P1] $\vec{r}(\theta) = (\cos\theta, \sin\theta, \cosh(\theta)) \quad \theta \in [0, 2\pi]$.

Calculemos el triedro de Frenet.

Obs. Utilizaremos notación de coordenadas cilíndricas

es decir: $\hat{\rho} = (\cos\theta, \sin\theta, 0) \quad \hat{\theta} = (-\sin\theta, \cos\theta, 0) \quad \hat{k} = (0,0,1)$

Obs. $\frac{d\hat{\rho}}{d\theta} = \hat{\theta}$ y $\frac{d\hat{\theta}}{d\theta} = -\hat{\rho}$, Además $\hat{\rho} \times \hat{\theta} = \hat{k}$, $\hat{\theta} \times \hat{k} = \hat{\rho}$, $\hat{k} \times \hat{\rho} = \hat{\theta}$.
(verificar!)

Calculemos el vector tangente:

$\vec{r}(\theta) = (\cos\theta, \sin\theta, \cosh(\theta)) = \hat{\rho} + \cosh(\theta)\hat{k}$

$\Rightarrow \frac{d\vec{r}}{d\theta} = \frac{d\hat{\rho}}{d\theta} + \frac{d}{d\theta}(\cosh(\theta)\hat{k}) = \hat{\theta} + \sinh(\theta)\hat{k}$ y $\left\|\frac{d\vec{r}}{d\theta}\right\| = \sqrt{1+\sinh^2(\theta)}$
$= \cosh(\theta)$

$\therefore \hat{T} = \frac{d\vec{r}}{d\theta} \Big/ \left\|\frac{d\vec{r}}{d\theta}\right\| = \frac{1}{\cosh(\theta)}(\hat{\theta} + \sinh(\theta)\hat{k})$

para $\hat{N}$ necesitamos calcular $\frac{d\hat{T}}{d\theta} = \frac{d}{d\theta}\left(\frac{1}{\cosh(\theta)} \cdot (\hat{\theta} + \sinh(\theta)\hat{k})\right)$

$$= \frac{(-\hat{\rho} + \cosh(\theta)\hat{k}) \cdot \cosh(\theta) - \sinh(\theta)(\hat{\theta} + \sinh(\theta)\hat{k})}{\cosh^2(\theta)}$$

$$= \frac{1}{\cosh^2(\theta)}\left(-\cosh(\theta)\hat{\rho} - \sinh(\theta)\hat{\theta} + \hat{k}\big(\overbrace{\cosh^2(\theta) - \sinh^2(\theta)}^{1}\big)\right)$$

$$= \frac{1}{\cosh^2(\theta)}\left(\hat{k} - \cosh(\theta)\hat{\rho} - \sinh(\theta)\hat{\theta}\right)$$

$\Rightarrow \left\|\frac{d\hat{T}}{d\theta}\right\| = \frac{1}{\cosh^2(\theta)} \cdot \sqrt{\underbrace{1+\cosh^2\theta+\sinh^2\theta}_{\cosh^2\theta}}$
$= \frac{\sqrt{2}}{\cosh(\theta)}$

notar que $\kappa = \frac{\left\|\frac{d\hat{T}}{d\theta}\right\|}{\left\|\frac{d\vec{r}}{d\theta}\right\|} = \frac{\frac{\sqrt{2}}{\cosh(\theta)}}{\cosh(\theta)} = \frac{\sqrt{2}}{\cosh^2(\theta)}$

Finalmente: $\hat{N} = \frac{d\hat{T}}{d\theta} \Big/ \left\|\frac{d\hat{T}}{d\theta}\right\| = \frac{\frac{1}{\cosh^2(\theta)}\left(\hat{k} - \cosh(\theta)\hat{\rho} - \sinh(\theta)\hat{\theta}\right)}{\frac{\sqrt{2}}{\cancel{\cosh(\theta)}}} = \boxed{\frac{1}{\sqrt{2}}\left(\frac{1}{\cosh(\theta)}\hat{k} - \hat{\rho} - \tanh(\theta)\hat{\theta}\right) = \hat{N}}$

Para el Binormal:

$$\hat{B} = \hat{T} \times \hat{N} = \frac{1}{\cosh(\theta)} \left(\hat{\theta} + \sinh(\theta)\hat{k}\right) \times \frac{1}{\sqrt{2}} \left(\frac{1}{\cosh(\theta)} \hat{k} - \hat{\rho} - \tanh(\theta)\hat{\theta} \right)$$

$$= \frac{1}{\sqrt{2}\cosh(\theta)} \left(\frac{1}{\cosh(\theta)} \underbrace{(\hat{\theta} \times \hat{k})}_{\hat{\rho}} - \underbrace{\hat{\theta} \times \hat{\rho}}_{-\hat{k}} - \sinh(\theta) \underbrace{(\hat{k} \times \hat{\rho})}_{\hat{\theta}} - \frac{\sinh^2\theta}{\cosh\theta} \underbrace{(\hat{k} \times \hat{\theta})}_{-\hat{\rho}} \right)$$

$$= \frac{1}{\sqrt{2}\cosh(\theta)} \left(\hat{\rho} \underbrace{\left(\frac{1}{\cosh(\theta)} + \frac{\sinh^2\theta}{\cosh(\theta)} \right)}_{\cosh(\theta)} - \sinh(\theta)\hat{\theta} + \hat{k} \right)$$

$$= \frac{1}{\sqrt{2}\cosh(\theta)} \left(\cosh(\theta)\hat{\rho} - \sinh(\theta)\hat{\theta} + \hat{k} \right) = \hat{N} + \sqrt{2}\hat{\rho}.$$

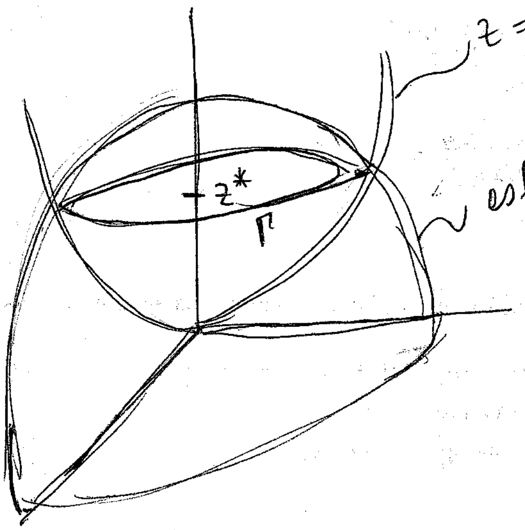
Finalmente, para la torsión:

$$\tau(\theta) = -\frac{d\hat{B}/d\theta}{\left\| d\vec{r}/d\theta \right\|} \cdot \hat{N} \quad \text{pero} \quad \frac{d\hat{B}}{d\theta} = \frac{d\hat{N}}{d\theta} + \sqrt{2}\hat{\theta}$$

$$\text{por Frenet} \quad \frac{d\hat{N}}{d\theta} = \frac{\frac{d\hat{N}}{ds}}{\left\|\frac{ds}{d\theta}\right\|} \overset{\text{Frenet}}{=} \frac{\tau\hat{B} - \kappa\hat{T}}{\cosh(\theta)} + \sqrt{2}\hat{\rho}.$$

el resto del cálculo queda propuesto :)

P2 | a) Grafiquemos:



$$\Gamma = \{(x,y,z) \mid x^2+y^2 = z \ \wedge \ x^2+y^2+z^2 = 1\}$$

Como se cumplen ambas ecc. se tiene:

$$z^2 + z = 1 \iff z^2 + z - 1 = 0$$

$$z = \frac{-1 \pm \sqrt{1+4}}{2}$$

$$\boxed{z_1 = \frac{-1+\sqrt{5}}{2} \ \text{ y } \ z_2 = \frac{-1-\sqrt{5}}{2}}$$

Nos quedamos con $\boxed{z^* = \frac{-1+\sqrt{5}}{2}}$ pues

$z = x^2+y^2$ nos dice que $z > 0$!!

además esa misma ecc. nos dice que Γ es una circunferencia de ecc:

$x^2+y^2 = \frac{\sqrt{5}-1}{2}$ a altura $z^* = \frac{\sqrt{5}-1}{2}$

Así pues, una paramet. para Γ es:

$$\vec{r}(\theta) = (\sqrt{z^*} \cos\theta, \sqrt{z^*} \operatorname{sen}\theta, z^*) \quad \theta \in [0, 2\pi].$$

b) Veamos el Triedro de Frenet:

notar que $\vec{r}(\theta) = \sqrt{z^*}\,\hat{\rho} + z^*\hat{k} \Rightarrow \frac{d\vec{r}}{d\theta} = \sqrt{z^*}\,\hat{\theta}$ y $\left\|\frac{d\vec{r}}{d\theta}\right\| = \sqrt{z^*}$

$$\Rightarrow \boxed{\hat{T} = \frac{d\vec{r}}{d\theta} \Big/ \left\|\frac{d\vec{r}}{d\theta}\right\| = \hat{\theta}}$$

$$\boxed{\begin{aligned} \hat{N} &= d\hat{T}/d\theta \,/\, \|d\hat{T}/d\theta\| = -\hat{\rho} / \|-\hat{\rho}\| = -\hat{\rho} \\ \hat{B} &= \hat{T} \times \hat{N} = \hat{\theta} \times (-\hat{\rho}) = \hat{k}. \end{aligned}}$$

Obs. Si no usamos los vectores $\hat{\theta}, \hat{\rho}, \hat{k}$:

$\vec{r}(\theta) = (\sqrt{z^*}\cos\theta, \sqrt{z^*}\operatorname{sen}\theta, z^*) \Rightarrow \frac{d\vec{r}}{d\theta} = (-\sqrt{z^*}\operatorname{sen}\theta, \sqrt{z^*}\cos\theta, 0)$

$\left\|\frac{d\vec{r}}{d\theta}\right\| = \sqrt{(\sqrt{z^*})^2(\operatorname{sen}^2\theta + \cos^2\theta)} = \sqrt{z^*}.$

$\Rightarrow \hat{T} = (-\operatorname{sen}\theta, \cos\theta, 0)$
$\left(= \frac{d\vec{r}}{d\theta} \Big/ \left\|\frac{d\vec{r}}{d\theta}\right\|\right)$

luego $\frac{d\vec{T}}{d\theta} = \frac{d}{d\theta}(-\operatorname{sen}\theta, \cos\theta, 0) = (-\cos\theta, -\operatorname{sen}\theta, 0)$

y $\frac{d\hat{T}}{d\theta} = \sqrt{\operatorname{sen}^2\theta + \cos^2\theta} = 1$

$\Rightarrow \hat{N} = \frac{d\hat{T}/d\theta}{\|d\hat{T}/d\theta\|} = (-\cos\theta, -\operatorname{sen}\theta, 0) = (-1)(\cos\theta, \operatorname{sen}\theta, 0)$

y Finalmente $\hat{B} = \hat{T} \times \hat{N} = (-\operatorname{sen}\theta, \cos\theta, 0) \times (-1)(\cos\theta, \operatorname{sen}\theta, 0)$

$$= (-1)\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -\operatorname{sen}\theta & \cos\theta & 0 \\ \cos\theta & \operatorname{sen}\theta & 0 \end{vmatrix} = (-1)\hat{k}(-\operatorname{sen}^2\theta - \cos^2\theta) = \hat{k}.$$

igual que antes.

$\therefore \hat{T} = (-\operatorname{sen}\theta, \cos\theta, 0) = \hat{\theta} \quad \hat{N} = (-\cos\theta, -\operatorname{sen}\theta, 0) = -\hat{\rho}$

$\hat{B} = (0, 0, 1) = \hat{k}$.

c) $\rho(x,y,z) = x+y+z$. Queremos masa y CG de Γ. Sea $r = \sqrt{z^*} = \sqrt{\frac{\sqrt{5}-1}{2}}$

$$M(\Gamma) = \int_\Gamma \rho\, dl \underset{\uparrow}{=} \int_0^{2\pi} (r\cos\theta + r\operatorname{sen}\theta + z^*) \cdot r\, d\theta = r z^* \cdot 2\pi + r^2 \underbrace{\int_0^{2\pi} \cos\theta + \operatorname{sen}\theta\, d\theta}_{=0 \text{ (integro en un período!)}}$$

$= 2\pi r z^*$.

$\gamma(\theta) = (r\cos\theta, r\operatorname{sen}\theta, z^*) \quad \theta \in [0, 2\pi]$

$\gamma'(\theta) = (-r\operatorname{sen}\theta, r\cos\theta, 0) \Rightarrow \|\gamma'(\theta)\| = r$

$$M(\Gamma) = 2\pi \cdot \sqrt{\frac{\sqrt{5}-1}{2}} \cdot \left(\frac{\sqrt{5}-1}{2}\right) = 2\pi \left(\frac{\sqrt{5}-1}{2}\right)^{3/2}$$

CG: $$X_G = \frac{1}{M}\int_\Gamma x\rho\, dl = \frac{1}{M}\int_0^{2\pi} r\cos\theta(r\cos\theta + r\operatorname{sen}\theta + z^*) \cdot r\, d\theta$$

$$= \frac{1}{M}\int_0^{2\pi} (r^3\cos^2\theta + r^3\operatorname{sen}\theta\cos\theta + r^2 z^*)\, d\theta$$

$$Y_G = \frac{1}{M}\int_0^{2\pi} (r^3\cos\theta\operatorname{sen}\theta + r^3\operatorname{sen}^2\theta + r^2 z^*)\, d\theta$$

$$Z_G = \frac{1}{M}\int_0^{2\pi} (r^2(\cos\theta + \operatorname{sen}\theta) z^* + r(z^*)^2)\, d\theta.$$

(aquí ya es solo explicitar el cálculo, háganlo!)

P3 $\vec{\sigma} : [0, L] \to \mathbb{R}^3$ param en long. de arco. $\sigma \in C^3$.

a) Pdq $\tau(s) = \dfrac{(\vec{\sigma}'(s) \times \vec{\sigma}''(s)) \cdot \vec{\sigma}'''(s)}{\|\vec{\sigma}''(s)\|}$.

Sol. Sabemos que $\tau(s) = -\dfrac{d\hat{B}}{ds} \cdot \hat{N}(s)$, además: $\hat{B} = \hat{T} \times \hat{N}$.

$\hat{T}(s) = \vec{\sigma}'(s)$ (long. de arco!) $\Rightarrow \hat{N}(s) = \dfrac{\vec{\sigma}''(s)}{\|\vec{\sigma}''(s)\|}$

$\therefore \hat{B}(s) = \dfrac{\vec{\sigma}'(s) \times \vec{\sigma}''(s)}{\|\vec{\sigma}''(s)\|} \Rightarrow \dfrac{d\vec{B}}{ds} = \dfrac{d}{ds}\left(\dfrac{1}{\|\vec{\sigma}''(s)\|} \cdot \vec{\sigma}'(s) \times \vec{\sigma}''(s)\right)$

$$= \frac{d}{ds}\left(\frac{1}{\|\vec{\sigma}''(s)\|}\right) \cdot \vec{\sigma}'(s) * \vec{\sigma}''(s)$$

$$+ \frac{1}{\|\vec{\sigma}''(s)\|} \cdot \frac{d}{ds}\left(\vec{\sigma}'(s) \times \vec{\sigma}''(s)\right)$$

$$= \frac{d}{ds}\left(\frac{1}{\|\vec{\sigma}''(s)\|}\right) \cdot \vec{\sigma}'(s) \times \vec{\sigma}''(s) + \frac{1}{\|\vec{\sigma}''(s)\|}\left(\underbrace{\vec{\sigma}'' \times \vec{\sigma}''}_{=0} + \vec{\sigma}' \times \vec{\sigma}'''\right)$$

$$\therefore \frac{d\vec{B}}{ds} = \frac{d}{ds}\left(\frac{1}{\|\vec{\sigma}''\|}\right) \cdot (\vec{\sigma}' \times \vec{\sigma}'') + \frac{1}{\|\vec{\sigma}''(s)\|} \vec{\sigma}' \times \vec{\sigma}'''$$

Luego: $\dfrac{d\vec{B}}{ds} \cdot \vec{N}(s) = \dfrac{d}{ds}\left(\dfrac{1}{\|\vec{\sigma}''\|}\right) \overset{\text{prod usual}}{\cdot} (\vec{\sigma}' \times \vec{\sigma}'') \overset{\text{prod pto.}}{\cdot} \dfrac{\vec{\sigma}''(s)}{\|\vec{\sigma}''(s)\|} + \dfrac{1}{\|\vec{\sigma}''\|}(\vec{\sigma}' \times \vec{\sigma}''') \cdot \dfrac{\vec{\sigma}''}{\|\vec{\sigma}''\|}$

Como $(A \times B) \cdot C = (C \times A) \cdot B = (B \times C) \cdot A$

Entonces: $(\underbrace{\vec{\sigma}'}_{A} \times \underbrace{\vec{\sigma}''}_{B}) \cdot \underbrace{\vec{\sigma}''}_{C} = (B \times C) \cdot A = (\underbrace{\vec{\sigma}'' \times \vec{\sigma}''}_{=0}) \cdot \vec{\sigma}' = 0$!

$$\therefore \frac{d\vec{B}}{ds} \cdot \vec{N}(s) = \frac{1}{\|\vec{\sigma}''\|^2}(\underbrace{\vec{\sigma}'}_{A} \times \underbrace{\vec{\sigma}'''}_{B}) \cdot \underbrace{\vec{\sigma}''}_{C} = \frac{1}{\|\vec{\sigma}''\|^2}(C \times A) \cdot B$$

$$= -\frac{1}{\|\vec{\sigma}''\|^2}(A \times C) \cdot B = \frac{-1}{\|\vec{\sigma}''\|^2}(\vec{\sigma}' \times \vec{\sigma}'')\vec{\sigma}'''$$

$$\therefore \tau(s) = -\frac{db}{ds} \circ N(s) = \frac{1}{\|\vec{\sigma}''(s)\|^2}\left(\vec{\sigma}'(s) \times \vec{\sigma}''(s)\right) \circ \vec{\sigma}'''(s).$$

y se concluye.

b) Para usar la fórmula se debe param en longitud de arco!

Es decir, debemos despejar $s(t)$

Si $\vec{r}(t) = (\cos t, \operatorname{sen} t, t) \quad t \in [0, 4\pi]$

$\Rightarrow \vec{r}\,'(t) = (-\operatorname{sen} t, \cos t, 1) \Rightarrow \|\vec{r}\,'(t)\| = \sqrt{2}$

Luego: $\displaystyle\int_0^t \|\vec{r}\,'(t)\| \, dt = \int_0^t \sqrt{2}\, d\tau = \sqrt{2}\, t =: s(t)$

$\Rightarrow t = \dfrac{s}{\sqrt{2}}$

$\therefore$

$$\vec{\sigma}(s) = \left(\cos\left(\frac{s}{\sqrt{2}}\right), \operatorname{sen}\left(\frac{s}{\sqrt{2}}\right), \frac{s}{\sqrt{2}}\right) \quad s \in [0, \underbrace{4\sqrt{2}\pi}_{\ell(\Gamma)}].$$

$$\Rightarrow \vec{\sigma}'(s) = \left(-\frac{1}{\sqrt{2}} \operatorname{sen}\left(\frac{s}{\sqrt{2}}\right), \frac{1}{\sqrt{2}}\cos\left(\frac{s}{\sqrt{2}}\right), \frac{1}{\sqrt{2}}\right)$$

$$\vec{\sigma}''(s) = \left(-\frac{1}{2}\cos\left(\frac{s}{\sqrt{2}}\right), -\frac{1}{2}\operatorname{sen}\left(\frac{s}{\sqrt{2}}\right), 0\right) \Rightarrow \|\vec{\sigma}''(s)\| = \frac{1}{2}$$

$$\vec{\sigma}'''(s) = \left(\frac{1}{2\sqrt{2}}\operatorname{sen}\left(\frac{s}{\sqrt{2}}\right), -\frac{1}{2\sqrt{2}}\cos\left(\frac{s}{\sqrt{2}}\right), 0\right)$$

$$\Rightarrow \vec{\sigma}' \times \vec{\sigma}'' = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -\frac{1}{\sqrt{2}}\operatorname{sen}\left(\frac{s}{\sqrt{2}}\right) & \frac{1}{\sqrt{2}}\cos\left(\frac{s}{\sqrt{2}}\right) & \frac{1}{\sqrt{2}} \\ -\frac{1}{2}\cos\left(\frac{s}{\sqrt{2}}\right) & -\frac{1}{2}\operatorname{sen}\left(\frac{s}{\sqrt{2}}\right) & 0 \end{vmatrix} = \left(\frac{1}{2\sqrt{2}}\operatorname{sen}\left(\frac{s}{\sqrt{2}}\right), -\frac{1}{2\sqrt{2}}\cos\left(\frac{s}{\sqrt{2}}\right), \frac{1}{2\sqrt{2}}\right)$$

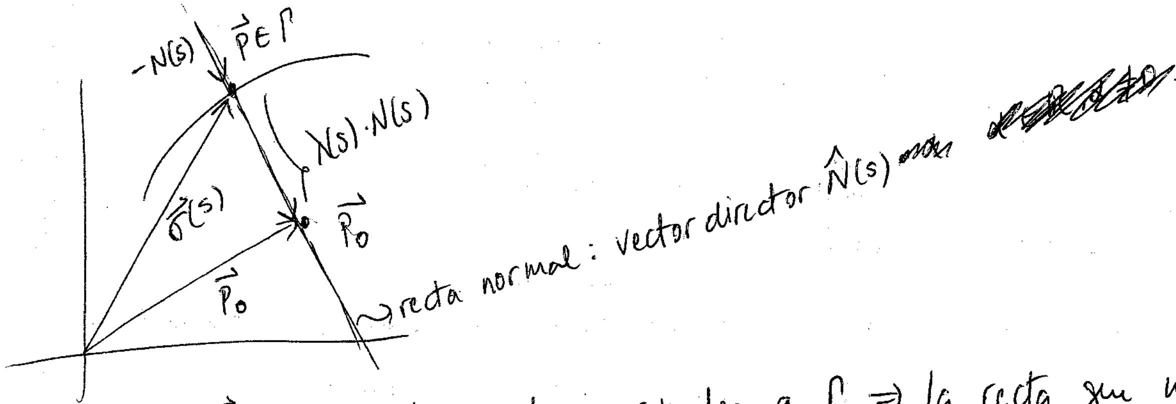
$$\Rightarrow (\vec{\sigma}' \times \vec{\sigma}'') \circ \vec{\sigma}''' = \left(\frac{1}{2\sqrt{2}}\operatorname{sen}\left(\frac{s}{\sqrt{2}}\right), -\frac{1}{2\sqrt{2}}\cos\left(\frac{s}{\sqrt{2}}\right), \frac{1}{2\sqrt{2}}\right) \circ \left(\frac{1}{2\sqrt{2}}\operatorname{sen}\left(\frac{s}{\sqrt{2}}\right), -\frac{1}{2\sqrt{2}}\cos\left(\frac{s}{\sqrt{2}}\right), 0\right)$$

$$= \left(\frac{1}{2\sqrt{2}}\right)^2 \operatorname{sen}^2\left(\frac{s}{\sqrt{2}}\right) + \left(\frac{1}{2\sqrt{2}}\right)^2 \cos^2\left(\frac{s}{\sqrt{2}}\right) + 0$$

$$= \left(\frac{1}{2\sqrt{2}}\right)^2$$

$$\Rightarrow \frac{(\vec{\sigma}' \times \vec{\sigma}'') \cdot \vec{\sigma}'''}{\|\vec{\sigma}''\|^2} = \left(1/2\sqrt{2}\right)^2 \Big/ \left(\frac{1}{2}\right)^2 = \left(\frac{1}{\sqrt{2}}\right)^2 = \boxed{\frac{1}{2} = \tau(s)}$$ como se quería.

P4) a) Hagamos un dibujo de la situacion en un instante s dado:



Como por $\vec{P_0}$ pasan las rectas normales a $\Gamma \Rightarrow$ la recta que une a $\vec{P_0}$ con $\vec{P_0}$ tiene como vector director a alguna ponderación de $\hat{N}(s)$

$\therefore$ vector que va de $\vec{P_0}$ a $\vec{P}$ será: $\lambda(s)\hat{N}(s)$, con $\lambda(s)$ la ponderación masaria para unir los puntos!

Luego, si defino $\varphi : [0, \ell(\Gamma)] \longrightarrow \mathbb{R}$ como $\varphi(s) = \lambda(s)$ tengo lo deseado. ($\exists$ de φ)

Para concluir basta notar la igualdad vectorial:

Vector $\vec{P_0}$ = Vector $\vec{\sigma}(s)$ + vector $\lambda(s) \cdot \hat{N}(s)$

ie. $\vec{P_0} = \vec{\sigma}(s) + \lambda(s) N(s) = \vec{\sigma}(s) + \varphi(s)\hat{N}(s)$ $\square$

b) Derivemos la expresión recién encontrada:

$$\frac{d}{ds}(\vec{P_0}) = \frac{d}{ds}(\vec{\sigma}(s)) + \frac{d}{ds}(\varphi(s) N(s))$$

$$0 = \hat{T}(s) + \varphi'(s)\hat{N}(s) + \varphi(s)\frac{d}{ds}(\hat{N}(s))$$

$$0 \overset{\text{Frenet}}{=} \hat{T}(s) + \varphi'(s)\hat{N}(s) + \varphi(s)\left(-\kappa(s)\hat{T}(s) + \tau(s)\hat{B}(s)\right)$$

$$\Rightarrow \quad 0 = \hat{T}(s)(1 - \varphi(s)\kappa(s)) + \hat{N}(s)(\varphi'(s)) + \hat{B}(s)(\varphi(s)\tau(s)).$$

Por indep. lineal de $\hat{T}, \hat{N}, \hat{B}$ se concluye el sistema deseado!

c) Notar que basta verificar que $\tau(s) \equiv 0 \quad \forall s$.

En efecto, del sistema encontrado: $\varphi'(s) \equiv 0 \Rightarrow \varphi(s) \equiv cte$.

más aun, como $1 - \kappa(s)\varphi(s) = 0 \Rightarrow \kappa(s)\varphi(s) = 1 \Rightarrow \varphi(s)$ no puede ser 0 (sino $\kappa(s) \cdot 0 = 1 \rightarrow$ ☼)

$\therefore \varphi(s) \equiv C \neq 0$, y como $\tau(s) \cdot \varphi(s) = 0 \Leftrightarrow \tau(s) \cdot \underbrace{C}_{\neq 0} = 0$

$\Rightarrow \tau(s) \equiv 0$.

$\therefore$ La curva es plana (pues $\tau(s) = 0$, resultado visto en clase!)

d) Γ es un arco de circunf.

Veamoslo por definición, es decir, veamos que $\exists \vec{P_0}$ tq $\forall \vec{P} \in \Gamma$ $\|\vec{P_0} - \vec{P}\| = cte$.

En efecto, de a):

$$\vec{P_0} = \vec{\sigma}(s) + \varphi(s)\hat{N}(s)$$

$$\Rightarrow \|\vec{P_0} - \vec{\sigma}(s)\| = \|\varphi(s)\hat{N}(s)\| = \underbrace{|\varphi(s)|}_{=|cte| \neq 0 \text{ por lo visto en c)}} \underbrace{\|\hat{N}(s)\|}_{=1 \text{ por def.}}$$

$\therefore \|\vec{P_0} - \vec{\sigma}(s)\| = |cte| \neq 0 \quad \forall s$

$\therefore \vec{\sigma}(s)$ paramet. a un arco de circunf. Como se deseaba mostrar.

$\square$

P5 a) Γ hélice $\Rightarrow$ $\hat{e} = \alpha \hat{T} + \beta \hat{B}$. Con α y β a determinar.

Para ello primero probemos que $\hat{e} \cdot \hat{N} = 0$, en efecto: ($\hat{e}$ es fijo por cierto)

Como Γ es hélice caracterizada por eje $\hat{e}$ y ángulo θ:

$$\Leftrightarrow \quad \hat{T}(s) \cdot \hat{e} = \cos\theta \quad \text{(el ángulo entre } \hat{T} \text{ y } \hat{e} \text{ es } \theta!)$$

$$\Leftrightarrow \quad \frac{d}{ds}\left(\hat{T}(s) \cdot \hat{e}\right) = \frac{d}{ds}(\cos\theta) \underset{\substack{\uparrow \\ \theta \text{ cte.}}}{=} 0$$

$$\underbrace{\frac{d\hat{T}}{ds} \cdot \hat{e}}_{\text{(Frenet) } \underbrace{\kappa(s)}_{\neq 0 \text{ por hipot}} \hat{N} \cdot \hat{e}} + \underbrace{\hat{T}(s) \frac{d}{ds}(\hat{e})}_{=0 \ (\hat{e} \text{ fijo!})} = 0$$

$$\Leftrightarrow \underbrace{\kappa(s)}_{\neq 0} \hat{N} \cdot \hat{e} = 0$$

$$\therefore \quad \hat{N} \cdot \hat{e} = 0 \ \forall s.$$

Obs. hemos probado que Γ hélice $\Leftrightarrow \hat{e} \cdot \hat{N} = 0$

Luego, como $\hat{T}, \hat{N}, \hat{B}$ son ortogonales entre si y $\hat{e} \cdot \hat{N} = 0$ entonces, necesariamente $\hat{e}$ está contenido en un plano con vectores directores $\hat{T}$ y $\hat{B}$

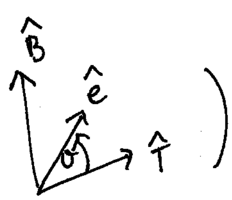
$$\therefore \quad \hat{e} = \alpha \hat{T} + \beta \hat{B} \qquad \text{¿como determinar } \alpha \text{ y } \beta\text{?}$$

Hagamos prod. punto con $\hat{T}$: $\underset{\substack{\| \\ \cos\theta}}{\hat{e} \cdot \hat{T}} = \alpha \underbrace{\hat{T} \cdot \hat{T}}_{\substack{\|\hat{T}\|^2 \\ \| \\ 1}} + \beta \underbrace{\hat{B} \cdot \hat{T}}_{=0} \ (\hat{B} \perp \hat{T})$

$$\therefore \quad \boxed{\alpha = \cos\theta}$$

por otro lado: $\hat{e} \times \hat{T} = \alpha \underbrace{\hat{T} \times \hat{T}}_{=0} + \beta \hat{B} \times \hat{T}$

$$\Rightarrow \hat{e} \times \hat{T} = \beta \underbrace{(\hat{B} \times \hat{T})}_{\hat{N}} \Rightarrow \underset{\substack{\| \\ |\sin\theta|}}{\|\hat{e} \times \hat{T}\|} = |\beta| \overset{\substack{1 \\ \|}}{\|\hat{N}\|} = |\beta|.$$

(o notar que ) así $\boxed{\beta = \sin\theta}$

b) Γ hélice $\Rightarrow$ κ/τ = cte.

Como Γ hélice $\Rightarrow$ $\hat{e} = \cos\theta\,\hat{T} + \operatorname{sen}\theta\,\hat{B}$ $\quad / \frac{d}{ds}$ $\quad$ (θ cte!)

$$0 = \cos\theta \frac{d\hat{T}}{ds} + \operatorname{sen}\theta \frac{d\hat{B}}{ds}$$

$$0 \underset{\text{Frenet}}{\overset{\uparrow}{=}} \cos\theta\,\kappa(s)\hat{N} + \operatorname{sen}\theta\left(-\tau(s)\hat{N}\right)$$

$$0 = \hat{N}\left(\cos\theta\cdot\kappa(s) - \tau(s)\operatorname{sen}\theta\right),$$ como $\hat{N}(s)$ no es nulo:

$\therefore$ $\quad \cos\theta\cdot\kappa(s) - \tau(s)\operatorname{sen}\theta = 0$

$$\boxed{\frac{\kappa(s)}{\tau(s)} = \frac{\operatorname{sen}\theta}{\cos\theta} = \operatorname{tg}\theta. \quad \forall s.}$$

$\Leftarrow$) Sup. que $\frac{\kappa}{\tau}$ = cte.

$$\Rightarrow \kappa(s) - \tau(s)\cdot C = 0 \Rightarrow (\kappa(s) - \tau(s)\cdot C)\hat{N} = 0$$

$$\underbrace{\kappa(s)\hat{N}}_{\frac{d\hat{T}}{ds}} - \tau(s)\cdot C\hat{N} = 0$$

$$\frac{d\hat{T}}{ds} + C\underbrace{(-\tau(s)\hat{N})}_{\frac{d\hat{B}}{ds}} = 0$$

o sea $\frac{d\hat{T}}{ds} + c\cdot\frac{d\hat{B}}{ds} = 0 \overset{\substack{c \text{ cte.}\\ \uparrow}}{\Leftrightarrow} \frac{d}{ds}(\hat{T} + c\hat{B}) = 0 \quad \forall s$

$\therefore$ $\exists$ un vector fijo, que llamaremos $\hat{e}$ tq

$$\hat{T} + c\hat{B} = \hat{e}$$

$$\Rightarrow (\hat{T} + c\hat{B})\cdot\hat{N} = \hat{e}\cdot\hat{N} \quad \Big(\Rightarrow 0 = \hat{e}\cdot\hat{N}$$

$$\underbrace{\hat{T}\cdot\hat{N}}_{=0} + c\underbrace{\hat{B}\cdot\hat{N}}_{=0}$$

$\hat{N}\perp\hat{T} \wedge \hat{N}\perp\hat{B}$

$\Rightarrow \Gamma$ es hélice $\underset{\text{obs. a)}}{\uparrow}$

y se concluye :)