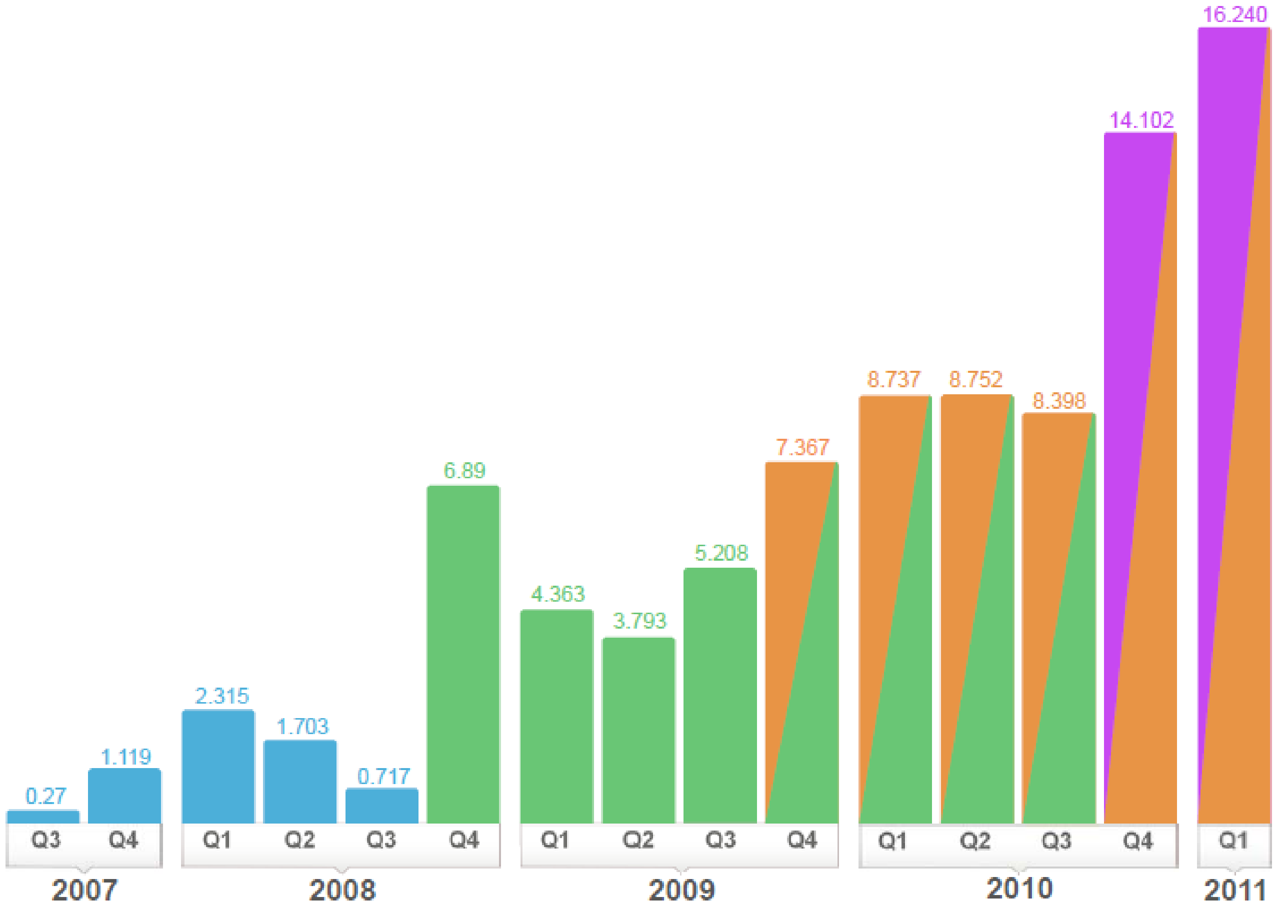
The Newsvendor Model

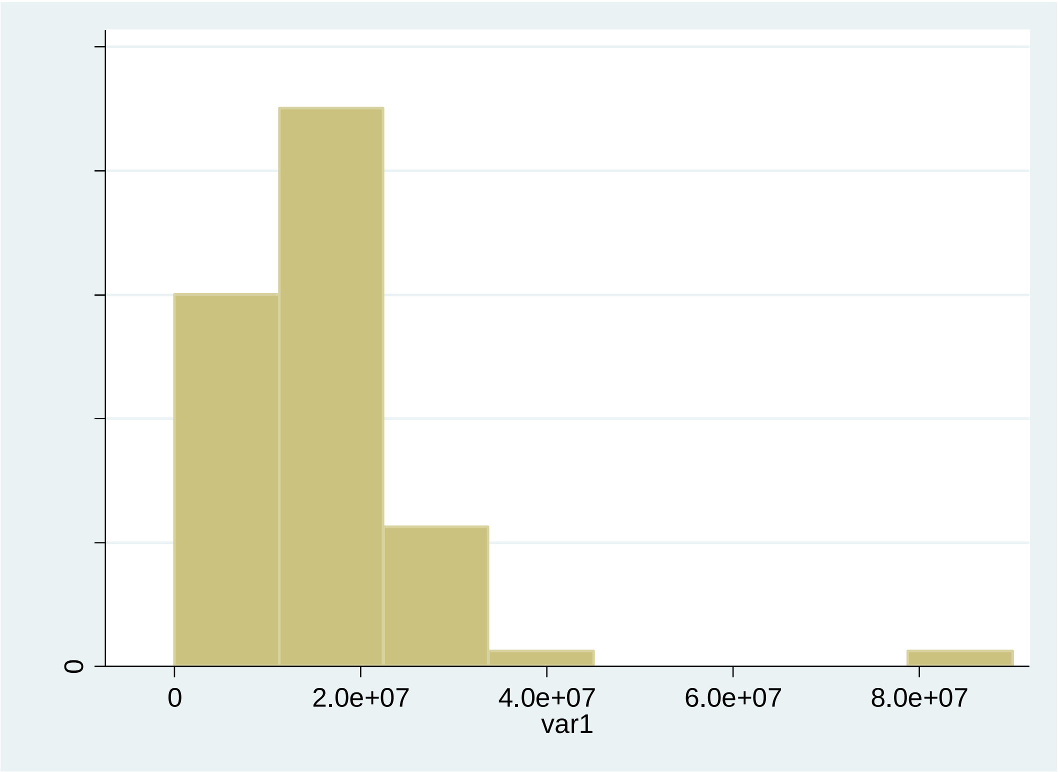
Class outline

- Motivating examples.
- Steps to implement the Newsvendor model.
- Finding the quantity that maximizes expected profit.
- Performance measures in the Newsvendor model.

iPhone Sales in Fiscal Year 2011 Q3?

Historical Sales Data

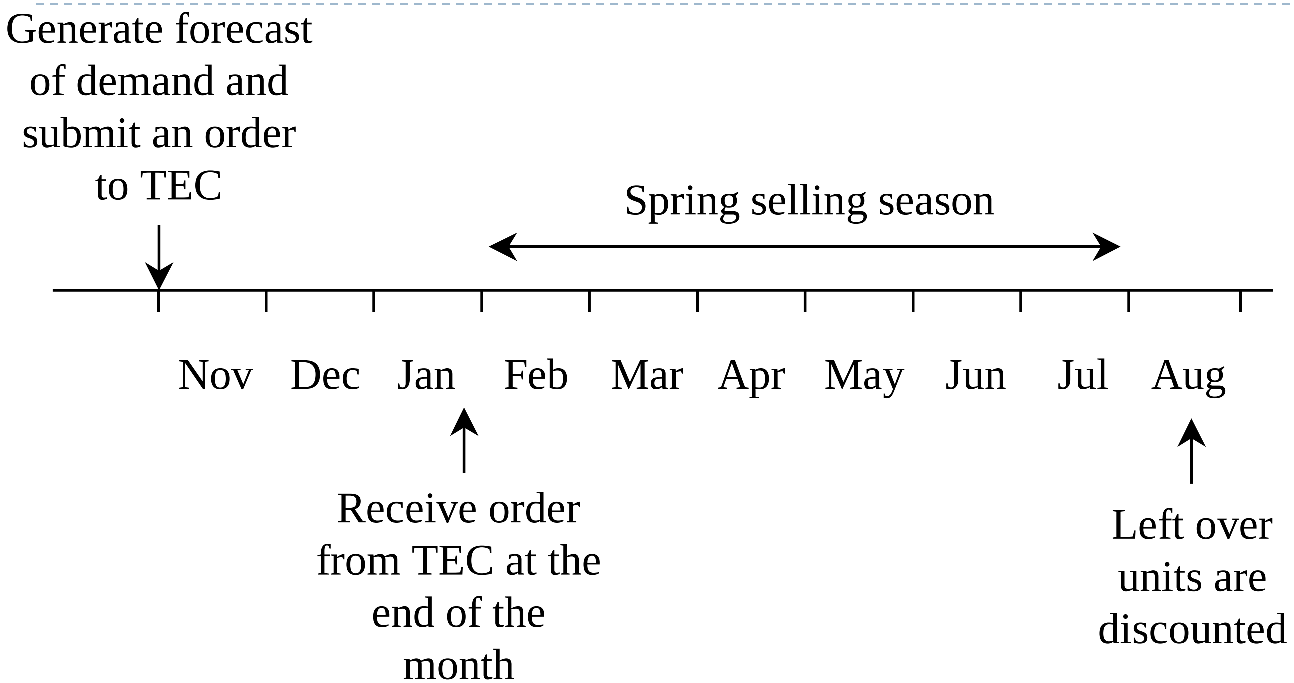




Obs	Mean	Std. Dev.	Min	Max
71	1.60e+07	1.13e+07	9000	9.00e+07

O’Neill’s Hammer 3/2 wetsuit

Hammer 3/2 timeline and economics



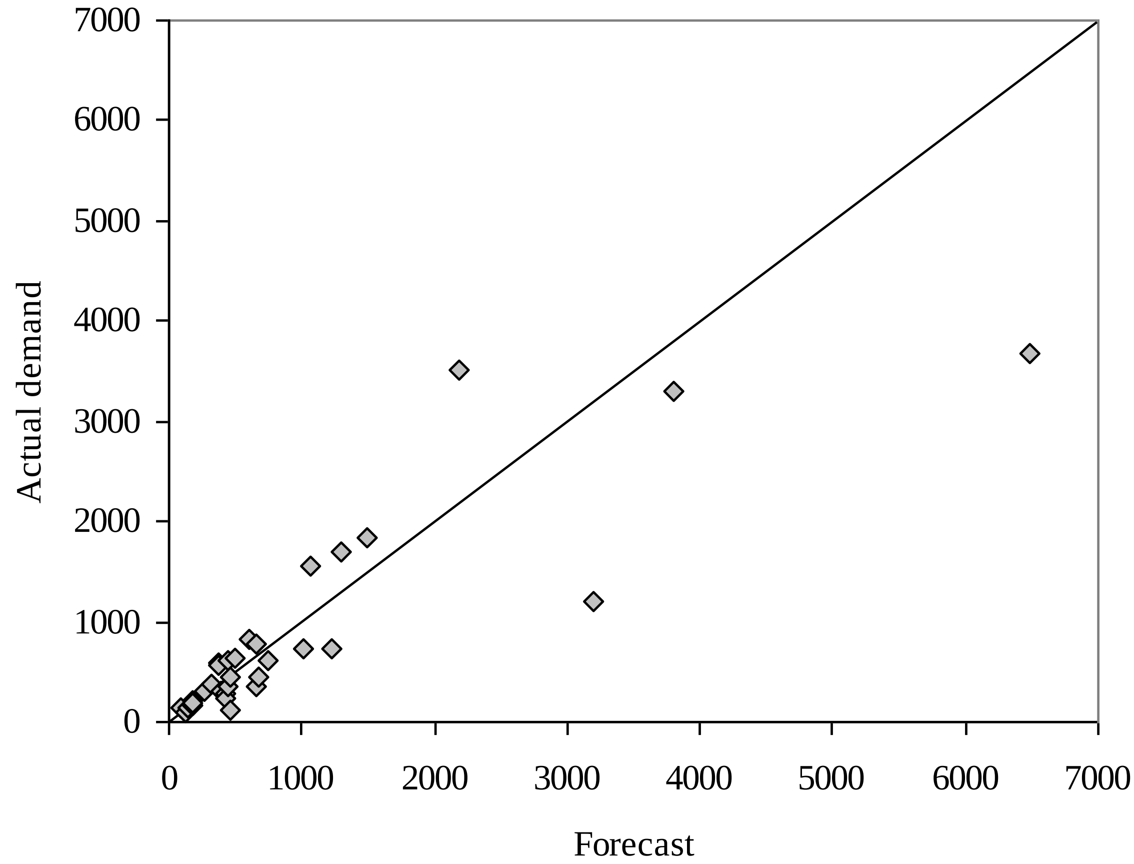
Economics:

- Each suit sells for p = \$180
- TEC charges c = \$110 per suit
- Discounted suits sell for v = \$90

- The "too much/too little problem":
  - Order too much and inventory is left over at the end of the season
  - Order too little and sales are lost.

- Marketing's forecast for sales is 3200 units.

Historical forecast performance at O'Neill



Forecasts and actual demand for surf wet-suits from the previous season.
Each point is a different style.

The Newsvendor model: Implementation

Newsvendor model can be used to make a single-shot commitment in anticipation to an uncertain outcome.

Gather Economic Inputs: Co and Cu
Selling price, production cost, salvage value

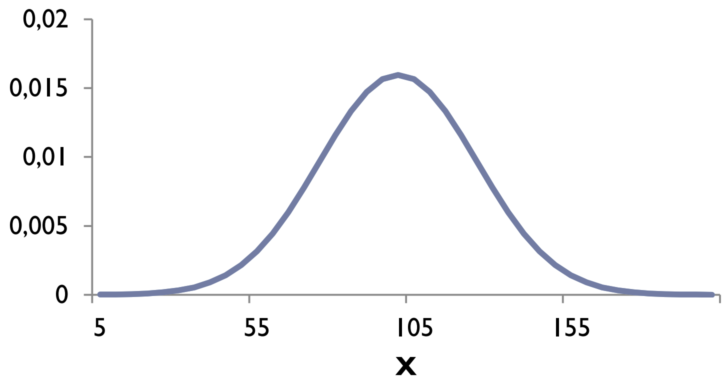
Demand model: Distribution function F(.)
From historical data, experts assessment, market info.

Choose an objective:
-Maximize Expected Profit

Optimal Order Quantity (Q)

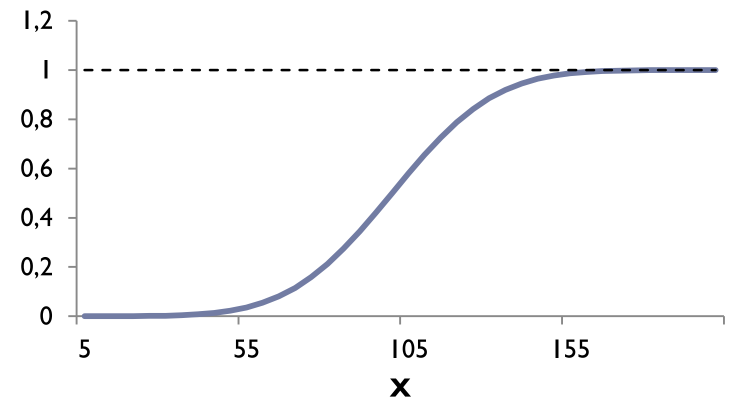
Input 1: Demand distribution

Density Function (Histogram)



Examples:
•Normal with mean= 100 and std. dev. =25.
•Histogram of historical data.

Distribution Function (F(x))



F(x) = Probability that demand is less than or equal to *x*.

Based on historical performance of forecasts, O'Neill uses **normal distribution** with **mean 3192** and **standard deviation 1181** to represent demand for the Hammer 3/2 during the Spring season.

Input 2: "Too much" and "too little" costs

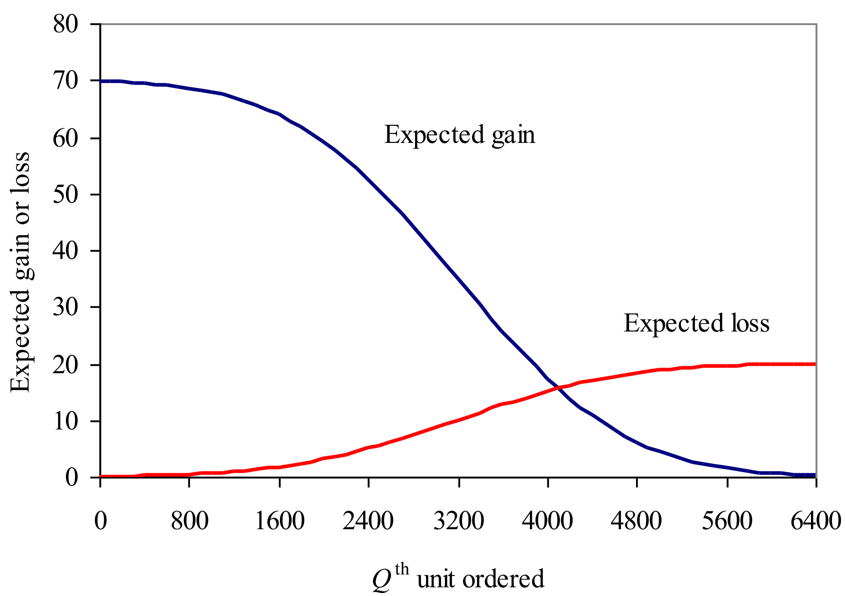
- C_o = overage cost
  - The consequence of ordering one more unit than what you would have ordered had you known demand.
  - In other words, suppose you had left over inventory (you over ordered). C_o is the increase in profit you would have enjoyed had you ordered one fewer unit.
  - For the Hammer 3/2 $C_o = Cost - Salvage\ value = c - v = 110 - 90 = 20$

- C_u = underage cost
  - The consequence of ordering one fewer unit than what you would have ordered had you known demand.
  - In other words, suppose you had lost sales (you under ordered). C_u is the increase in profit you would have enjoyed had you ordered one more unit.
  - For the Hammer 3/2 $C_u = Price - Cost = p - c = 180 - 110 = 70$

The Newsvendor Model: Choosing an order quantity to maximize expected profit.

Balancing the risk and benefit of ordering a unit

- Ordering one more unit increases the chance of overage …
  - Expected loss on the Q^{th} unit = C_o x $F(Q)$
  - $F(Q)$ = Distribution function of demand = $Prob(Demand <= Q)$
- … but the benefit/gain of ordering one more unit is the reduction in the chance of underage:
  - Expected gain on the Q^{th} unit = C_u x $(1-F(Q))$



- As more units are ordered, the expected benefit from ordering one unit decreases while the expected loss of ordering one more unit increases.

Newsvendor expected profit maximizing order quantity

- To maximize expected profit order Q units so that the expected loss on the Q^{th} unit equals the expected gain on the Q^{th} unit:

$$C_o \times F(Q) = C_u \times (1 - F(Q))$$

- Rearrange terms in the above equation:

$$F(Q) = \frac{C_u}{C_o + C_u}$$

- The ratio $C_u / (C_o + C_u)$ is called the *critical ratio*.

- Hence, to maximize profit, choose Q such that the probability we satisfy all demand (i.e., demand is Q or lower) equals the critical ratio.

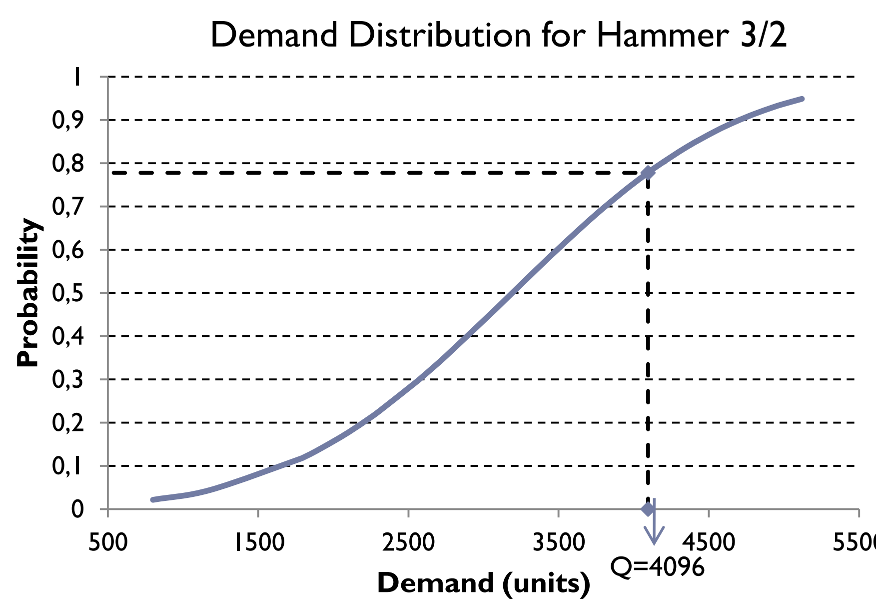
Profit maximizing order quantity using the Normal distribution: Hammer 3/2 example.

- Demand distribution: Normal (μ=3192, σ=1181)
- Economic data: *p=$180, c=$110, v=$90.*
  - *Cu = $70, Co=$20.*
  - *Critical ratio = Cu / (Co + Cu) = .778*
- *In Excel*: find the ordering quantity Q by inverting a Normal Distribution with mean μ and standard deviation σ.

=NORMINV(.778, 3192, 1181)

Critical Ratio μ σ

Q = 4096



Order quantity for Hummer 3/2 using Standard Normal Table

- Demand distribution: Normal (μ=3192, σ=1181)
- Economic data: *p=$180, c=$110, v=$90.*
  - *Cu = $90, Co=$20.*
  - *Critical ratio = Cu / (Co + Cu) = .778*
- Look critical ratio in Standard Normal distribution table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389

Approximate using higher value: z= 0.77

(it is just a coincidence that z=0.77 is so close to the critical ratio)

- Convert z-statistic into order quantity *Q*:

$$Q = \mu + z \times \sigma$$
$$= 3192 + 0.77 \times 1181 = 4101$$

Performance Measures in the Newsvendor Model

Newsvendor model: Performance measures

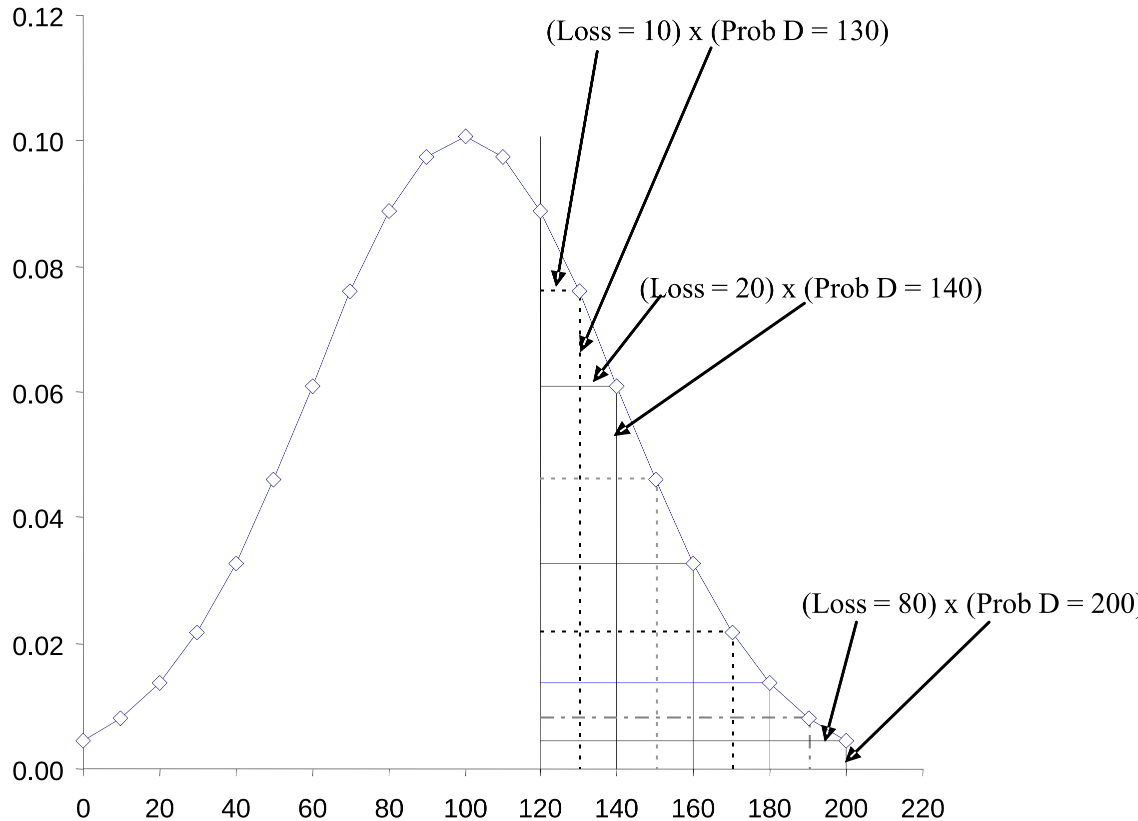
- For any order quantity we would like to evaluate the following performance measures:
  - *Expected lost sales*
    - The expected number of units demand exceeds the order quantity
  - *Expected sales*
    - The expected number of units sold.
  - *Expected left over inventory*
    - The expected number of units left over after demand (but before salvaging)
  - *Expected profit*
  - *Expected fill rate*
    - The fraction of demand that is satisfied immediately
  - *In-stock probability*
    - Probability all demand is satisfied
  - *Stockout probability*
    - Probability some demand is lost

Expected lost sales: graphical explanation

- Suppose demand can be one of the following values {0,10,20,…,190,200}
- Suppose Q = 120
- Let D = actual demand
- What is expected lost sales?
- If D<=Q, lost sales = 0
- If D =130,

  lost sales = D – Q = 10

- Expected lost sales =
  10 x Prob{D = 130} +
  20 x Prob{D = 140} +
  … +
  80 x Prob{D = 200}



Expected lost sales of Hammer 3/2s with Q = 3500

- For example:
  - if demand is 3800 and Q = 3500, then lost sales is 300 units.
  - if demand is 3200 and Q = 3500, then lost sales is 0 units.
- Definition:
  - Expected lost sales is the average number of lost sales over all possible demand outcomes.
- If demand is normally distributed:
  - Step 1: normalize the order quantity to find its z-statistic.

$$z = \frac{Q-\mu}{\sigma} = \frac{3500-3192}{1181} = 0.26$$

  - Step 2: Look up in the Standard Normal Loss Function Table the expected lost sales for a standard normal distribution with that z-statistic: $L(0.26)=0.2824$
    - or, in Excel

$$L(z) = Normdist(z,0,1,0) - z*(1 - Normsdist(z))$$

  - Step 3: Evaluate lost sales for the actual normal distribution:

$$Expected \ \ lost\ sales \ = \ \sigma \times L(z) = 1181 \times 0.2824 = 334$$

Measures that follow expected lost sales:

Expected sales = μ - *Expected lost sales* = 3192 – 334 = 2858

Expected Left Over Inventory = *Q* - *Expected Sales*

= 3500 – 2858 = 642

$$\begin{aligned} Expected\ \ profit &= \left[\left(Price\text{-}Cost \right) \times Expected\ sales \right] \\ &\quad - \left[\left(Cost\text{-}Salvage\ value \right) \times Expected\ left\ over\ inventory \right] \\ &= \left(\$70 \times 2858 \right) - \left(\$20 \times 642 \right) = \$187,221 \end{aligned}$$

$$\begin{aligned} \text{Expected fill rate} &= \frac{\text{Expected sales}}{\text{Expected demand}} = \frac{\text{Expected sales}}{\mu} \\ &= 1 - \frac{\text{Expected lost sales}}{\mu} = \frac{2858}{3192} \\ &= 89.6\% \end{aligned}$$

Note: the above equations hold for <u>any demand distribution</u>

Newsvendor model summary

- The model can be applied to settings in which …
  - There is a single decision opportunity (order/production/replenishment).
  - Demand is uncertain.
  - There is a "too much-too little" challenge:
    - If demand exceeds the order quantity, sales are lost.
    - If demand is less than the order quantity, there is left over inventory.
- Firm must have a demand model that includes an expected demand and uncertainty in that demand.
  - With the normal distribution, uncertainty in demand is captured with the standard deviation parameter.
- At the order quantity that maximizes expected profit the probability that demand is less than the order quantity equals the critical ratio:
  - The expected profit maximizing order quantity balances the "too much-too little" costs.