AUX 3 PAUTA

P1 (a) Los eventos G_i son disjuntos porque no se puede ganar premio en 2 máquinas.

$$\mathbb{P}(G_1) = p$$

$$\mathbb{P}(G_2) = \underbrace{\mathbb{P}(G_2 \mid G_1^c)}_{\text{QUE GANE EN 2 DADO QUE NO GANE EN 1}} \underbrace{\mathbb{P}(G_1^c)}_{\text{que no gane en 1}} = p \cdot (1-p).$$

$$\mathbb{P}(G_3) = \mathbb{P}(G_3 \mid G_1^c \cap G_2^c) \cdot \mathbb{P}(G_2^c \mid G_1^c)\, \mathbb{P}(G_1^c)$$

$$= p \cdot (1-p)(1-p) = p(1-p)^2$$

$$\Rightarrow \mathbb{P}(G_i) = p\,(1-p)^{i-1} \qquad i \in \{1,2,3\}.$$

(b) $\mathbb{P}(\text{"TATÁN NO GANE"}) = 1 - \mathbb{P}(\text{"TATÁN GANA EN ALGUNA MÁQUINA"}).$

$$= 1 - \left(\mathbb{P}(G_1) + \mathbb{P}(G_2) + \mathbb{P}(G_3)\right)$$

$$= 1 - \left(p + p(1-p) + p(1-p)^2\right) = (1-p)\left(1 + p + p(1-p\right.$$

$$= (1-p)^3$$

(c) Supongamos que G_1 y G_2 son independientes

$$\Rightarrow \mathbb{P}(G_1 \cap G_2) = \mathbb{P}(G_1)\mathbb{P}(G_2)$$

Pero $G_1 \cap G_2 = \emptyset$ por (a) $\rightarrow \mathbb{P}(G_1 \cap G_2) = 0$

y $\mathbb{P}(G_1) \cdot \mathbb{P}(G_2) = p \cdot p(1-p) > 0$

$\therefore$ G_1 y G_2 no son independientes

Análogo para otros pares de eventos.

(d) Sea G: "TATÁN HA GANADO".

$$\mathbb{P}(G) = \mathbb{P}(G_1) + \mathbb{P}(G_2) + \mathbb{P}(G_3) = p + p(1-p) + p(1-p)^2$$
$$= p\left(1 + (1-p) + (1-p)^2\right)$$

Nos piden entonces

Como $G_i \subseteq G \Rightarrow G_i \cap G = G_i$

$i \in \{1,2,3\}$

$$\mathbb{P}(G_i | G) = \frac{\mathbb{P}(G_i \cap G)}{\mathbb{P}(G)} \overset{\downarrow}{=} \frac{\mathbb{P}(G_i)}{\mathbb{P}(G)} = \frac{p(1-p)^{i-1}}{p\left(1 + (1-p) + (1-p)^2\right)}$$

$$= \frac{(1-p)^{i-1}}{1 + (1-p) + (1-p)^2} \quad \forall i \in \{1,2,3\}$$

(e) Como hay infinitas máquinas se tiene que

$$G = \bigcup_{n=1}^{\infty} G_n$$

$$\mathbb{P}(G) \overset{\text{son disjuntos}}{=} \sum_{i=1}^{\infty} \mathbb{P}(G_i) = \sum_{i=1}^{\infty} p(1-p)^{i-1} \overset{\text{modificamos índices}}{=} p \sum_{i=0}^{\infty} (1-p)^i$$

$$\overset{(*)}{=} p \cdot \frac{1}{1-(1-p)} = 1$$

(*) Recordar que $\sum_{i=0}^{\infty} x^i = \frac{1}{1-x} \quad \forall |x| < 1$