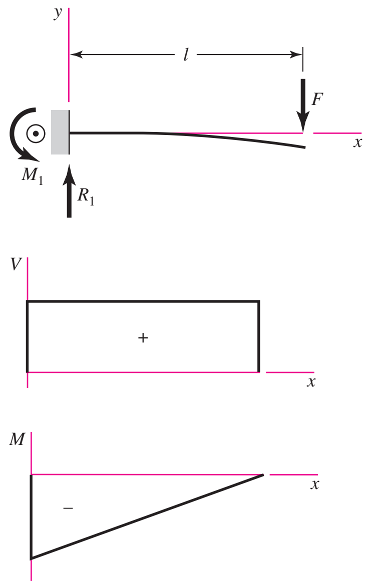
Table A–9

Shear, Moment, and Deflection of Beams (*Note: Force and moment reactions are positive in the directions shown; equations for shear force V and bending moment M follow the sign conventions given in Sec. 3–2.*)

1 Cantilever—end load



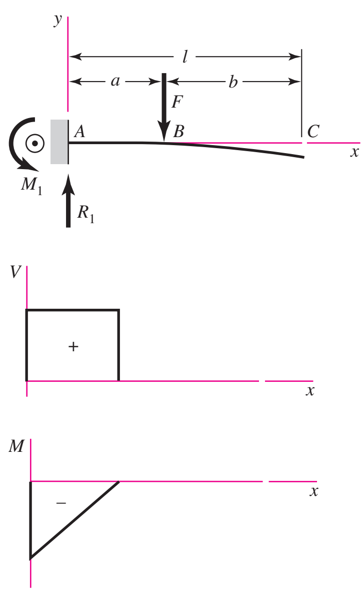
$$R_1 = V = F \qquad M_1 = Fl$$

$$M = F(x - l)$$

$$y = \frac{Fx^2}{6EI}(x - 3l)$$

$$y_{\max} = -\frac{Fl^3}{3EI}$$

2 Cantilever—intermediate load



$$R_1 = V = F \qquad M_1 = Fa$$

$$M_{AB} = F(x - a) \qquad M_{BC} = 0$$

$$y_{AB} = \frac{Fx^2}{6EI}(x - 3a)$$

$$y_{BC} = \frac{Fa^2}{6EI}(a - 3x)$$

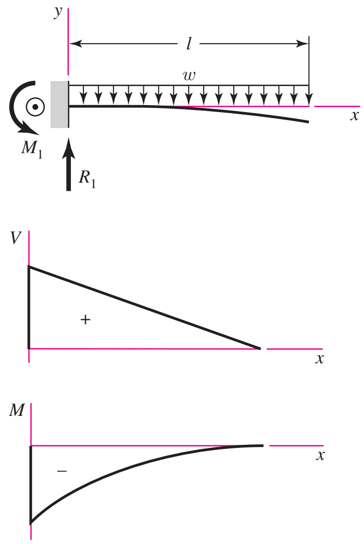
$$y_{\max} = \frac{Fa^2}{6EI}(a - 3l)$$

(continued)

Table A–9

Shear, Moment, and Deflection of Beams *(Continued)*
(*Note:* Force and moment reactions are positive in the directions shown; equations for shear force *V* and bending moment *M* follow the sign conventions given in Sec. 3–2.)

3 Cantilever—uniform load



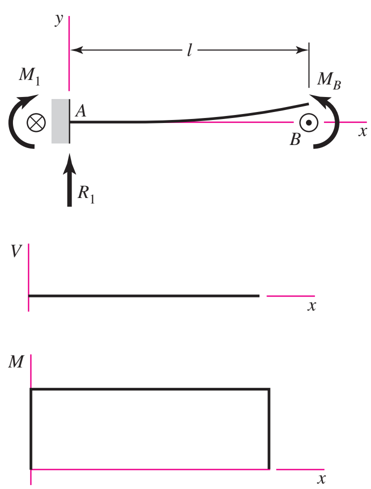
$$R_1 = wl \qquad M_1 = \frac{wl^2}{2}$$

$$V = w(l - x) \qquad M = -\frac{w}{2}(l - x)^2$$

$$y = \frac{wx^2}{24EI}(4lx - x^2 - 6l^2)$$

$$y_{max} = -\frac{wl^4}{8EI}$$

4 Cantilever—moment load



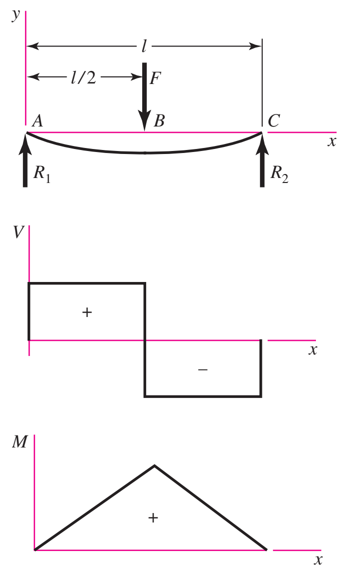
$$R_1 = V = 0 \qquad M_1 = M = M_B$$

$$y = \frac{M_B x^2}{2EI} \qquad y_{max} = \frac{M_B l^2}{2EI}$$

Table A–9

Shear, Moment, and Deflection of Beams *(Continued)*
(*Note:* Force and moment reactions are positive in the directions shown; equations for shear force *V* and bending moment *M* follow the sign conventions given in Sec. 3–2.)

5 Simple supports—center load



$$R_1 = R_2 = \frac{F}{2}$$

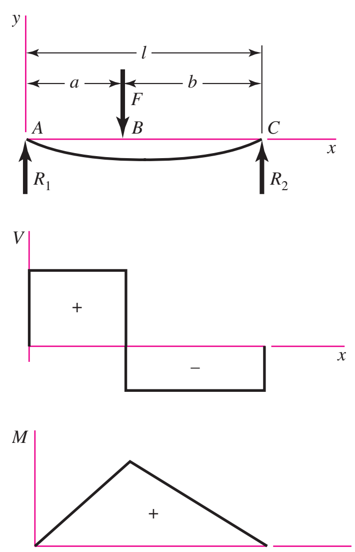
$$V_{AB} = R_1 \qquad V_{BC} = -R_2$$

$$M_{AB} = \frac{Fx}{2} \qquad M_{BC} = \frac{F}{2}(l - x)$$

$$y_{AB} = \frac{Fx}{48EI}(4x^2 - 3l^2)$$

$$y_{\max} = -\frac{Fl^3}{48EI}$$

6 Simple supports—intermediate load



$$R_1 = \frac{Fb}{l} \qquad R_2 = \frac{Fa}{l}$$

$$V_{AB} = R_1 \qquad V_{BC} = -R_2$$

$$M_{AB} = \frac{Fbx}{l} \qquad M_{BC} = \frac{Fa}{l}(l - x)$$

$$y_{AB} = \frac{Fbx}{6EIl}(x^2 + b^2 - l^2)$$

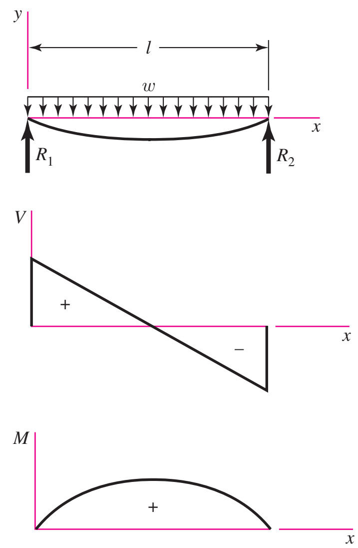
$$y_{BC} = \frac{Fa(l - x)}{6EIl}(x^2 + a^2 - 2lx)$$

(continued)

Table A–9

Shear, Moment, and Deflection of Beams *(Continued)*
(*Note:* Force and moment reactions are positive in the directions shown; equations for shear force V and bending moment M follow the sign conventions given in Sec. 3–2.)

7 Simple supports—uniform load



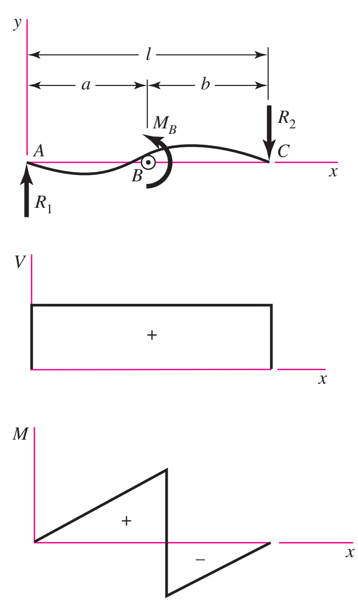
$$R_1 = R_2 = \frac{wl}{2} \qquad V = \frac{wl}{2} - wx$$

$$M = \frac{wx}{2}(l - x)$$

$$y = \frac{wx}{24EI}(2lx^2 - x^3 - l^3)$$

$$y_{\max} = -\frac{5wl^4}{384EI}$$

8 Simple supports—moment load



$$R_1 = R_2 = \frac{M_B}{l} \qquad V = \frac{M_B}{l}$$

$$M_{AB} = \frac{M_B x}{l} \qquad M_{BC} = \frac{M_B}{l}(x - l)$$

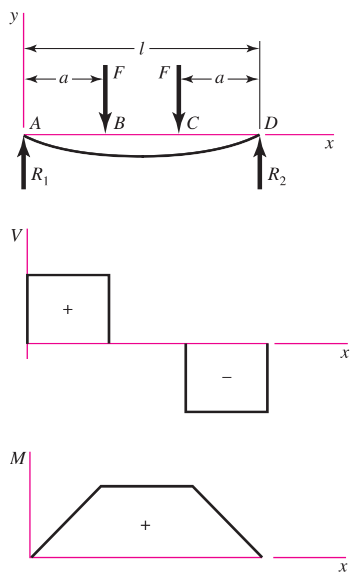
$$y_{AB} = \frac{M_B x}{6EIl}(x^2 + 3a^2 - 6al + 2l^2)$$

$$y_{BC} = \frac{M_B}{6EIl}[x^3 - 3lx^2 + x(2l^2 + 3a^2) - 3a^2 l]$$

Table A–9

Shear, Moment, and Deflection of Beams *(Continued)*
(*Note:* Force and moment reactions are positive in the directions shown; equations for shear force *V* and bending moment *M* follow the sign conventions given in Sec. 3–2.)

9 Simple supports—twin loads



$$R_1 = R_2 = F \qquad V_{AB} = F \qquad V_{BC} = 0$$

$$V_{CD} = -F$$

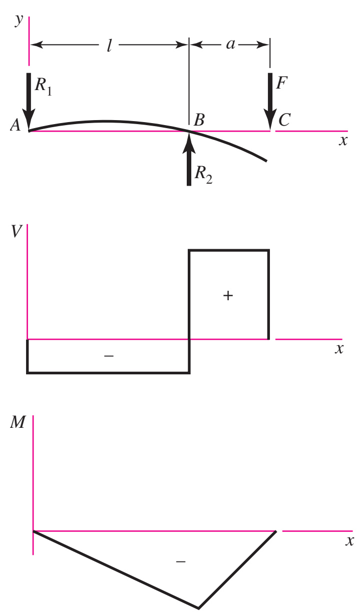
$$M_{AB} = Fx \qquad M_{BC} = Fa \qquad M_{CD} = F(l - x)$$

$$y_{AB} = \frac{Fx}{6EI}(x^2 + 3a^2 - 3la)$$

$$y_{BC} = \frac{Fa}{6EI}(3x^2 + a^2 - 3lx)$$

$$y_{\max} = \frac{Fa}{24EI}(4a^2 - 3l^2)$$

10 Simple supports—overhanging load



$$R_1 = \frac{Fa}{l} \qquad R_2 = \frac{F}{l}(l + a)$$

$$V_{AB} = -\frac{Fa}{l} \qquad V_{BC} = F$$

$$M_{AB} = -\frac{Fax}{l} \qquad M_{BC} = F(x - l - a)$$

$$y_{AB} = \frac{Fax}{6EIl}(l^2 - x^2)$$

$$y_{BC} = \frac{F(x - l)}{6EI}[(x - l)^2 - a(3x - l)]$$

$$y_c = -\frac{Fa^2}{3EI}(l + a)$$

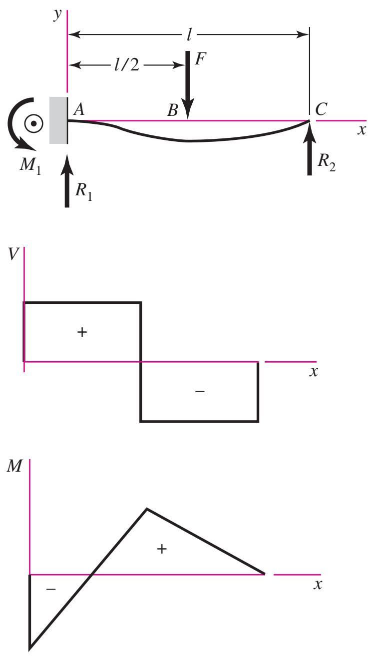
(continued)

Table A–9

Shear, Moment, and Deflection of Beams *(Continued)*

(*Note:* Force and moment reactions are positive in the directions shown; equations for shear force *V* and bending moment *M* follow the sign conventions given in Sec. 3–2.)

11 One fixed and one simple support—center load



$$R_1 = \frac{11F}{16} \qquad R_2 = \frac{5F}{16} \qquad M_1 = \frac{3Fl}{16}$$

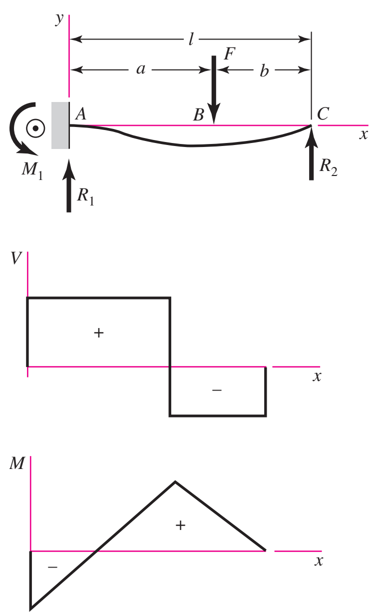
$$V_{AB} = R_1 \qquad V_{BC} = -R_2$$

$$M_{AB} = \frac{F}{16}(11x - 3l) \qquad M_{BC} = \frac{5F}{16}(l - x)$$

$$y_{AB} = \frac{Fx^2}{96EI}(11x - 9l)$$

$$y_{BC} = \frac{F(l - x)}{96EI}(5x^2 + 2l^2 - 10lx)$$

12 One fixed and one simple support—intermediate load



$$R_1 = \frac{Fb}{2l^3}(3l^2 - b^2) \qquad R_2 = \frac{Fa^2}{2l^3}(3l - a)$$

$$M_1 = \frac{Fb}{2l^2}(l^2 - b^2)$$

$$V_{AB} = R_1 \qquad V_{BC} = -R_2$$

$$M_{AB} = \frac{Fb}{2l^3}[b^2 l - l^3 + x(3l^2 - b^2)]$$

$$M_{BC} = \frac{Fa^2}{2l^3}(3l^2 - 3lx - al + ax)$$

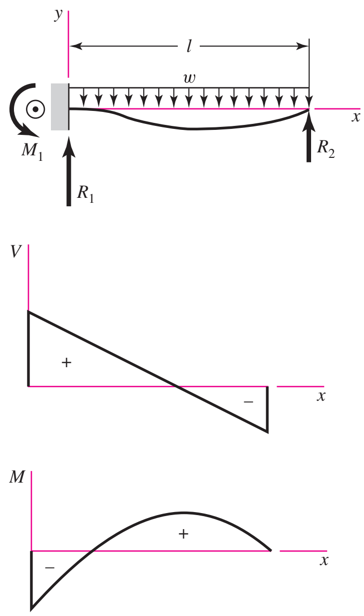
$$y_{AB} = \frac{Fbx^2}{12EIl^3}[3l(b^2 - l^2) + x(3l^2 - b^2)]$$

$$y_{BC} = y_{AB} - \frac{F(x - a)^3}{6EI}$$

Table A–9

Shear, Moment, and Deflection of Beams *(Continued)*
(*Note:* Force and moment reactions are positive in the directions shown; equations for shear force *V* and bending moment *M* follow the sign conventions given in Sec. 3–2.)

13 One fixed and one simple support—uniform load



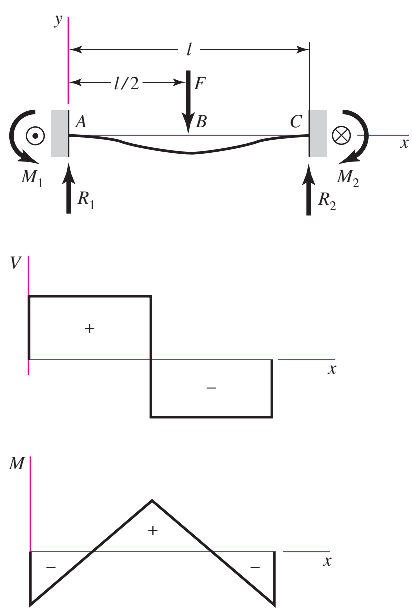
$$R_1 = \frac{5wl}{8} \qquad R_2 = \frac{3wl}{8} \qquad M_1 = \frac{wl^2}{8}$$

$$V = \frac{5wl}{8} - wx$$

$$M = -\frac{w}{8}(4x^2 - 5lx + l^2)$$

$$y = \frac{wx^2}{48EI}(l - x)(2x - 3l)$$

14 Fixed supports—center load



$$R_1 = R_2 = \frac{F}{2} \qquad M_1 = M_2 = \frac{Fl}{8}$$

$$V_{AB} = -V_{BC} = \frac{F}{2}$$

$$M_{AB} = \frac{F}{8}(4x - l) \qquad M_{BC} = \frac{F}{8}(3l - 4x)$$

$$y_{AB} = \frac{Fx^2}{48EI}(4x - 3l)$$

$$y_{max} = -\frac{Fl^3}{192EI}$$

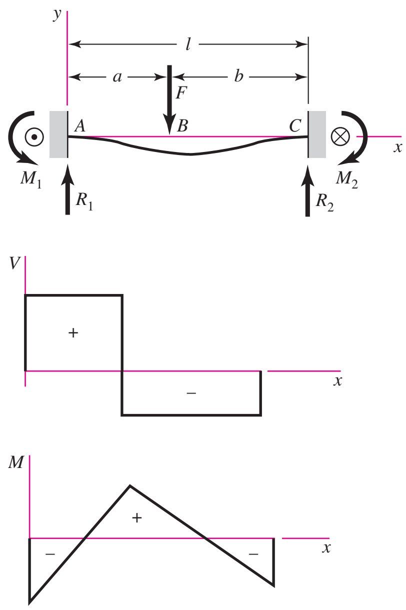
(continued)

Table A–9

Shear, Moment, and Deflection of Beams *(Continued)*

(Note: Force and moment reactions are positive in the directions shown; equations for shear force V and bending moment M follow the sign conventions given in Sec. 3–2.)

15 Fixed supports—intermediate load



$$R_1 = \frac{Fb^2}{l^3}(3a+b) \qquad R_2 = \frac{Fa^2}{l^3}(3b+a)$$

$$M_1 = \frac{Fab^2}{l^2} \qquad M_2 = \frac{Fa^2b}{l^2}$$

$$V_{AB} = R_1 \qquad V_{BC} = -R_2$$

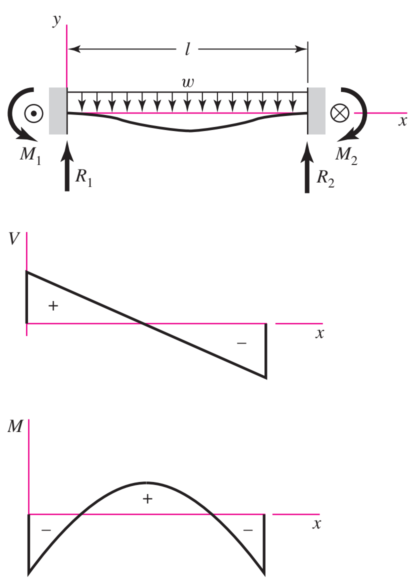
$$M_{AB} = \frac{Fb^2}{l^3}[x(3a+b) - al]$$

$$M_{BC} = M_{AB} - F(x-a)$$

$$y_{AB} = \frac{Fb^2x^2}{6EIl^3}[x(3a+b) - 3al]$$

$$y_{BC} = \frac{Fa^2(l-x)^2}{6EIl^3}[(l-x)(3b+a) - 3bl]$$

16 Fixed supports—uniform load



$$R_1 = R_2 = \frac{wl}{2} \qquad M_1 = M_2 = \frac{wl^2}{12}$$

$$V = \frac{w}{2}(l - 2x)$$

$$M = \frac{w}{12}(6lx - 6x^2 - l^2)$$

$$y = -\frac{wx^2}{24EI}(l-x)^2$$

$$y_{\max} = -\frac{wl^4}{384EI}$$