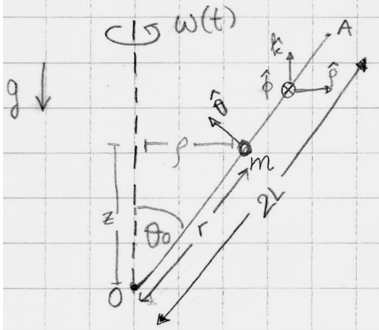
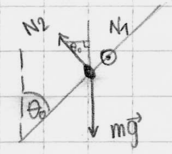
Pauta P2 Control 1, 2012



(a) Describiremos el mov. en coordenadas cilíndricas.

$$\vec{r} = \rho\hat{\rho} + z\hat{k}$$
$$\vec{v} = \dot{\rho}\hat{\rho} + \rho\dot{\phi}\hat{\phi} + \dot{z}\hat{k} = \dot{\rho}\hat{\rho} + \rho\omega\hat{\phi} + \dot{z}\hat{k}$$
$$\vec{a} = \ddot{\rho}\hat{\rho} + 2\dot{\rho}\omega\hat{\phi} + \rho\ddot{\phi}\hat{\phi} - \rho\dot{\phi}^2\hat{\rho} + \ddot{z}\hat{k}$$

Newton



$$\vec{N_2} = N_2 \sin\theta_0 \hat{k} - N_2 \cos\theta_0 \hat{\rho}$$

$$(\hat{\rho}): \quad m\ddot{\rho} - m\rho\omega^2 = -N_2 \cos\theta_0$$
$$(\hat{\phi}): \quad 2m\dot{\rho}\omega + m\rho\dot{\omega} = -N_1$$
$$(\hat{k}): \quad m\ddot{z} = mg + N_2 \sin\theta_0$$

ahora quiero escribir $\rho = r\sin\theta_0 \mapsto \dot{\rho} = v_0 \sin\theta_0 \mapsto \ddot{\rho} = 0$

$z = r\cos\theta_0 \mapsto \dot{z} = v_0\cos\theta_0 \mapsto \ddot{z} = 0$

Con eso,
$$(\hat{\rho}): \quad +mr\sin\theta_0\omega^2 = N_2\cos\theta_0$$
$$(\hat{\phi}): \quad 2mv_0\sin\theta_0\omega + mr\sin\theta_0\dot{\omega} = -N_1$$
$$(\hat{k}): \quad -mg = +N_2\sin\theta_0$$

$(\hat{k})$ en $(\hat{\rho})$:
$$\cancel{m}\underbrace{r}_{r_0+v_0t\,=\,L+v_0t}\sin\theta_0\,\omega^2 = -\cancel{m}g\frac{\cos\theta_0}{\sin\theta_0} \quad (*)$$

$$\mapsto \quad \omega(t) = \sqrt[2]{\frac{g}{L+v_0t}\cdot\frac{\cos\theta_0}{\sin^2\theta_0}} \quad //$$

(b) $r(t) = L + V_0 t \overset{\text{(Punto A, en que m sale de la varilla)}}{=} 2L$ cuando $t = \frac{L}{V_0}$

$$\underbrace{\phi - \phi_0}_{\phi_{RECORRIDO}} = \int_0^{L/V_0} \overset{\omega(t)}{\dot{\phi}}\, dt = \sqrt{\frac{g\cos\theta_0}{\sin^2\theta_0}} \cdot \left(\frac{2}{V_0} \sqrt{L + V_0 t} \right) \Big|_0^{L/V_0} = \sqrt{\frac{gL\cos\theta_0}{\sin^2\theta_0}} \cdot \frac{2}{V_0} \cdot (1 + \sqrt{2}) \quad //$$

(c) Derivando ⊛,

$$V_0 \cancel{\sin\theta_0}\, \omega^2 + (L + V_0 t) \cancel{\sin\theta_0} \cdot 2\omega \cdot \dot{\omega} = 0$$

$$\mapsto \quad \dot{\omega} = \frac{-V_0 \omega}{2(L + V_0 t)}$$

en $(\hat{\phi})$: $N_1 = -\omega \left(2mV_0 \cancel{\sin\theta_0} - m \cdot \cancel{(L + V_0 t)} \frac{\sin\theta_0 V_0}{2\cancel{(L + V_0 t)}} \right) = \frac{-3\omega m V_0 \sin\theta_0}{2}$

$$= \frac{-3mV_0 \cancel{\sin\theta_0}}{2} \sqrt{\frac{g\cos\theta_0}{(L + V_0 t)\sin^{\cancel{2}}\theta_0}} = \frac{-3mV_0}{2} \sqrt{\frac{g\cos\theta_0}{(L + V_0 t)}} \quad //$$

En cuanto a N_2, de $(\hat{k})$ ya sabíamos que es $\frac{-mg}{\sin\theta_0}$ //

$$\vec{N} = N_1 \hat{\phi} + N_2 \hat{\theta}$$

PD: En realidad, en esféricas también debería salir, con similar cantidad de trabajo, cosa de gustos... =P