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Scientific Background on the Nobel Prize in Physics 2012

MEASURING AND MANIPULATING INDIVIDUAL QUANTUM SYSTEMS

compiled by the Class for Physics of the Royal Swedish Academy of Sciences

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Measuring and Manipulating Individual Quantum Systems

Introduction

The behaviour of the individual constituents that make up our world – atoms (matter) and photons (light) – is described by quantum mechanics. These particles are rarely isolated and usually interact strongly with their environment. The behaviour of an ensemble of particles generally differs from isolated ones and can often be described by classical physics. From the beginning of the field of quantum mechanics, physicists used thought experiments to simplify the situation and to predict single quantum particle behaviour.

During the 1980s and 1990s, methods were invented to cool individual ions captured in a trap and to control their state with the help of laser light. Individual ions can now be manipulated and observed *in situ* by using photons with only minimal interaction with the environment. In another type of experiment, photons can be trapped in a cavity and manipulated. They can be observed without being destroyed through interactions with atoms in cleverly designed experiments. These techniques have led to pioneering studies that test the basis of quantum mechanics and the transition between the microscopic and macroscopic worlds, not only in thought experiments but in reality. They have advanced the field of quantum computing, as well as led to a new generation of high-precision optical clocks.

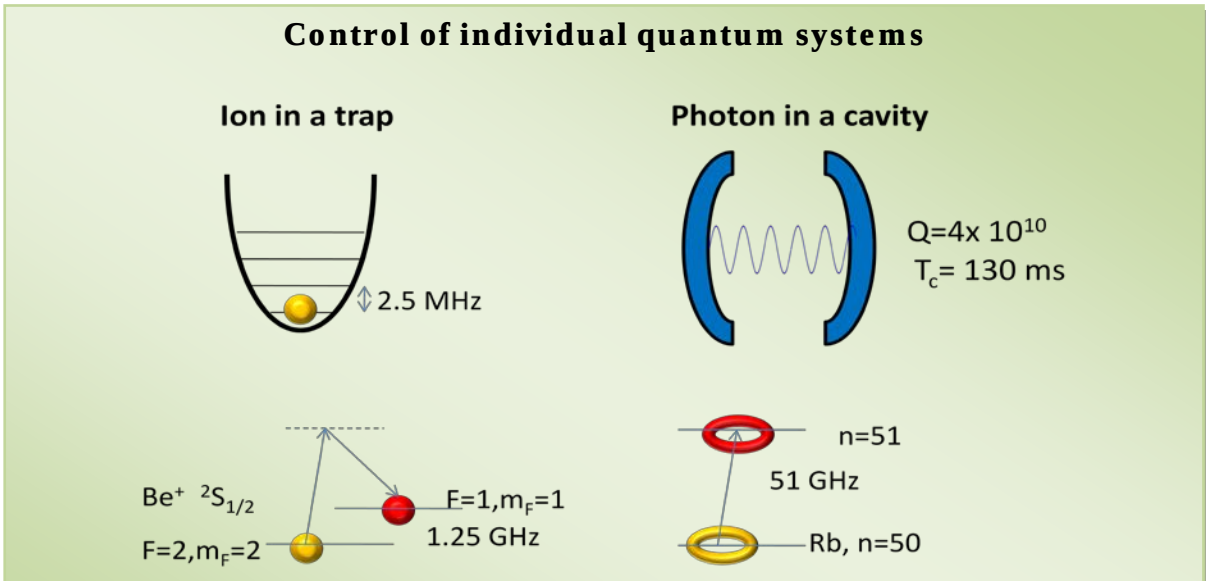

Fig. 1: Illustration of the two types of experiments discussed in this scientific background: On the left, an ion is captured in a harmonic trap. Its quantum state (both its internal state and its motion) is controlled by interaction with laser pulses as exemplified for the case of Be^+. On the right, a photon is (or several photons are) trapped in a high-Q microwave cavity. The field state is measured and controlled by interaction with highly excited Rb atoms.

This year's Nobel Prize in Physics honours the experimental inventions and discoveries that have allowed the measurement and control of individual quantum systems. They belong to two separate but related technologies: ions in a harmonic trap and photons in a cavity (see Fig. 1).

There are several interesting similarities between the two. In both cases, the quantum states are observed through quantum non-demolition measurements where two-level systems are coupled to a quantized harmonic oscillator – a problem described by the so-called Jaynes-Cummings Hamiltonian. The two-level system consists of an ion (with two levels coupled by laser light) or a highly excited atom (with two Rydberg levels coupled by a microwave field). The quantized harmonic oscillator describes the ion's motion in the trap or the microwave field in the cavity.

Here, we describe the implemented methods in the two cases, after a short background, and we present some important applications within science and technology.

Trapped ions

This research field started from techniques developed in the 1970s for trapping charged particles. Paul and Dehmelt were awarded the 1989 Nobel Prize in Physics "for the development of the ion trap technique". An important step towards the control of isolated ions was Doppler cooling, which was proposed by Hänsch and Schawlow (1975) for neutral atoms and by Wineland and Dehmelt (1975) for ions. The first experiments with ions were performed independently by Wineland and colleagues (Mg^+) and by Neuhauser *et al.* (Ba^+) in 1978. Wineland, Ekstrom and Dehmelt (1973) discussed the possibility of catching a single ion as early as 1973. This was achieved by Toschek's group in 1980 (Neuhauser *et al.*, 1980), who observed a single Ba^+ ion in a Paul trap, and by Wineland and Itano (1981), who caught a Mg^+ ion in a Penning trap. The group of Gabrielse has developed closely related techniques to cool single electrons captured in a Penning trap (Peil and Gabrielse, 1999).

Ion traps are created in ultrahigh vacuum using a combination of static and oscillating electric fields. There are traps where only one ion is captured, but also linear traps where a few ions are distributed on a line. A trapped ion has an oscillating movement, which is quantized at low temperature. An ion therefore has two sets of quantized levels: vibrational modes that characterize the motion in the trap (also called external states) and electronic levels that describe the internal quantum state of the ion. These levels can be coupled through light absorption or emission, and through a two-photon process, called Raman

transition. The ions can be observed through optical transitions that lead to strong light scattering when excited by a laser. They can be directly observed by eye or with a CCD camera (Fig. 2). Moreover, the internal state of the ion can be determined by observing quantum jumps. This was demonstrated by Nagourney *et al.* (1986) and by Wineland and colleagues (Bergquist *et al.*, 1986).

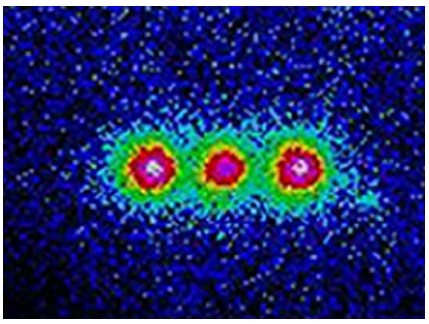

Fig. 2: Image of the fluorescence emitted by three trapped Be^+ ions (National Institute of Standards and Technology image gallery).

An important step in controlling the quantum state of an ion was cooling to the lowest energy of the trap using a technique called sideband cooling (Diedrich *et al.*, 1989; Monroe *et al.*, 1995a). Figure 3 shows several vibrational states of an ion in a trap for two different electronic levels ($|\downarrow>$ and $|\uparrow>$). The technique consists of exciting the ion, increasing the internal energy and decreasing the vibrational energy. This is done with a narrow-bandwidth laser with frequency $\omega_0 - \omega_\nu$, where ω_ν represents the frequency interval between two vibrational modes of the trap and ω_0 is the atomic frequency, *i.e.* the frequency difference between two electronic levels of the ion. The excited ion decays preferentially towards a state with the same vibrational quantum number ν. This reduces the ion energy and it gradually cools down to the $\nu = 0$ state. This technique, which was developed by Wineland and coworkers, allows the control of both internal and external degrees of freedom of the ion. By precisely monitoring the trap properties, Fock states of motion (with a well-defined ν) can be created, as well as various well-controlled superpositions of Fock states, *e.g.*, coherent or thermal states (Meekhof *et al.*, 1996).

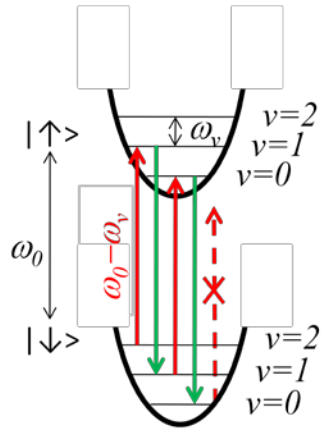

Fig. 3: Principle for sideband cooling (see text).

Another breakthrough was the development of techniques to transfer a quantum superposition of electronic states to a quantum superposition of vibrational modes of the trap (Monroe *et al.*, 1995b), inspired by a theoretical proposal by Cirac and Zoller (1995). Such a quantum superposition can then be transferred to another ion that shares the vibrational states with the first ion, as demonstrated in 2003 by Blatt and collaborators at the University of Innsbruck, Austria (Schmidt-Kaler *et al.*, 2003). This technique has been extensively used

by Wineland and coworkers for decoherence measurements and optical clocks, and is the basis of quantum gates based on trapped ions. We illustrate it with an example in Box 1.

Box 1. Creating and transferring a superposition of states of an ion

We consider an ion in a trap, in the lowest electronic state $|\downarrow>$, and in the lowest state of the trap $|0>$. The quantum system can be described as

$|\varphi_0> = |\downarrow>|0>$.

The ion can be excited by a laser pulse so that a coherent superposition of states is created:

$|\varphi_0> \rightarrow |\varphi_1> = (\alpha|\downarrow> + \beta|\uparrow>)\,|0>$.

A "red sideband" π-puls (with a frequency equal to $\omega_0 - \omega_1$; see Fig. 3) interacts with the ion. Because the ion is in the lowest vibration mode, only the state $|\uparrow>|0>$ is affected. It goes to $|\downarrow>|1>$, so that

$|\varphi_1> \rightarrow |\varphi_2> = \alpha|\downarrow>|0> + \beta|\downarrow>|1> = |\downarrow>(\alpha|0> + \beta|1>)$.

The superposition has been transferred to the ion's vibration mode. If the trap also contains another ion, this ion will share the vibration modes with the first ion. In the same way, the superposition now can be transferred to the internal state of the second ion.

Photons in a cavity

The research field called cavity quantum electrodynamics (CQED) started in the 1980s to study how the properties of an atom (especially spontaneous emission) were affected when the atom is placed in an optical or microwave cavity (for a review of early work, see Haroche and Kleppner, 1989). The suppression of spontaneous emission when the cavity size approaches the emitted light wavelength was observed successfully by Kleppner and his group (Hulet *et al.*, 1985), DeMartini *et al.* (1987) and Haroche's group at Yale University (Jhe *et al.*, 1987). The next step in this research was to study the light amplification in a resonant cavity, with early input from Haroche and collaborators in the microwave region (Goy *et al.*, 1983). A group at the Max Planck Institute for Quantum Optics in Garching, Germany, led by Walther, demonstrated a one-atom micromaser (Meschede *et al.*, 1985), while Haroche and his group showed evidence for a micromaser with two photons (Brune *et al.*, 1987). Kimble developed CQED in the optical domain (for a review, see Miller *et al.*, 2005), achieving the so-called strong coupling of atom-field interaction in the cavity (Thompson *et al.*, 1992; Hood *et al.*, 1998), in parallel with Haroche's work in the microwave domain (Brune *et al.*, 1996a). CQED in the optical domain combines cavity field dynamics with laser cooling and trapping techniques, and has interesting applications in quantum optics and quantum information (McKeever *et al.*, 2004). Cavity-QED has also inspired

research using superconducting circuits which has been named Circuit-QED (Schoelkopf and Girvin, 2008).

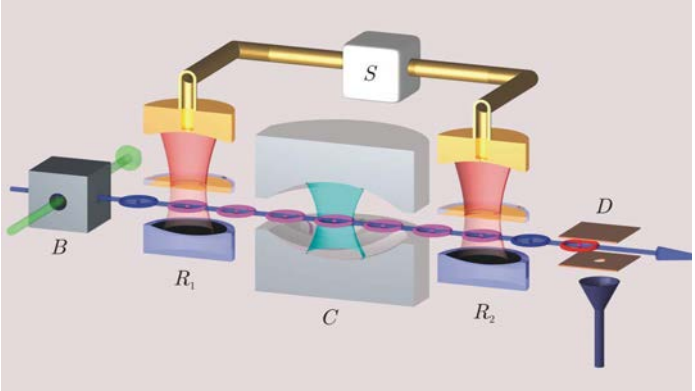

Fig. 4: Experimental setup to study microwave field states with the help of circular Rydberg atoms (see text).

The main experimental component used by Haroche, Raimond, Brune and their collaborators is a microwave cavity (Fig. 4) that consists of two spherical mirrors separated by a distance of 2.7 cm, made of a superconducting material (Nb) and cooled to very low temperature~0.8 K. Technological progress in the mirrors' quality led at the beginning of the past decade to a cavity with an extremely high Q value ($4x10^{10}$), *i.e.* implying a very long lifetime of a photon in the cavity, of ~130 ms. In such a cavity, a photon travels about 40,000 km before it disappears.

The field in the cavity is probed by Rb atoms that are prepared in a circular Rydberg state (*e.g.*, $n = 50$, $l = |m| = 49$). Such atoms have a large area, with a radius of 125 nm, and are very strongly coupled to the field in the cavity. The transition $n = 50$ ($|\downarrow>$) to $n = 51$ ($|\uparrow>$) has almost the same frequency as the microwave field in the cavity (51 GHz). Two cavities R_1 and R_2 (see Fig. 4) are used to create and analyze a controlled quantum superposition between $|\downarrow>$ and $|\uparrow>$. A selective field ionization detector (D) detects the state of the atom. Photons produced by a coherent source are coupled to the cavity via a waveguide. The atoms are sent one at a time into the cavity at a controlled velocity and thereby have a controlled time of interaction. In most experiments performed by Haroche's group, the atom and field have slightly different frequencies. An atom travelling in the cavity does not absorb photons, but its energy levels shift due to the dynamical Stark effect, inducing a phase variation of the microwave field. This phase shift is of the opposite sign, depending on whether the atom is in the $|\downarrow>$ or $|\uparrow>$ state, leading to an entanglement of the atomic and field states (Brune *et al.*, 1996b).

In 1990, Haroche and coworkers suggested a method to measure the number of photons in the cavity in a quantum non-demolition measurement (Brune *et al.*, 1990). Recently, they were able to demonstrate it experimentally (Gleyzes *et al.*, 2007; for a related experiment, see Nogues *et al.*, 1999). Individual photons are captured in a cavity and observed via the

interaction with atoms. The principle of the measurement is explained in more detail in Box 2. This has led to experiments where the "progressive collapse" of a wave function has been observed by means of non-destructive quantum measurements. In these experiments, the number of photons can be followed as it evolves during the measurement (Guerlin *et al.*, 2007).

Box 2. Measuring one photon in a cavity without destroying it

An atom in the state $|\uparrow\rangle$ is prepared in B (see Fig. 4). In R_1, a $\pi/2$ pulse creates a superposition of atomic states $|\downarrow\rangle$ and $|\uparrow\rangle$ so that $|\varphi_1\rangle = (|\downarrow\rangle + |\uparrow\rangle)/\sqrt{2}$.

After traveling through the cavity C, the superposition becomes $|\varphi\rangle = (|\downarrow\rangle + e^{i\phi}|\uparrow\rangle)/\sqrt{2}$, where ϕ is the phase accumulated by the superposition of states during the travel. The atomic dipole created by the superposition of states behaves as a clock, and the phase represents the needle position of the clock after travel through the cavity. The microwave field in the cavity has a frequency slightly detuned from the atomic frequency. An atom crossing the cavity will not absorb or emit photons, but its energy levels will be shifted. Consequently, the phase ϕ will change, depending on the number of photons in the cavity.

The atom's travel time can be chosen so that if the cavity contains no photons, $\phi = 0 \bmod 2\pi$. When the atom arrives in R_2, it is in the superposition $|\varphi_1\rangle$. In R_2. it interacts with another $\pi/2$ pulse so that $|\varphi_1\rangle \rightarrow |\downarrow\rangle$, which is detected by D. The detuning of the atomic frequency relative to the field in the cavity can be chosen so that if the cavity contains one photon, $\phi = \pi$. When the atom arrives in R_2 after having interacted with one photon, it is in the superposition $|\varphi_2\rangle = (|\downarrow\rangle - |\uparrow\rangle)/\sqrt{2}$. When interacting with the $\pi/2$ pulse, $|\varphi_2\rangle \rightarrow |\uparrow\rangle$. Thus it is possible to measure non-destructively whether there is 0 or 1 photon in the cavity. This type of measurement can be extended to a few photons by repeating similar measurements, using different phases and several atoms.

Experimental investigation of Schrödinger's cat paradox

A central question in quantum physics is the transition between the quantum and the classical world. This question is illustrated in a popular way by the so-called Schrödinger's cat paradox. This name refers to a thought experiment proposed by Schrödinger in 1935, emphasizing the difficulty in applying the concepts of quantum mechanics to everyday life (see Fig. 5). It poses the question: When, as time proceeds, does a quantum system stop existing as a superposition of states and become one or the other? The quantum-classical

boundary has been studied by many physicists since the beginning of quantum mechanics in the 1930s (see, *e.g.*, Zurek, 1991, and the review by Leggett *et al.*, 1987).

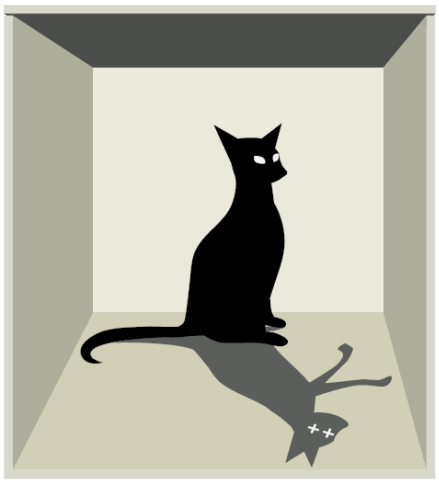

Fig. 5: The Schrödinger's cat.

The control achieved by the groups led by Haroche and Wineland on single quantum systems allowed them to perform Schrödinger's cat-like experiments in the laboratory, using photons and ions (see a review by Haroche, 1998). In an experiment proposed (Davidovich *et al.*, 1996) and performed by Haroche's group (Brune *et al.*, 1996b), a superposition of cat-like microwave field states was created by entangling a Rydberg atom with the cavity field. Such a superposition is very fragile and can be destroyed easily via coupling to the environment (in this case, by photons escaping the cavity). The decoherence of this superposition, *i.e.* its evolution towards a statistical mixture, could be measured as a function of time and the properties of the superposition of states. Wineland and coworkers performed similar experiments using ion trap technology. They created "cat states" consisting of single trapped ions entangled with coherent states of motion (Monroe *et al.*, 1996) and observed their decoherence (Myatt *et al.*, 2000). Recently, Haroche and coworkers created cat states, measured them and made a movie of how they evolve from a superposition of states to a classical mixture (Deléglise *et al.*, 2008). This extraordinary control has also led them to implement quantum feedback schemes in which the effects of decoherence are measured and corrected for, thus "stabilizing" a quantum state, *e.g.*, a given Fock state (Sayrin *et al.*, 2011).

Quantum computers

In a seminal theoretical article published in 1995, Cirac and Zoller suggested a way to build a quantum computer with trapped ions. Quantum bits (qubits) are encoded into hyperfine levels of trapped ions, which interact very weakly with the environment and therefore have long lifetimes. Two or more ions can be coupled through the center-of-mass motion (as presented in Box 1). Wineland and his group were the first to carry out experimentally a two-qubit operation (the Controlled NOT gate, CNOT) between motion and spin for Be^+ ions (Monroe *et al.*, 1995b). Since then, the field of quantum information based on trapped ions has progressed considerably. In 2003, Blatt and collaborators in Innsbruck, Austria, achieved

a CNOT operation between two Ca^+ ions (Schmidt-Kaler *et al.*, 2003). Today, the most advanced quantum computer technology is based on trapped ions, and has been demonstrated with up to 14 qubits and a series of gates and protocols (see Blatt and Wineland, 2008, for a review). Developing large devices capable of carrying out calculations beyond what is possible with classical computers will require solving substantial challenges in the future.

Optical Clocks

An important application of Wineland's research with trapped ions is optical clocks. Clocks based on a transition in the optical domain are interesting because the frequency of the transition, which is in the visible or ultraviolet range, is several orders of magnitude higher than that of the Cs clocks operating in the microwave range. Optical clocks developed by Wineland and coworkers (Diddams *et al.*, 2001; Rosenband *et al.*, 2008; Chou *et al.*, 2010a) currently reach a precision just below 10^{-17}, two orders of magnitude more accurate than the present frequency standard based on Cs clocks.

An optical ion clock uses a narrow (forbidden) transition in a single ion, insensitive to perturbations. The ion also needs to have strong allowed transitions for efficient cooling and detection. Wineland and colleagues developed a new technique, called quantum logic spectroscopy, based on entanglement of two ion species, as explained in Box 1. In this technique, one ion provides the spectroscopy transition [*e.g.*, $^1S_0 \rightarrow {}^3P_1$ in $^{27}Al^+$ (267 nm)], while the other one (*e.g.*, $^9Be^+$) has the strong cooling transition (Schmidt *et al.*, 2005). The precision of two different optical clocks can be compared with the help of the frequency comb technique invented by Hänsch and Hall (2005 Nobel Prize in Physics).

The accuracy recently achieved by the optical clocks has allowed Wineland and coworkers to measure relativistic effects, such as time dilation at speeds of a few kilometers per hour or the difference in gravitational potential between two points with a height difference of only about 30 cm (Chou *et al.*, 2010b).

Summary

David Wineland and Serge Haroche have invented and implemented new technologies and methods allowing the measurement and control of individual quantum systems with high accuracy. Their work has enabled the investigation of decoherence through measurements of

the evolution of Schrödinger's cat-like states, the first steps towards the quantum computer, and the development of extremely accurate optical clocks.

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