Problema 1.

a) $z = f(x,y) = \operatorname{sen}(\pi x y^2) + e^{x^2y^3}$

$$\frac{\partial f}{\partial x} = \pi y^2 \cos(\pi x y^2) + 2xy^3 e^{x^2y^3}$$

$$\frac{\partial f}{\partial y} = 2\pi y x \cos(\pi x y^2) + 3x^2y^2 e^{x^2y^3}$$

i) $\nabla f(1,1) = \left(-\pi + 2e, -2\pi + 3e\right) = \left(2e-\pi, 3e-2\pi\right)$

ii) $D_{\hat{u}} f(1,1)$ $\quad u = (2,1) \quad \hat{u} = \frac{1}{\sqrt{5}}(2,1) = \left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$

$$D_{\hat{u}} f(1,1) = \nabla f(1,1) \cdot \hat{u}$$

$$= \left(2e-\pi, 3e-2\pi\right) \cdot \left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$$

$$D_u f(1,1) = \left(4e - 2\pi + 3e - 2\pi\right)\frac{1}{\sqrt{5}} = \frac{1}{\sqrt{5}}(7e - 4\pi) = 6{,}4616$$

iii) diferencial $\vec{A} \cdot \vec{h} = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy = dz$

$$dz = df\left((1,1); \left(\tfrac{1}{20}, \tfrac{1}{30}\right)\right) = (2e-\pi)\frac{1}{20} + (3e-2\pi)\frac{1}{30}$$

$$dz = \frac{3(2e-\pi)}{60} + \frac{2(3e-2\pi)}{60} = \frac{6e - 3\pi + 6e - 4\pi}{60} = \frac{12e - 7\pi}{60}$$

$$dz = 0{,}177137$$

iv) incremento $\Delta z = f(\vec{x}+\vec{h}) - f(\vec{x}_0)$

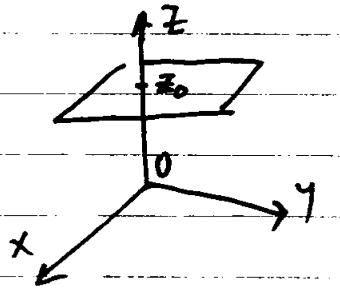
$S: \; z = 4x + 2y - x^2 + xy - y^2$

$\frac{\partial z}{\partial x} = 4 - 2x + y$ Punto (x_0, y_0, z_0)

$\frac{\partial z}{\partial y} = 2 + x - 2y$

Plano $(x-x_0)\frac{\partial z}{\partial x} + (y-y_0)\frac{\partial z}{\partial y} - (z-z_0) = 0$

$$z = z_0 + (x-x_0)(4-2x_0+y_0) + (y-y_0)(2+x_0-2y_0)$$



el plano tiene que ser de la forma $z = z_0$

o sea $\begin{cases} 4 - 2x_0 + y_0 = 0 \\ 2 + x_0 - 2y_0 = 0 \end{cases}$

$\begin{cases} 2x - y = 4 \\ x - 2y = -2 \end{cases} \to x = 2y - 2$

$2(2y-2) - y = 4$

$4y - 4 - y = 4$

$y = $

$y = \frac{8}{3}$ $\quad x = \frac{16}{3} - 2 = \frac{10}{3}$

$x = \frac{10}{3} \quad y = \frac{8}{3}$

$$z = \frac{40}{3} + \frac{16}{3} - \frac{100}{9} + \frac{80}{9} - \frac{64}{9} = \frac{1}{9}[120 + 48 - 100 + 80 - 64]$$

$z = \frac{84}{9}$

único punto $P\left(\frac{10}{3}, \frac{8}{3}, \frac{84}{9}\right)$

$$x = u + 5v$$
$$y = 2u - 3v$$

$$\frac{\partial x}{\partial u} = 1 \quad \frac{\partial x}{\partial v} = 5 \qquad \frac{\partial y}{\partial u} = 2 \quad \frac{\partial y}{\partial v} = -3$$

Teor.11 $$\boxed{\frac{\partial f}{\partial u} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial u}} = \frac{\partial f}{\partial x} + 2\frac{\partial f}{\partial y}$$

$$\frac{\partial^2 f}{\partial u^2} = \frac{\partial}{\partial u}\left(\frac{\partial f}{\partial x} + 2\frac{\partial f}{\partial y}\right)$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x} + 2\frac{\partial f}{\partial y}\right)\frac{\partial x}{\partial u} + \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x} + 2\frac{\partial f}{\partial y}\right)\frac{\partial y}{\partial u}$$

$$= \frac{\partial^2 f}{\partial x^2} + 2\frac{\partial^2 f}{\partial x \partial y} + 2\frac{\partial^2 f}{\partial y \partial x} + 4\frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2 f}{\partial u^2} = \frac{\partial^2 f}{\partial x^2} + 4\frac{\partial^2 f}{\partial x \partial y} + 4\frac{\partial^2 f}{\partial y^2} \qquad (1)$$

$$\frac{\partial^2 f}{\partial v^2} = \frac{\partial}{\partial v}\left(\frac{\partial f}{\partial v}\right) = \frac{\partial}{\partial v}\left(\frac{\partial f}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial v}\right)$$

$$= \frac{\partial}{\partial v}\left(5\frac{\partial f}{\partial x} - 3\frac{\partial f}{\partial y}\right)$$

$$= \frac{\partial}{\partial x}\left(5\frac{\partial f}{\partial x} - 3\frac{\partial f}{\partial y}\right)\frac{\partial x}{\partial v} + \frac{\partial}{\partial y}\left(5\frac{\partial f}{\partial x} - 3\frac{\partial f}{\partial y}\right)\frac{\partial y}{\partial v}$$

$$= 5\left(5\frac{\partial^2 f}{\partial x^2} - 3\frac{\partial^2 f}{\partial x \partial y}\right) - 3\left(5\frac{\partial^2 f}{\partial y \partial x} - 3\frac{\partial^2 f}{\partial y^2}\right)$$

$$= 25\frac{\partial^2 f}{\partial x^2} - 15\frac{\partial^2 f}{\partial x \partial y} - 15\frac{\partial^2 f}{\partial y \partial x} + 9\frac{\partial^2 f}{\partial y^2}$$

$$= 25\frac{\partial^2 f}{\partial x^2} - 30\frac{\partial^2 f}{\partial x \partial y} + 9\frac{\partial^2 f}{\partial y^2} \qquad (2)$$

$$\frac{\partial^2 z}{\partial u \partial v} = \frac{\partial}{\partial u}\left(\frac{\partial z}{\partial v}\right) = \frac{\partial}{\partial u}\left(5\frac{\partial z}{\partial x} - 3\frac{\partial z}{\partial y}\right)$$

$$= \frac{\partial}{\partial x}\left(5\frac{\partial z}{\partial x} - 3\frac{\partial z}{\partial y}\right)\frac{\partial x}{\partial v} + \frac{\partial}{\partial y}\left(5\frac{\partial z}{\partial x} - 3\frac{\partial z}{\partial y}\right)\frac{\partial y}{\partial v}$$

$$= 5\frac{\partial}{\partial x}\left(5\frac{\partial z}{\partial x} - 3\frac{\partial z}{\partial y}\right) - 3\frac{\partial}{\partial y}\left(5\frac{\partial z}{\partial x} - 3\frac{\partial z}{\partial y}\right)$$

$$= 25\frac{\partial^2 z}{\partial x^2} - 15\frac{\partial^2 z}{\partial x \partial y} - 15\frac{\partial^2 z}{\partial y \partial x} + 9\frac{\partial^2 z}{\partial y^2}$$

$$\frac{\partial^2 z}{\partial u \partial v} = 25\frac{\partial^2 z}{\partial x^2} - 30\frac{\partial^2 z}{\partial x \partial y} + 9\frac{\partial^2 z}{\partial y^2} \quad (3)$$

$$(1)\times 4 + (3)\times 2 + (2) =$$

$$4\frac{\partial^2 z}{\partial x^2} + 16\frac{\partial^2 z}{\partial x \partial y} + 16\frac{\partial^2 z}{\partial y^2} \qquad (1)\times 4$$

$$50\frac{\partial^2 z}{\partial x^2} - 60\frac{\partial^2 z}{\partial x \partial y} + 18\frac{\partial^2 z}{\partial y^2} \qquad (3)\times 2$$

$$25\frac{\partial^2 z}{\partial x^2} - 30\frac{\partial^2 z}{\partial x \partial y} + 9\frac{\partial^2 z}{\partial y^2} \qquad (2)$$

sumas:

$$4\frac{\partial^2 z}{\partial u^2} + 2\frac{\partial^2 z}{\partial u \partial v} + \frac{\partial^2 z}{\partial v^2} = 79\frac{\partial^2 z}{\partial x^2} - 74\frac{\partial^2 z}{\partial x \partial y} + 43\frac{\partial^2 z}{\partial y^2} = 0$$

revisar!

a) $f(x,y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$

$$\left.\begin{aligned} \frac{\partial f}{\partial x} &= 3x^2 + 3y^2 - 6x = 0 \\ \frac{\partial f}{\partial y} &= 6xy - 6y = 0 \end{aligned}\right\} \quad \left.\begin{aligned} x^2+y^2-2x &= 0 \\ (x-1)y &= 0 \end{aligned}\right\}$$

$y = 0$
$x = 1$

si $y=0$ $\quad x^2 - 2x = 0$

$x = 2$ y $x = 0$ $\quad A(0,0)$ y $B(2,0)$

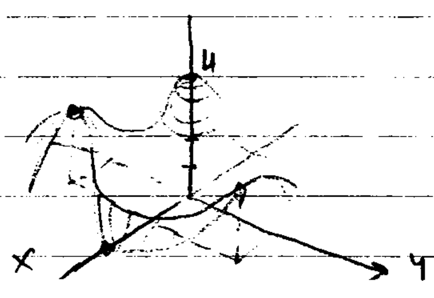
si $x = 1$ $\quad y^2 = 1$ $\quad C(1,1)$ y $D(1,-1)$

$y = \pm 1$

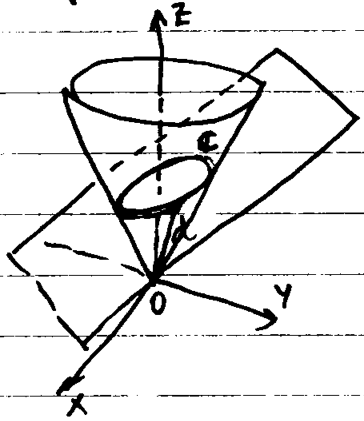
4 puntos críticos

Hessiano

	A(0,0)	B(2,0)	C(1,1)	D(1,-1)
$\frac{\partial^2 f}{\partial x^2} = 6x - 6$	-6	$+6$	0	0
$\frac{\partial^2 f}{\partial y^2} = 6x - 6$	-6	$+6$	0	0
$\frac{\partial^2 f}{\partial x \partial y} = 6y$	0	0	6	-6
$H(x,y) = 36(x-1)^2 - 6y$	36	36	-36	-36
	MÁX	mín	Silla	Silla
	$f_{máx} = 4$	$f_{mín} = 0$	$f = 2$	$f = 2$



plano $z = x+y-1$ y la superficie (cono) $x^2+y^2=z^2$



Solución. Se trata de

minimizar $f(x,y,z) = x^2+y^2+z^2$

con las restricciones

$$\varphi_1(x,y,z) = z-x-y+1=0$$
$$\varphi_2(x,y,z) = x^2+y^2-z^2=0$$

Lagrange: Se forma $w = f(x,y,z) + \lambda_1\varphi_1 + \lambda_2\varphi_2$

$$W = x^2+y^2+z^2+\lambda_1(z-x-y+1)+\lambda_2(x^2+y^2-z^2)$$

$$\begin{aligned}
\partial w/\partial x &= 2x - \lambda_1 + 2\lambda_2 x = 0\\
\partial w/\partial y &= 2y - \lambda_1 + 2\lambda_2 y = 0\\
\partial w/\partial z &= 2z + \lambda_1 - 2\lambda_2 z = 0\\
\varphi_1 &: \quad z-x-y+1 = 0\\
\varphi_2 &: \quad x^2+y^2-z^2=0
\end{aligned}$$

Incógnitas $x, y, z, \lambda_1, \lambda_2$

i) $(2+2\lambda_2)x - \lambda_1 = 0$

ii) $(2+2\lambda_2)y - \lambda_1 = 0$

de aquí $x=y$

iv) $z-x-y+1=0$

$$z-2y+1=0$$
$$z = 2y-1$$

v) $y^2+y^2-(2y-1)^2=0$

$$2y^2-4y^2+4y-1=0$$
$$2y^2-4y+1=0$$
$$y^2-2y+\tfrac{1}{2}=0$$

$$y = 1\pm\sqrt{1-\tfrac{1}{2}} = 1\pm\frac{\sqrt{2}}{2}$$

$$z = 2y-1$$
$$z = 2\pm\sqrt{2}-1$$
$$z = 1\pm\sqrt{2}$$

	A	B
$x=$	$1+\frac{\sqrt{2}}{2}$	$1-\frac{\sqrt{2}}{2}$
$y=$	$1+\frac{\sqrt{2}}{2}$	$1-\frac{\sqrt{2}}{2}$
$z=$	$1+\sqrt{2}$	$1-\sqrt{2}$

Hay una distancia máxima y una mínima

$A\left(1+\frac{\sqrt{2}}{2}, 1+\frac{\sqrt{2}}{2}, 1+\sqrt{2}\right)$ $B\left(1-\frac{\sqrt{2}}{2}, 1-\frac{\sqrt{2}}{2}, 1-\sqrt{2}\right)$

$OA = 11,66$ m.

$OB = 0,34$ m.