move directly raises profits for the firms. What is interesting, however, is that the development of spheres of influence also enables firms to collude more effectively on price. By shifting sales toward the more efficient firm in each of the two markets, profits on the equilibrium path rise, while the possible gains from deviating fall. (A greater fraction of a firm's gain comes in the market where it has higher costs.) Both effects relax the incentive constraint associated with sustaining any given price p.[26]

An interesting comparison can be made between the effect of multimarket contact on the ability to collude and the effect of nonbinding side payments on collusion in a single-market setting.[27] In particular, the set of outcomes that can be sustained here through multimarket contact is identical to that which can be sustained with a scheme of nonbinding side payments in a single-market context. Suppose, for example, that outcome $(p, \underline{\lambda})$ is sustainable with multimarket contact; that is, it satisfies (12). Now, consider a single-market scheme in which in each period, the firms first name price p, receiving shares of $\underline{\lambda}$ and $(1 - \underline{\lambda})$, and then, if no one has deviated, one firm makes a payment to the other. If at any time a firm deviates, both firms revert to pricing at cost in all future periods, and no further side payments are made. It is easy to see that if the firm making the side payment is going to deviate, it will do so at the start of the period. (It will not name price p if it plans to refuse to make the side payment.) Let S denote the side payment given or received by the inefficient firm. (S is positive if the inefficient firm receives the side payment and negative if it makes a payment.) Then, the relevant incentive constraint for the efficient firm is

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patents or assets may be market specific, but the firms may be legally barred from trading them so as to also prevent each other's entry.

[24] Note that while this highly stylized model involves *complete* withdrawal of the less efficient firm, this need not happen with more general cost structures. If, for example, the marginal costs of production for the less efficient firm were $\underline{c}$ up to some quantity $\bar{q}$ but were $\bar{c}$ thereafter, then an optimal collusive arrangement would involve the less efficient firm producing $\bar{q}$ units. More generally, the optimal stationary market share allotment for any given price minimizes industry production costs; as in the case analyzed in the text, this share allotment allows firms to collude more effectively on prices. It is worth noting, however, that there may be advantages to complete withdrawal arising from factors not present in our model. Withdrawal may, for example, improve the quality of monitoring. (For example, in geographic markets, it may be easier to detect a rival's entry into a city than to monitor its price if it is selling there.)

[25] Clearly, when $p^m(\underline{c})$ can be sustained, this stationary symmetric equilibrium is an optimal stationary symmetric-payoff equilibrium (since it yields the joint monopoly outcome). This is also true, however, when $p = p^m(\underline{c})$ does not satisfy (13). In particular, we show in Proposition B3 of Appendix B that in any stationary symmetric-payoff equilibrium, the highest price that can be sustained in either market must satisfy (13), and therefore no stationary symmetric-payoff equilibrium can yield larger profits than those derived in the text.

[26] An additional and distinct way in which multimarket contact can facilitate collusion is by eliminating the bargaining problems associated with single-market asymmetries. For example, we have noted that a collusive price might not arise in the single-market case even when it could ($\delta \geq 1/2$) because both firms might not benefit from this outcome. Multimarket contact can allow each firm to gain from collusion that raises aggregate profits by allowing each to gain in one market.

[27] We would like to thank Ken Hendricks and Paul Klemperer for suggesting that we think about this issue.

$$\phi(p|\underline{c}) \leq \left(\frac{1}{1-\delta}\right)[\underline{\lambda}(p-\underline{c})Q(p) - S], \tag{14}$$

and the incentive constraint for the inefficient firm is

$$\phi(p|\bar{c}) \leq \left(\frac{1}{1-\delta}\right)[(1-\underline{\lambda})(p-\bar{c})Q(p) + S]. \tag{15}$$

Since $(p, \underline{\lambda})$ satisfies (12), one can clearly find an S that satisfies both (14) and (15). ((12) is just the sum of (14) and (15).) More generally, when (nonbinding) side payments are allowed in a single-market context, their presence allows firms to effectively pool their incentive constraints in each period. (That is, they face an incentive constraint that is the sum of the individual firms' incentive constraints.) Thus, when firms differ across markets in a symmetric way (as is the case here), multimarket contact without side payments yields the same set of outcomes as does single-market interaction with side payments.[28]

The welfare implications of multimarket contact in this setting should also be noted. When collusion would arise in a single market (without side payments), multimarket contact unambiguously *improves* welfare, in contrast to the usual presumption; in this case, the movement toward spheres of influence both lowers costs (by setting $\underline{\lambda} = 1$) and (weakly) lowers prices (since the single-market collusive price lies in $[p^m(\underline{c}), p^m(\bar{c})]$). In contrast, if a collusive outcome would not arise in the single-market context (which is always the case whenever $\delta < 1/2$), welfare is unambiguously impaired: in both the single- and multimarket situations, $\underline{\lambda} = 1$, but prices are higher in the multimarket case. Thus, multimarket contact may or may not reduce welfare, even when it has real effects.[29]

As we have seen, when firms differ in their costs across markets, multimarket contact can facilitate the maintenance of collusive prices through the development of spheres of influence. Nevertheless, if firms cannot coordinate such an arrangement tacitly, they may be reluctant to do so overtly given the Sherman Act's *per se* ban on market division agreements. In such circumstances, they may seek other lawful means of accomplishing the same ends. As we shall now demonstrate, in the case of geographic markets with high transportation costs, a frequently observed form of horizontal reciprocal output agreement can serve exactly this purpose.

Horizontal reciprocal output agreements are common in a number of industries in which transportation costs are significant.[30] In the typical reciprocal output agreement, a firm with a production facility in market A and a presence in market B will agree to swap output on a unit-for-unit basis with a firm whose production facility is in market B and who also sells in market A. Effectively, this provides output at a lower cost to the firm that is inefficient in each market.

Consider a situation in which the inefficient firm initially has a share, $\underline{\lambda}$, in each market,

[28] Note that we could alternatively model side payments as occurring simultaneously with price choices. Which choice is more appropriate depends on the structure of the market that we are trying to capture. For example, if competition occurs in discrete lumps (as in government procurement auctions) but side payments can occur at any time, then the timing described in the text is appropriate. Alternatively, if the delay in reaction to a deviation occurs because changing prices takes time, while making side payments does not, then our assumed timing is again appropriate. If, on the other hand, the delay in punishment occurs because of a detection lag, then simultaneous modelling is more appropriate. With simultaneous modelling the equivalence discussed in the text does not hold. In particular, side payments no longer completely pool incentive constraints in the single-market context because the payment also appears on the left-hand side of either (14) or (15). (Whichever firm receives the side payment still receives it in the period in which it first deviates.)

[29] Note, though, that in the case of geographic markets with high transportation costs, there is no simple policy, such as prevention of mergers, that can eliminate the effects of multimarket contact. (The essence of this situation is that differing costs arise precisely because firms do not have a plant in each market.)

[30] See *Blue Bell Co. v. Frontier Refining Co.*, 213 F. 2d 354 (10th Cir. 1954) for an example in the oil industry and, in the corrugated container industry, Baker (1986).

and suppose that antitrust considerations preclude further specialization. Price $p \leq p^m(\underline{c})$ is sustainable without a reciprocal agreement if and only if

$$[\underline{\lambda}(p - \bar{c})Q(p) + (1 - \underline{\lambda})(p - \underline{c})Q(p)] \leq \left(\frac{\delta}{1 - \delta}\right)[\underline{\lambda}(p - \underline{c})Q(p) + (1 - \underline{\lambda})(p - \bar{c})Q(p)]. \quad (16)$$

(This is the same constraint as in (12).) Now consider an agreement in which the efficient firm in each market provides the inefficient firm with $(1 - \underline{\lambda})Q(p)$ units of output in each period. In what follows, we distinguish contractual from noncontractual agreements.

Consider first a noncontractual agreement. To start, we suppose that in each period the firms first name their prices, and then they simultaneously announce whether they are willing to trade; a trade takes place only if both firms agree.[31] Optimal punishments in this setting involve a reversion to the single-market punishment and the cessation of all future trade. Clearly, neither firm will deviate and say "no" to trade if there has been no prior deviation, so we need only consider the incentive to undercut the collusive price. By not deviating, each firm earns a discounted payoff of $\left(\frac{1}{1 - \delta}\right)(p - \underline{c})Q(p)$, while deviation yields $[(p - \underline{c})Q(p) + (p - \bar{c})Q(p)]$, since the output trade ceases in the period of deviation. Thus, price p is sustainable if and only if condition (13) is satisfied, i.e., under exactly the same conditions as those for the development of spheres of influence.

The ability of the firms to refuse to trade in period t if a firm has deviated in that period is important for this equivalence. If trade must occur either before or at the same time price choices are made, this lessens the effectiveness of the reciprocal output trade because a firm that undercuts the collusive price now benefits from the trade in the period of deviation. (Recall footnote 28.) Nevertheless, the reciprocal trade still increases the firms' abilities to collude on price. In this case, price p is sustainable if and only if

$$[\underline{\lambda}(p - \bar{c})Q(p) + (1 - \underline{\lambda})(p - \underline{c})Q(p)] \leq \left(\frac{\delta}{1 - \delta}\right)(p - \underline{c})Q(p). \quad (17)$$

Finally, consider a contractual agreement in which the firms agree to trade in every future period. Once again, optimal punishments yield a payoff of zero in every period after the deviation. (Now both firms name prices equal to $\underline{c}$, and the inefficient firm sells $(1 - \underline{\lambda})Q(\underline{c})$ units of output in each period.) The condition under which a price, p, is sustainable with a contractual agreement is identical to (17).[32]

The key point is that the reciprocal agreement creates additional surplus for the firms that can be dissipated if either deviates from the collusive price.[33,34] As with spheres of influence, the welfare implications of these agreements are ambiguous.

[31] We ignore any difficulties in enforcing the trade if both parties say "yes." In a sense, our "contractual" versus "noncontractual" distinction really represents long-term (many-period) versus short-term (single-period) contracting.

[32] This illustrates that a breakdown in a noncontractual agreement is actually inessential to our result when trade is simultaneous with price choices. The surplus can be dissipated equally well through price choices. Likewise, although we have assumed that the contractual agreement remains in force, in principle, we could allow the firms to tear up the agreement and get the same result. The only important point about the contractual agreement is that it precludes the cessation of trade in the same period as a deviation, since the deviator can refuse to tear up the agreement.

[33] Baker (1986) quotes a former paper industry executive as indicating that the threat of exclusion from the existing network of linerboard exchanges was an effective means of enforcing cooperation in the corrugated container industry. It is interesting to note that our analysis also suggests a theory of detente, in which gains from trade can help sustain more cooperative behavior in the military sphere. For a development of this idea, see Alt and Eichengreen (1987).

[34] It should be clear that similar benefits can be had from supply agreements in a single-market context. Indeed, the outcomes that can arise with "at cost" supply arrangements are equivalent to those when nonbinding

Absolute advantage. We now shift our attention to the case in which a single firm maintains a cost advantage over its competitor in two distinct markets. In particular, the efficient firm produces output at a cost $\underline{c}$ per unit, while the inefficient firm produces at $\bar{c} > \underline{c}$. As before, demand in each market k is given by $Q(p_k)$. Since we have already described the single-market outcomes, we turn immediately to the multimarket case.

Our goal is to show that multimarket contact expands the range of environments in which firms can sustain collusive equilibria. Accordingly, we assume that $\delta < 1/2$, so that single-market outcomes are necessarily competitive. (See Proposition B2 in the Appendix.)

To construct a collusive multimarket equilibrium, we begin by fixing current output prices, $p_k \in [\bar{c}, p^m(\bar{c})]$, for each k. Without loss of generality, let $p_A \geq p_B$. Let $\underline{\lambda}_k$ denote the efficient firm's share of market k, and define

$$\underline{\pi}_k = (p_k - \underline{c})Q(p_k)$$

and

$$\bar{\pi}_k = (p_k - \bar{c})Q(p_k),$$

$k = A, B$. By deviating from the prescribed prices, the inefficient firm could obtain current profits arbitrarily close to $\bar{\pi}^d = \bar{\pi}_A + \bar{\pi}_B$. For the efficient firm, current period (deviation) profits are $\underline{\pi}^d \equiv \phi(p_A|\underline{c}) + \phi(p_B|\underline{c})$.

As before, we focus on stationary equilibria. Consider an allocation $(p_A, p_B, \underline{\lambda}_A, \underline{\lambda}_B)$. Since optimal punishments entail zero profits, this is sustainable if and only if

$$\bar{\pi}^d \leq (1-\delta)^{-1}[\underline{\lambda}_A\underline{\pi}_A + \underline{\lambda}_B\underline{\pi}_B]$$

and

$$\underline{\pi}^d \leq (1-\delta)^{-1}[(1-\underline{\lambda}_A)\bar{\pi}_A + (1-\underline{\lambda}_B)\bar{\pi}_B].$$

From these inequalities, it is easy to check that the set of sustainable collusive allocations is empty when $\delta < 1/2$ and $p_A = p_B$. Henceforth, we take $p_A > p_B$. It is helpful to rewrite the incentive constraints as

$$\underline{\lambda}_A(\underline{\pi}_A/\underline{\pi}_B) + \underline{\lambda}_B \geq (1-\delta)(\underline{\pi}^d/\underline{\pi}_B) \tag{18}$$

and

$$\underline{\lambda}_A(\bar{\pi}_A/\bar{\pi}_B) + \underline{\lambda}_B \leq (1 + \bar{\pi}_A/\bar{\pi}_B). \tag{19}$$

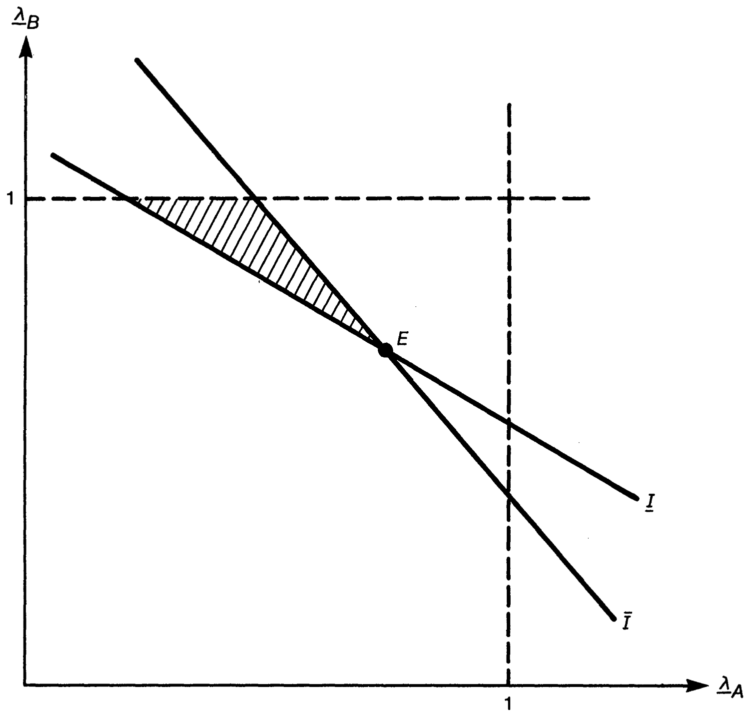
When $p_A > p_B$, it is straightforward to check that $\bar{\pi}_A/\bar{\pi}_B > \underline{\pi}_A/\underline{\pi}_B$. Accordingly, for fixed (p_A, p_B), we can graph the sustainable market shares. (See Figure 1.) $\underline{I}$ ($\bar{I}$) represents the incentive constraint for the efficient (inefficient) firm. Equation (18) implies that for $\delta < 1/2$, $\underline{I}$ must intersect the vertical axis above $\underline{\lambda}_B = 1$. Thus, the set of sustainable market shares must look like the shaded area between the two incentive constraints. This suggests that the inefficient firm will have a tendency to specialize in the high-price market.

As δ rises, $\underline{I}$ shifts down, while $\bar{I}$ shifts up, so that the point of intersection, E, moves to the southeast. Some degree of collusion first becomes sustainable when the vertical coordinate of E reaches unity. (At this point, the horizontal component must exceed zero, since $\underline{I}$ crosses the vertical axis above $\underline{\lambda}_B = 1$.) For the moment, we take $p_A \leq p^m(\underline{c})$. Using

side payments occur. While such supply arrangements would typically raise a concern with antitrust authorities in a single-market context because of their "appearance" of being like side payments, reciprocal output agreements—which yield identical results here—are typically allowed.

FIGURE 1

SUSTAINABLE MARKET SHARES



(18) and (19) to compute E, we find that it is possible to sustain the prices (p_A, p_B) in a stationary equilibrium if and only if

$$\delta \geq \delta^* \equiv [1 + (1 + \underline{\pi}_B/\underline{\pi}_A)(1 + \bar{\pi}_B/\bar{\pi}_A)^{-1}]^{-1}.$$

Since $\underline{\pi}_A/\underline{\pi}_B < \bar{\pi}_A/\bar{\pi}_B$, it follows immediately that $\delta^* < ½$. Thus, multimarket contact does expand the range of environments in which some collusion is sustainable.

More generally, for any fixed (p_A, p_B), we can say a bit more about the set of likely outcomes. Note that the efficient firm's isoprofit curves have slope $\underline{\pi}_A/\underline{\pi}_B$ and are therefore parallel to $\underline{I}$. Similarly, the inefficient firm's isoprofit curves have slope $\bar{\pi}_A/\bar{\pi}_B$, and are therefore parallel to $\bar{I}$. Thus, we can Pareto improve any allocation in the shaded area in Figure 1, unless that allocation lies on the northern frontier. It follows that the inefficient firm should completely specialize in the high-price market.

To summarize, when $\delta \in [\delta^*, ½]$, it is possible to sustain some degree of collusion in the multimarket game. Firms set different prices in otherwise identical markets, and the inefficient firm specializes in the high-price market. Specialization again suggests the formation of spheres of influence. Note that this outcome entails both inefficient production and noncompetitive pricing.

When $\delta > ½$, it is possible to sustain collusion even in a single-market game. However, any stationary equilibrium must involve a positive market share for the inefficient firm, and therefore, both firms can be made better off through the specialization that multimarket contact makes possible.[35]

[35] For sufficiently large δ, however, there are nonstationary single-market equilibria which mimic the multimarket solutions. In essence, price alternates between a high and low value, and each firm produces most or all of the output in every other period.

6. Nonconvexities: spheres of influence

■ We now modify the model of Section 3 by introducing a fixed cost of production, F. Thus, when firm i produces $q_{ik} > 0$ in market k, its total costs of production are given by $C(q_{ik}) = F + cq_{ik}$. We assume that firms bear this fixed cost only if they choose to produce, so $C(0) = 0$. Otherwise, we maintain all previous assumptions and notation. To avoid possible confusion, we note that π^m is defined as single-firm monopoly profits, $(p^m Q(p^m) - C(Q(p^m)))$, rather than the joint level of profits earned by two operating firms that collude fully, $(\pi^m - F)$. We assume that a single market can support both firms under a collusive arrangement, i.e., that $\pi^m > F$.

We begin our analysis by noting that the single-market, static price competition game has a unique equilibrium and that this equilibrium yields zero profits for both firms.[36] Repetition of this equilibrium generates optimal punishments for both firms; as in the previous sections, we may assume, without loss of generality, that firms revert to this equilibrium following any deviation from some proposed path.

Consider first the single-market (stationary symmetric-payoff) outcome. Let π denote the level of profits earned by each firm in each period along the equilibrium path (net of its fixed cost). An optimal deviation would yield current profits arbitrarily close to $2\pi + F$; subsequently, the deviator would earn zero profits in the punishment phase. On balance, deviation is unprofitable as long as

$$\delta \geq 1 - \frac{\pi}{2\pi + F} \equiv \delta^*(\pi).$$

Note that $\delta^*(\pi)$ is decreasing in π. Thus, it is (perhaps counterintuitively) easiest to sustain the fully collusive profit level, $(\pi^m - F)/2$. The required discount factor is

$$\tilde{\delta} \equiv \delta^*((\pi^m - F)/2) = 1/2 + \frac{F}{2\pi^m} > 1/2.$$

If $\delta < \tilde{\delta}$, no collusion is possible, while if $\delta \geq \tilde{\delta}$, a fully collusive outcome is sustainable.

Now consider the two-market case. Suppose firms attempt to sustain the global optimum by specializing. That is, each firm sets the monopoly price and meets all of the demand in its "home" market. In its competitor's home market, each firm sets a price strictly greater than p^m and produces nothing. Cooperation yields profits of π^m in each period. By deviating, a firm can increase its profits to (almost) $2\pi^m$ but will earn nothing thereafter. Accordingly, this collusive outcome is sustainable in equilibrium as long as $\delta \geq 1/2$; multimarket contact again expands the range of environments in which collusion is feasible. It is also possible to show that when $\delta < 1/2$, all perfect equilibria yield zero profits.

Note that when $\delta < 1/2$, multimarket contact has no effect on resource allocation. When $1/2 \leq \delta < \tilde{\delta}$, multimarket contact leads to higher prices without improving productive efficiency and is therefore socially undesirable. On the other hand, if $\tilde{\delta} < \delta$, multimarket contact does not alter prices but does increase productive efficiency, and is therefore unambiguously desirable. Finally, note that, as in Section 5, multimarket contact is associated with the development of "spheres of influence."

[36] Specifically, each firm announces the price p^c defined implicitly by the following single-firm, zero-profit condition: $(p^c - c)Q(p^c) - F = 0$. Consumers resolve their indifference by demanding all output from the same firm. Consequently, both firms earn zero profits, and neither has an incentive to deviate. One might object to this zero-profit equilibrium on the grounds that the convenient coordination of consumers' decisions is implausible. However, one can view the game here as a limiting approximation, either along the lines discussed in footnote 8, or alternatively by interpreting the continuous strategy space as approximating a large but finite number of strategies. With discrete price choices, for example, one could sustain an approximate zero-profit equilibrium without encountering the coordination problem: simply have one firm set the lowest price that yields nonnegative profits and have the other firm set its price "one penny" higher.

One might object to this analysis on the grounds that we have restricted our attention to stationary paths. For the single-market case, stationary equilibria are necessarily inefficient, since both firms must produce in every period. Nonstationary paths are of particular interest here, since they allow for the possibility that only one firm produces at a time. For analytic completeness, we have shown in Bernheim and Whinston (1987) that the consideration of nonstationary paths, which allow for the possibility that only one firm incurs the fixed cost in any period, does not qualitatively alter our results.

7. Differentiated products

■ Up to this point, we have assumed that products are homogeneous within markets. In this section, we turn our attention to the case of differentiated products.[37] Our discussion focuses on two issues of interest that did not arise in the homogeneous product case.

□ **Optimal allocation of market power.** In the models with homogeneous products considered above, collusion in a single-market context was an all-or-nothing occurrence. If the discount factor was above a certain threshold level, then a fully monopolistic outcome was possible; if it was below this threshold, then no collusive price could be sustained. When markets differed, multimarket contact could increase profits only if firms could successfully cartelize one of the markets in isolation. Slack in the incentive constraints from this market could then be used to increase profits elsewhere.

Product heterogeneity within each market adds considerable complexity, since the maximum sustainable price typically increases continuously as the discount factor, δ, rises. At any given δ, the maximum degree of sustainable collusion may differ between markets, according to demand and cost conditions. Thus, even when firms cannot sustain a fully collusive outcome in any market, they may be able to gain by shifting market power between markets (i.e., lowering prices in one market and raising them in another).

To address this possibility, consider a simple model of symmetric product differentiation. Once again, there are two markets (A and B) and two firms (1 and 2). The sales of firm i in market k are given by the function $Q_k(p_{ik}, p_{jk})$ ($j \neq i$), which is symmetric in its arguments. The constant unit cost of production for both firms in market k is c_k. We assume that a firm's profits in market k, $(p_{ik} - c_k)Q_k(p_{ik}, p_{jk})$, are concave in (p_{ik}, p_{jk}) for $k = A, B$. Now define

$$\hat{\pi}_k(p) \equiv \max_z \, (z - c_k)Q_k(z, p)$$

and

$$\pi_k(p) \equiv (p - c_k)Q_k(p, p).$$

$\hat{\pi}_k(p)$ gives a firm's one-period deviation profits in market k when the price charged by its rival is p, while $\pi_k(p)$ is the firm's one-period profit when both firms charge a price of p. It is easy to verify that our concavity assumption implies that $\pi''_k(p) < 0$ and that $\hat{\pi}''_k(p) > 0$.

Under these assumptions the optimal symmetric-payoff equilibria in both the single- and multimarket settings can be shown to be symmetric and stationary (involve firms naming a single price in each market in every period).[38] The most profitable stationary equilibrium for the firms solves

[37] The points we cover in this section also apply to the case of quantity (Cournot) competition with homogeneous products. In fact, an earlier version of this article (Bernheim and Whinston (1986)) analyzed the Cournot case.

[38] The symmetry of price choices within each market derives from the concavity assumption. In particular, if $\{(p_{1A}(t), p_{2A}(t), p_{1B}(t), p_{2B}(t))\}_{t=0}^{\infty}$ is a sustainable sequence of prices that yields symmetric payoffs, then

$$\max_{p_A, p_B} [\pi_A(p_A) + \pi_B(p_B)]$$

subject to

$$[\hat{\pi}_A(p_A) + \hat{\pi}_B(p_B)] + \delta \underline{v} \leq \left(\frac{1}{1-\delta}\right)[\pi_A(p_A) + \pi_B(p_B)].$$

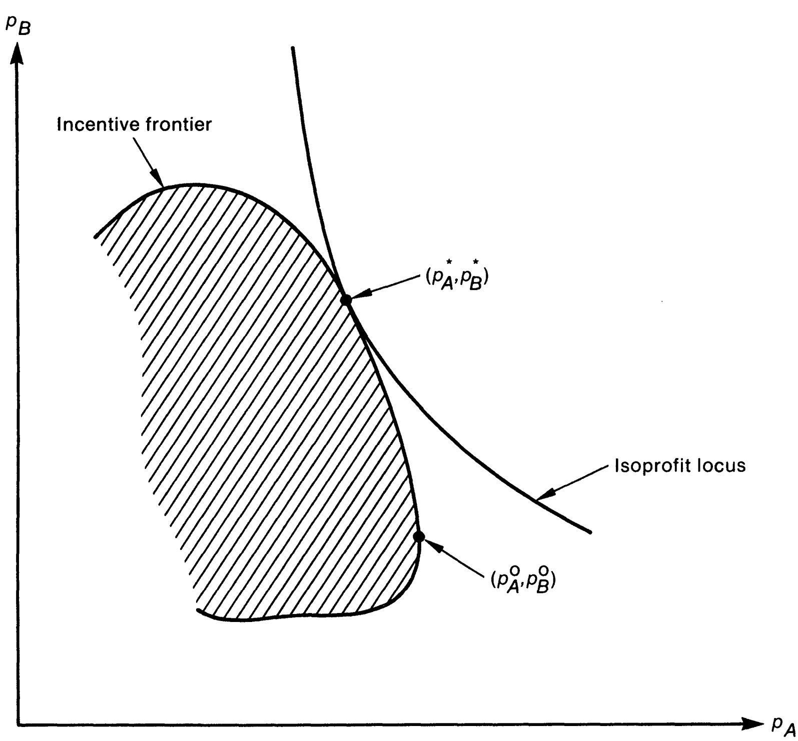
Assuming that full collusion is not sustainable, we know that the constraint must bind, which implicitly defines p_B as a function of p_A. The feasible price frontier is depicted in Figure 2. The optimal collusive scheme finds the highest isoprofit contour that intersects this frontier. The solution is depicted in Figure 2 and satisfies the necessary condition

$$\frac{\pi_A'(p_A^*)}{\hat{\pi}_A'(p_A^*)} = \frac{\pi_B'(p_B^*)}{\hat{\pi}_B'(p_B^*)}.$$ [39]

Thus, the optimal collusive allocation equalizes the ratio of the marginal profit from collusion to the marginal profit from deviation across markets. Suppose, for example, that a monopolistic price is just sustainable (with no slack) in market A alone, but that complete cartelization of market B is not possible. Then, since $\pi_A' = 0$ at the single-market solution in market A, optimal multimarket collusion involves a decline in price in market A and an increase in price in market B. More generally, if $\hat{\pi}_k'' > 0$ for $k = A, B$, then multimarket

FIGURE 2

OPTIMAL ALLOCATION OF MARKET POWER



$\{(\hat{p}_A(t), \hat{p}_A(t)), (\hat{p}_B(t), \hat{p}_B(t))\}_{t=0}^{\infty}$, where $\hat{p}_k(t) \equiv \frac{1}{2}[p_{1k}(t) + p_{2k}(t)]$ $(k = A, B)$ is a sustainable sequence that yields (weakly) larger symmetric payoffs. (Profits rise, while deviation profits fall.) Stationarity is demonstrated using an argument parallel to that in Abreu (1986).

[39] $\hat{\pi}_k'' > 0$ $(k = A, B)$ implies that this condition is also sufficient for a profit maximum. (The frontier in Figure 2 will be strictly convex.)

contact leads to a price increase in the market in which the single-market outcome entails a higher value of $(\pi'_k/\hat{\pi}'_k)$ and a price decrease in the other market.

It is of interest to note that, in contrast to the cases considered in Section 4, multimarket contact across differing heterogeneous product markets may actually increase welfare when products are heterogeneous within markets, since price increases in some markets are offset by price decreases in others.[40]

□ **Punishments.** One issue that does not arise in the homogeneous product case is the effect of multimarket contact on punishments. Since optimal single-market punishments always yield zero profits, multimarket contact could not increase the severity of punishments.

With differentiated products, this is no longer the case. To see this, we consider once again a two-firm, two-market model with symmetric differentiation. Unfortunately, as Abreu (1986) discusses, globally optimal punishments are often intractable. We therefore restrict our attention here to the case of symmetric punishment paths, in which the firms charge the same prices within each market during any period (i.e., $p_{1k}(t) = p_{2k}(t) \equiv p_k(t)$ for all t). Following Abreu (1986), it is not difficult to show that when the optimal multimarket punishment yields positive profits, it involves a "stick-and-carrot" structure: firms first engage in a one-period price war and then revert to the most collusive symmetric price path.[41] The prices that prevail during the price war, $(\underline{p}_A, \underline{p}_B)$, minimize $[\pi_A(p) + \pi_B(p)]$ subject to the constraint that

$$\sum_{k=A,B} \hat{\pi}_k(\underline{p}_k) + \delta[\pi_k(\underline{p}_k) + \delta\bar{v}_k] \leq \sum_{k=A,B} [\pi_k(\underline{p}_k) + \delta\bar{v}_k],$$

where $\bar{v}_k$ is the discounted payoff from the most collusive symmetric path in market k. Differentiation once again yields the necessary condition that

$$[\pi'_A(\underline{p}_A)/\hat{\pi}'_A(\underline{p}_A)] = [\pi'_B(\underline{p}_B)/\hat{\pi}'_B(\underline{p}_B)],$$

which will not generally hold at the most severe single-market (symmetric) punishments. Thus, at least within the class of symmetric punishments, multimarket contact can enable firms to construct more severe punishments, which, in turn, further enhances collusive outcomes.

Finally, we argue that in the presence of multimarket contact, it is often possible to arrange more severe punishments for single-market competitors. To see this, consider first the globally optimal punishment in a single-market context with three symmetric firms. If firm 3 deviates, then the first-period prices along the optimal punishment path must solve

$$\min_{p_1, p_2, p_3} \pi(p_3 \mid p_1, p_2) \tag{20}$$

subject to

$$\hat{\pi}(p_2, p_3) - \pi(p_1 \mid p_2, p_3) \leq \delta(u_1 - \underline{v}_1), \tag{21}$$

$$\hat{\pi}(p_1, p_3) - \pi(p_2 \mid p_1, p_3) \leq \delta(u_2 - \underline{v}_2), \tag{22}$$

$$\hat{\pi}(p_1, p_2) - \pi(p_3 \mid p_1, p_2) \leq \delta(u_3 - \underline{v}_3), \tag{23}$$

where u_i is firm i's continuation value on the punishment path, $\underline{v}_i$ is the value of the optimal punishment of firm i after it deviates, $\pi(p_i \mid p_j, p_l)$ is firm i's profit if it charges p_i and the

[40] Unfortunately, it is difficult to say anything general about the welfare effect of the movement from the single-market outcomes to the multimarket solution. We did perform some limited simulations using demands (x_1, x_2) generated by a representative consumer with quasi-linear preferences over x_1, x_2, and income (I) of the form $u(x_1, x_2, I) = I + \alpha(x_1 + x_2) - (\lambda/2)[x_1^2 + x_2^2] - \beta x_1 x_2$. We found multimarket contact to have a small positive effect on welfare (less than a 3% increase in all cases). The changes in prices due to this contact, however, were in some cases quite large. The difference between the prices in the two markets decreased by as much as 80% after the establishment of multimarket contact.

[41] More precisely, a sufficient condition for the one-period punishment is that $\lim_{p \to 0} Q_k(p, p) = \infty$.

other two firms charge p_j and p_l, and $\hat{\pi}(p_j, p_l)$ is firm i's one-shot deviation profit in that situation. If the punishment value to firm 3 is not zero, then either (21) or (22)—the incentive constraints for firms 1 and 2, respectively—must be binding at an optimum. Suppose not. By continuity, we can perturb p_1 and p_3 without violating (21) or (22). A small decrease in p_1 causes $\hat{\pi}(p_1, p_2)$ to fall. Since this relaxes constraint (23), we can choose p_3 appropriately to induce a lower value of $\pi(p_3 \mid p_1, p_2)$.

Since either constraint (21) or (22) binds, it is clear that multimarket contact can make the punishment for firm 3 worse. To see this, suppose that firms 1 and 2 are involved in a second duopolistic market that can be completely cartelized. (The incentive constraint for this market is slack on the fully collusive path.) Then, if firm 3 deviates, firms 1 and 2 can shift into a punishment mode for the three-firm market *only;* therefore, while they are punishing firm 3, firms 1 and 2 continue to reap monopoly profits in the other market. If, however, either firm 1 or 2 subsequently deviates, it is punished as harshly as possible in *both* markets. The slack in the duopolistic market's incentive constraint relaxes constraints (21) and (22) and thereby facilitates a more severe punishment of firm 3.

8. Conclusion

■ In the preceding sections we have analyzed multimarket contact and collusive behavior in a variety of formal models. Three primary conclusions emerge from this investigation. First, multimarket contact can have real effects; in a wide range of circumstances, it relaxes the incentive constraints that limit the extent of collusion. Second, firms gain from multimarket contact by behaving in ways that have been noted in previous empirical discussions of multimarket firms. This suggests that multimarket contact may indeed have effects in practice. Third and finally, even when multimarket contact does have real effects, these effects are not necessarily socially undesirable.

Ultimately, the question of whether multimarket contact does have significant effects must be resolved through empirical research. Recently, there have been a number of attempts to address this empirical question. (See footnote 2.) Most of this work involves cross-sectional analyses of differences in performance (e.g., prices or profits) across industries, in which one or several measures of multimarket contact are included as explanatory variables.[42] These studies faced the formidable task of trying to distinguish between the effects of internal and external factors on performance. (Here, market definition is particularly critical.) In general, the literature has found a significant multimarket effect, although the sign of this effect has tended to vary across studies. One implication from our analysis, however, is that the effect of multimarket contact on the price or profits of any one industry depends greatly on the set of markets over which the firms have contact and on the characteristics of active (and potentially active) firms. For example, when firms are identical and markets differ, prices and profits may rise in some markets but fall in others as a result of multimarket contact. Similarly, when firms differ in their costs, multimarket contact can cause prices to either rise or fall (depending upon the discount factor). Thus, our analysis suggests that identifying the effects of multimarket contact on the price or profit level of an industry may require significantly more complex explanatory variables than have thus far been used in the literature.

A somewhat different approach was used in a recent experimental investigation conducted by Phillips and Mason (1988). Their study examined the effects of multimarket contact across two Cournot duopoly markets which fit into the class of markets covered in Section 7 above. (Recall footnote 39.) Phillips and Mason's experimental procedure consists of running separate single-market experiments of the two markets and comparing these

[42] Of the various articles listed in footnote 2, only Gelfand and Spiller (1986) was not a cross-sectional study. They analyzed time series data on two interrelated Uraguayan banking markets (U.S. dollar and new pesos loans) and found multimarket effects. Examples of typical measures of multimarket contact can be found in Heggestad and Rhoades (1978) and in Scott (1982).

outcomes with those that arise in a multimarket setting. Interestingly, Phillips and Mason's findings correspond closely to our theoretical predictions: multimarket contact causes the price in their "monopolistic" market (low realized $\pi'/\hat{\pi}'$ ratio in the single-market experiment) to fall and the price in the other market to rise.

Our analysis also suggests other strategies for empirically examining the effects of multimarket contact. In particular, since market-specific events affect a multimarket incentive constraint, one would expect to observe correlations between prices in otherwise unrelated markets.[43] The identification of a large (independent) shock to one market could therefore offer a natural experiment for examining the theory.

In the preceding analysis, we have investigated the effects of multimarket contact by contrasting single-market outcomes with those arising in the presence of parallel diversification. It is worth inquiring about the extent to which parallel diversification is important to our results; that is, what is the effect of multimarket operation absent multimarket contact? To investigate this question, imagine that we have two markets and only one multimarket firm. Suppose, first, that this firm is a *monopolist* in one of the markets. Then the outcomes in the two markets would be no different than if all firms in both markets were single-product competitors. However, when the multiproduct firm faces single-product competitors in *both* markets, multiproduct operation pools the incentive constraints of the multiproduct firm. This can expand the set of possible outcomes, though to a lesser extent than with multimarket contact. For example, in Section 4, firm 1 could still potentially transfer slack from market A to market B even if firm 2 operated only in market A. Note, though, that this would require that market A still revert to punishment mode in the event of a deviation in market B. In practice, the likelihood of this occurring in the presence of a significant number of single-product firms seems questionable in part because these firms may not even observe outcomes in market B. Thus, we are somewhat doubtful of the likelihood of effects arising from multimarket operation absent multimarket contact. Of course, the concern raised above can be a problem whenever single-market competitors are present. Thus, we suspect that in practice, the presence of significant single-market competitors will tend to retard the effects of multimarket contact. Clearly, further formal analysis of these issues seems desirable.

Appendix A

■ The proof of Proposition 1 for the unrestricted case follows.

Proof of Proposition 1 for the unrestricted case (including extensions). Consider any optimal multimarket equilibrium, $\{p^0_{ik}(t), \pi^0_{ik}(t)\}_{t=0}^{\infty}$, $i = 1, 2$, $k = A, B$. We construct two single-market equilibria, which together yield each firm exactly the profits it obtained in the multimarket equilibrium. This implies that multimarket contact yields no gain to the firms, since they may do equally well by treating each market in isolation. The important property shared by both the basic model and its extensions is that there exist positive constants (Θ_A, Θ_B) such that $\Theta_A + \Theta_B = 1$ and $\Pi^m_k(t) = \Theta_k[\Pi^m_A(t) + \Pi^m_B(t)]$ for $k = A, B$, where $\Pi^m_k(t)$ is the monopoly profit level in market k in period t.

Let $\pi^0_i(t) \equiv \pi_{iA}(t) + \pi_{iB}(t)$ and $\Pi^0(t) \equiv \pi^0_1(t) + \pi^0_2(t)$. Note that in any optimal collusive scheme, we must have $\pi^0_{ik}(t) \geq 0$ for all i, k, and t: if not, each firm's profits would be raised weakly, and one firm's profits would be raised strictly by setting both firms' prices equal to the cost in period t in market k. Since deviation profits would be unaffected, this change would satisfy each firms' incentive constraint.

To construct these single-market equilibria, we begin by choosing $\hat{p}_k(t)$ to be the lowest price that satisfies

$$[\hat{p}_k(t) - c_{kt}]Q_{kt}(\hat{p}_k(t)) = \Theta_k\Pi^0(t),$$

where c_{kt} is market k's marginal cost in period t and $Q_{kt}(\cdot)$ is market k's demand function in period t. Note that such a $\hat{p}_k(t)$ must exist and that, from the discussion in the previous paragraph, $\hat{p}_k(t) \geq c_{kt}$. Next, we choose $\hat{\lambda}_i(t) \equiv \pi^0_i(t)/\Pi^0(t)$. Note that $\hat{\lambda}_i(t)[\hat{p}_k(t) - c_{kt}]Q_{kt}(\hat{p}_k(t)) = \Theta_k\pi^0_i(t)$.

The equilibrium in market k has both firms naming price $\hat{p}_k(t)$ in period t and has firm i receiving a market share of $\hat{\lambda}_i(t)$ in period t in both markets. This outcome will be a single-market equilibrium in market k if and only if, for all t and $i = 1, 2$,

[43] The closest article to this sort of test is that of Gelfand and Spiller (1986).

$$[\hat{p}_k(t) - c_{kt}]Q_{kt}(\hat{p}_k(t)) \leq \sum_{\tau=t}^{\infty} \hat{\lambda}_i(\tau)[\hat{p}_k(\tau) - c_{k\tau}]Q_{kt}(\hat{p}_k(\tau))\delta^{\tau-t}. \tag{A1}$$

Substituting, we get

$$\Theta_k \Pi^0(t) \leq \sum_{\tau=t}^{\infty} \Theta_k \pi_i^0(\tau)\delta^{\tau-t}. \tag{A2}$$

But, cancelling the Θ_k, we see that this condition is implied by the condition that must hold if $\{p_{ik}^0(t), \pi_{ik}^0(t)\}_{t=0}^{\infty}$, $i = 1, 2$, $k = A, B$ is a multimarket equilibrium. *Q.E.D.*

Appendix B

■ Propositions B1, B2, B3, their proofs, and Corollary B1 follow.

Proposition B1. For the stationary multimarket models discussed in Section 3 (including the extensions), there exists an optimal symmetric-payoff equilibrium that has stationary prices and stationary market shares.

Proof. The proof involves two steps. First, in any stationary model in which an optimal symmetric-payoff equilibrium satisfies these two conditions—(a) that equilibrium path actions can be restricted to a compact set and payoff functions are continuous functions of these variables, and (b) that the equilibrium involves equal payoffs for the two players in every period—there exists a *stationary* optimal symmetric-payoff equilibrium. The argument, which we omit here, closely parallels the proof of Theorem 9 in Abreu (1986). Second, these conditions are satisfied by the models in Section 3. To see this, note first that for the reasons discussed in the text, there is always an optimal symmetric-payoff equilibrium in which $p_{ik}^*(t) = p_{jk}^*(t) \equiv p_k^*(t) \in [c_k, p_k^m]$ for all k and t. Letting $S = \{(p, p) | p \in [c_k, p_k^m]\}$, we can restrict the equilibrium path price choices of the two firms in market k to lie in this set. Since profits are continuous in the price vector on S, the first condition is satisfied. Now, if $\lambda_{ik}^*(t)$ is firm i's market share in market k at time t in this equilibrium, it must be that for all t and $i = 1, 2$,

$$\sum_{k=A,B} [p_k^*(t) - c_k]Q_k(p_k^*(t)) \leq \sum_{k=A,B} \{\sum_{\tau=t}^{\infty} \delta^\tau \lambda_{ik}^*(\tau)[p_k^*(\tau) - c_k]Q_k(p_k^*(\tau))\}. \tag{A3}$$

Summing over i implies that

$$\sum_{k=A,B} [p_k^*(t) - c_k]Q_k(p_k^*(t)) \leq \sum_{k=A,B} \{\sum_{\tau=t}^{\infty} \delta^\tau (1/2)[p_k^*(\tau) - c_k]Q_k(p_k^*(\tau))\}. \tag{A4}$$

But (A4) is exactly the incentive constraint that applies if we alter the market shares to give each firm half of each market in every period (while not altering the prices). Thus, an optimal symmetric-payoff equilibrium exists that involves equal payoffs for the two players in every period, so the second condition is also satisfied. *Q.E.D.*

Proposition B2. Consider a single market with N firms, $i = 1, \ldots, N$. Firm i has a constant marginal cost of production of c_i, where $c_N \geq c_{N-1} \geq \ldots \geq c_1$, and there are no fixed costs. Then, for all $n \leq N$ if $\delta < \dfrac{n-1}{n}$, no subgame perfect equilibrium has sales at a price greater than c_n in any period.

Proof. Suppose not. Let $p_t \equiv \min \{p_i(t)\}_{i=1}^N$ for all t, and define $\bar{p} \equiv \sup p_t$. Then, $\bar{p} > c_N$. Now, if we have a perfect equilibrium, $\{p_i(t), \pi_i(t)\}_{t=0}^{\infty}$ $(i = 1, \ldots, N)$, with associated output shares in period t of $\{\lambda_i(t)\}_{i=1}^N$, then the following condition must hold in every period t for $i = 1, \ldots, n$:

$$\phi(p_t | c_i) \leq \sum_{\tau=t}^{\infty} \delta^{\tau-t}(p_\tau - c_i)Q(p_\tau)\lambda_i(\tau), \tag{A5}$$

where $\phi(p_t | c_i) \equiv \max_{s \leq p} (s - c_i)Q(s)$. Note that $\phi(\cdot)$ is continuous. (A5) implies that

$$\phi(p_t | c_i) \leq \sum_{\tau=t}^{\infty} \delta^{\tau-t}\phi(p_\tau | c_i)(p_\tau)\lambda_i(\tau). \tag{A6}$$

Since $\phi_i(\cdot)$ is nondecreasing and $\phi_i(\bar{p}) > 0$, we then have

$$\frac{\phi(p_t | c_i)}{\phi(\bar{p} | c_i)} \leq \sum_{\tau=t}^{\infty} \delta^{\tau-t}\lambda_i(\tau). \tag{A7}$$

Summing over $i = 1, \ldots, n$ and noting that $\sum_{i=1}^{n} \lambda_i(\tau) \leq 1$ yields

$$\sum_{i=1}^{n} \frac{\phi(p_t | c_i)}{\phi(\bar{p} | c_i)} \leq \frac{1}{1-\delta}. \tag{A8}$$

By the definition of $\bar{p}$, the left-hand side of (A8) can be made arbitrarily close to n by choosing t appropriately. When $\delta \leq \frac{n-1}{n}$, however, $\frac{1}{1-\delta} < n$, so we must have a contradiction to (A8) for some t. *Q.E.D.*

Corollary B1. Consider a single market with N identical firms with constant marginal costs and no fixed costs. If $\delta < \frac{N-1}{N}$, then any subgame perfect equilibrium gives every firm zero discounted profits.

Proposition B3. In the model of Section 5, any price above $\bar{c}$ arising in a stationary symmetric-payoff equilibrium must satisfy condition (13) in the text.

Proof. Consider an equilibrium with outcome $(p_A, p_B, \lambda_A, \lambda_B)$ where, without loss of generality, $p_A \geq p_B$ and $P_A \in (\bar{c}, p^m(\bar{c})]$. The incentive constraints for the two firms are

$$(1-\delta)[\phi(p_A|\underline{c}) + \phi(p_B|\bar{c})] \leq [\lambda_A(P_A - \underline{c})Q(p_A) + (1-\lambda_B)(p_B - \bar{c})Q(p_B)] \qquad \text{(firm 1)}$$

and

$$(1-\delta)[\phi(p_A|\bar{c}) + \phi(p_B|\underline{c})] \leq [(1-\lambda_A)(p_A - \bar{c})Q(p_A) + \lambda_B(p_B - \underline{c})Q(p_B)]. \qquad \text{(firm 2)}$$

In a symmetric-payoff equilibrium we must have that

$$\lambda_A(p_A - \hat{c})Q(p_A) - \lambda_B(p_B - \hat{c})Q(p_B) = \tfrac{1}{2}[(p_A - \bar{c})Q(p_A) - (p_B - \bar{c})Q(p_B)], \qquad \text{(A9)}$$

where $\hat{c} \equiv (\underline{c}/2) + (\bar{c}/2)$. Now, consider the change in the firms' payoffs when λ_A and λ_B are raised keeping prices fixed and the firms' payoffs equal. Using (A9) to determine the change in λ_A that is required when λ_B is raised, we can calculate the change in firm 2's profit per period, π_2, to be

$$\frac{d\pi_2}{d\lambda_B} = Q(p_B)\left[\left(\frac{p_B - \underline{c}}{p_A - \bar{c}}\right) - \left(\frac{p_B - \hat{c}}{p_A - \hat{c}}\right)\right](p_A - \bar{c}) > 0.$$

Thus, raising λ_B and λ_A in this manner raises profits and therefore also satisfies the incentive constraints. Furthermore, this implies that a necessary condition for sustaining prices (p_A, p_B) is that the firms' incentive constraints are satisfied when $\lambda_B = 1$ and that

$$\lambda_A = \left[\frac{(p_A - \bar{c})Q(p_A) + (p_B - \underline{c})Q(p_B)}{2(p_A - \hat{c})Q(p_A)}\right] \equiv \hat{\lambda}_A,$$

the level implied by (A9) when $\lambda_B = 1$. (Note that this may imply a $\lambda_A > 1$; although this is not actually feasible, the necessary condition we derive is still valid.) Now, if firm 2's incentive constraint is satisfied when $\lambda_A = \hat{\lambda}_A$ and $\lambda_B = 1$, then, by the definition of $\phi(\cdot\,|\,\cdot)$, it must be that

$$(1-\delta)[(p_A - \bar{c})Q(p_A) + (p_B - \underline{c})Q(p_B)] \leq -\hat{\lambda}_A(p_A - \bar{c})Q(p_A) + [(p_A - \bar{c})Q(p_A) + (p_B - \underline{c})Q(p_B)].$$

Substituting for $\hat{\lambda}_A$ and rearranging yields

$$\frac{(p_A - \bar{c})Q(p_A)}{2(p_A - \hat{c})Q(p_A)} \leq \delta,$$

or

$$(p_A - \bar{c}) \leq \left(\frac{\delta}{1-\delta}\right)(p_A - \underline{c}),$$

which is condition (13) in the text. *Q.E.D.*

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