Pauta P1- C2

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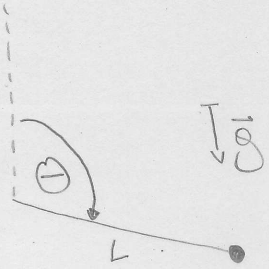
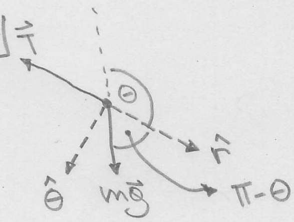


a) CI| $r \equiv L \Rightarrow \dot r = \ddot r = 0$, $\theta(t=0) = \pi/2$, $\dot\phi(t=0) = \dfrac{V_0}{L}$

$\Rightarrow$ DCL|



$\sum \vec F = m\vec g + \vec T = mg(\cos(\pi-\theta)\,\hat r + \text{sen}(\pi-\theta)\,\hat\theta) - T\hat r$

$\cos(\pi-\theta) = \cos\pi\cos\theta + \text{sen}\,\pi\,\text{sen}\,\theta = -\cos\theta$

$\text{sen}(\pi-\theta) = \text{sen}\,\pi\cos\theta - \cos\pi\,\text{sen}\,\theta = \text{sen}\,\theta$

$\Rightarrow \hat r)\ -mL(\dot\theta^2 - \dot\phi^2\sin^2\theta) = -mg\cos\theta - T$

$\hat\theta)\ mL(\ddot\theta - \dot\phi^2\sin\theta\cos\theta) = mg\,\text{sen}\,\theta$

$\hat\phi)\ \dfrac{m}{L\sin\theta}\dfrac{d}{dt}(\dot\phi L^2\sin^2\theta) = 0 \iff L^2\dot\phi\sin^2\theta = \underbrace{L^2\dfrac{V_0}{L}\sin^2(\pi/2)}_{\vec l(t=0)\ \text{se conserva}}$

$$\Rightarrow \boxed{\dot\phi = \frac{V_0}{L}\frac{1}{\sin^2\theta}}\ (1)$$

(1) en $\hat\theta) \Rightarrow$

$$\ddot\theta = \frac{V_0^2}{L^2}\frac{\cos\theta}{\text{sen}^3\theta} + \frac{g\,\text{sen}\,\theta}{L}$$

$$\dot\theta\frac{d\dot\theta}{d\theta} = \frac{V_0^2}{L^2}\frac{\cos\theta}{\text{sen}^3\theta} + \frac{g\,\text{sen}\,\theta}{L} \quad \Big\backslash \int_{\theta(t=0)}^{\theta} d\theta$$

$$\frac{\dot\theta^2}{2} - \cancel{\frac{\dot\theta^2(t=0)}{2}}^{\,0} = \frac{V_0^2}{L^2}\left[\frac{1}{2}\frac{1}{\cancel{\text{sen}^2\pi/2}}1 - \frac{1}{2}\frac{1}{\text{sen}^2\theta}\right] + \frac{2g}{L}\left[\cancel{\cos\pi/2}^{\,0} - \cos\theta\right]$$

$$\boxed{\dot\theta^2 = \frac{V_0^2}{L^2}\left[1 - \frac{1}{\text{sen}^2\theta}\right] - \frac{2g}{L}\cos\theta}$$

a) Se impone $\dot\theta(3\pi/4) = 0$

$\Rightarrow 0 = \dfrac{V_0^2}{L^2}[1-2] + \dfrac{2g}{L}\dfrac{\sqrt2}{2} \Rightarrow \dfrac{V_0^2}{L^{\cancel{2}}} = \dfrac{g\sqrt2}{\cancel{L}} \Rightarrow \boxed{V_0 = \sqrt{\sqrt2\, gL}}$ //

b) El mov. está confinado a $\theta \in [\pi/2, 3\pi/4]$. En los extremos superior e inferior están la min. y max. tensión, respectivamente. Además cumplen $\dot\theta = 0$

$\Rightarrow T(\theta=\pi/2) = T_{min} = m\cancel{L}\dfrac{V_0^2}{L^{\cancel{2}}}\dfrac{1}{\sin^2\pi/2} - mg\cos\pi/2$

$= m\dfrac{V_0^2}{L} = \sqrt2\, mg$

$T(\theta=3\pi/4) = T_{max} = m\dfrac{V_0^2}{L}2 + mg\dfrac{\sqrt2}{2}$

$\Rightarrow T_{max} = 2\sqrt2\, mg + mg\sqrt2/2$ //

$T_{max} = \dfrac{5}{2}\sqrt2\, mg$

F.