Auxiliar 8 IN709

Ejercicio 1

Considere el modelo de regresión lineal sin constante y con una sola variable explicativa estocástica x_t:

$$y_t = \beta x_t + \varepsilon_t \qquad t = 1.....T$$

Se hacen los supuestos siguientes:
$\frac{\sum x_t \varepsilon_t}{T} \xrightarrow{p} E(x_t\varepsilon_t) = \gamma_1 \neq 0$, y $\frac{\sum x_t^2}{T} \xrightarrow{p} E(x_t^2) = \mu_2 \neq 0$.

(a) **[0.5 puntos]** Demuestre que el estimador de mínimos cuadrados ordinarios de β no es consistente.

(b) **[0.5 puntos]** Sea z_t un instrumento valido por x_t tal que: $E(z_t\varepsilon_t) = 0\ \forall t$ y $\frac{\sum z_t x_t}{T} \xrightarrow{p} E(z_t x_t) = \gamma_2 \neq 0$.Demuestre que el estimador de variables instrumentales de β es: $\hat{\beta}_{IV} = \frac{\sum z_t y_t}{\sum z_t x_t}$.

(c) **[0.5 puntos]** Demuestre que $\hat{\beta}_{IV}$ es un estimador consistente de β.

(d) **[1 punto]** Se supone que sea la variable dependiente sea la independiente estan medidas con error, y que las variables observadas sean $y_t^* = y_t + v_t$ y $x_t^* = x_t + w_t$, donde $E(v_t) = E(w_t) = 0$, $Var(v_t) = \sigma_v^2$ y $Var(w_t) = \sigma_w^2\ \forall t$. Se supone también que $E(x_t w_t) = E(y_t v_t) = E(\varepsilon_t w_t) = E(\varepsilon_t v_t) = E(w_t v_t) = 0$ y que el instrumento z_t está incorrelacionado con los terminos de error v_t y w_t: $E(z_t v_t) = E(z_t w_t) = 0$. Demuestre que el estimador $\hat{\beta}_{IV}$ en este caso sigue siendo un estimador consistente de β.

Ejercicio 2

Sea el modelo poblacional:

$$Y = XB + e$$

Donde:

$X\ tiene\ dimensión\ nxK$

$Y\ tiene\ dimensión\ nx1$

$E(e/X) \neq 0$

Si existe Z, que cumple que está correlacionado con Z y no con e, demuestre que $\widehat{\beta}_{2sls} = \widehat{\beta}_{IV}$ en caso de que el modelo esté exactamente identificado.

Ejercicio 3

1.1 Source of Endogeneity 1:

In the linear model with a lagged dependent variable,

$$y_t = x_t'\beta + \gamma y_{t-1} + \varepsilon_t$$

suppose the error terms have first-order serial correlation, i.e.,

$$\varepsilon_t = \rho\varepsilon_{t-1} + u_t$$

where u_t is an *i.i.d.* sequence with zero mean, variance σ^2, and is independent of x_s for all t and s. For this model, show that the classical OLS estimator will be biased and inconsistent for β and γ

Ejercicio 4

1.2 Source of Endogeneity 2:

Suppose $y_i = y_i^* + u_i$ and $x_i = x_i^* + v_i$, where $i = \{1, .., N\}$. Assume y_i^* and x_i^* are nonstochastic scalars and (u_i, v_i) is a bivariate *i.i.d.* random variable with mean $E(u_i) = E(v_i) = 0$ and constant variances $V(u_i) = \sigma_u^2$, and $V(v_i) = \sigma_v^2$, respectively and covariance $cov(u_i, v_i) = \sigma_{uv} \neq 0$. Assume that $N^{-1}\sum_{i=1}^{N}(x_i^*)^2 \xrightarrow{p} r$ with r a constant scalar. Assume $y_i^* = \beta x_i^*$, but vectors y^* and x^* are not observable so that we must estimate β on the basis of vectors y and x.

1. For this model, show that the classical OLS estimator will be biased and inconsistent for β.
2. If $\{y^*, x^*\}$ are the true GDP growth and unemployment rate respectively. However we observe in the data a noisy measure of them denoted $\{y, x\}$. Moreover during recessions we think that the measure problem is more serious in both series. Use the model above to answer: What will be the effect on the estimated β? Do we have an upper bound or lower bound of β?

Ejercicio 5

1.3 Exercise on Asymptotics

There exists a scalar V_0 such that $\sqrt{N}\hat{\theta} \longrightarrow_d \mathcal{N}(0, V_0)$, where $E(\hat{\theta}) = \theta_0 = 0$. Find the asymptotic distribution of $\left(\frac{N}{V_0}\right)\hat{\theta}^2$