Recurrencias & Applicaciones

- Torre de Hanoi
- Dish Pile Problem

- Programacion Dinamica

Torre de Hanoi

```
Mover (n, A, B, C) {
  if (n==1) MoverUnoDisco(A, C);
  else(if n>1) {
      Mover (n-1, A, C, B)
      Mover (1, A, B, C)
      Mover (n-1, B, A, C)
  }
}
```

Complejidad

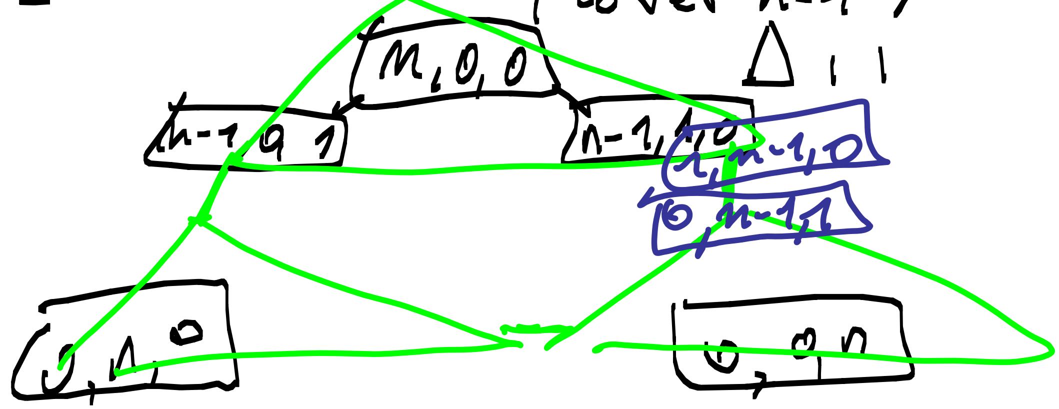
- Cota Superior

$f(n) = \#$ movimientos de Mover(n, A, B, C)

$= 2f(n-1) + 1$

$= 2^n - 1 \overset{?}{=} 2(2^{n-1} - 1) + 1$

- Cota Inferior
  - → "adversario" → Mover $n-1$, Mover 1, Mover $n-1$ $\Rightarrow 2^n - 1$
  - → grafica



II Disk Pile Problem

n_i: discos de tamaño i

$i \in [1..h]$, $n = \sum n_i$ discos

Complejidad

- peor caso por n fijo
  - cota superior: $2^n - 1$
  - cota inferior: $2^n - 1$
- peor caso por n, h fijos $\rightsquigarrow$ $n_1 = n-h+1$, $\forall i > 1$ $n_i = 1$ $\Rightarrow 2^{h-1}(n-h+1) + 2^{h-1} - 1$
- peor caso por $h, n_1, \ldots, n_h$ fijos

$$f(h, n_1, \ldots, n_h) = 2\underbrace{f(h-1, n_1, \ldots, n_{h-1})}_{2f(h-2, n_1, \ldots, n_{h-2}) + n_{h-1}} + n_h = \boxed{\sum_{i=1}^{h} 2^{h-i} n_i}$$

VII Recurrencias

$$\left(\sum_{i=1}^{h} 2^{h-i}\right) n \Big/ h = \frac{n}{h}\left(\sum_{j=0}^{h-1} 2^{j}\right)$$

$$= \frac{n}{h}\left(2^{h}-1\right)$$

$$\boxed{\sum_{i=1}^{n} q^{i}} = \frac{1-q^{n}}{1-q}\, q$$

$$= \text{primero termino}\ \frac{1-\text{razon}^{\#\text{term}}}{1-\text{razon}}$$

$$\boxed{\sum_{i=1}^{n} i} = \frac{n(n+1)}{2}$$

$$\boxed{\sum_{i=1}^{n} i^{2} \to}\ ?$$

VI Recurrencias Lineales

ⓐ Fibonacci

$$F_0 = 1$$

$$F_1 = 1$$

$$F_n = F_{n-2} + F_{n-1}$$

$$\longrightarrow \lambda^n = \lambda^{n-2} + \lambda^{n-1}$$

$$\longrightarrow \lambda^n - \lambda^{n-2} - \lambda^{n-1} = 0$$

$$\lambda^{n-2}(\lambda^2 - 1 - \lambda) = 0$$

$$\begin{cases} \lambda = 0 \\ \text{o } \lambda^2 - \lambda - 1 = 0 \end{cases}$$

$$\Longrightarrow \lambda = \frac{1 \pm \sqrt{5}}{2}$$

ⓑ General

$$F_n = a_1 F_{n-1} + a_2 F_{n-2} + \cdots a_k F_{n-k} + a_0$$

$$\begin{pmatrix} F_n \\ F_{n-1} \\ \vdots \\ F_{n-k+1} \end{pmatrix} = \begin{pmatrix} a_1 & a_2 & \cdots & a_k & a_0 \\ 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \cdots & & & & 1 \end{pmatrix} \begin{pmatrix} F_{n-1} \\ \vdots \\ F_{n-k} \end{pmatrix} = \begin{pmatrix} F_k \\ \vdots \\ F_1 \end{pmatrix} A^{n-k+1}$$