



# Characteristics of developing waves as a function of atmospheric conditions, water properties, fetch and duration

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## ABSTRACT

For any specific wind speed, waves grow in period, height and length as a function of the wind duration and fetch until maximum values are reached, at which point the waves are considered to be fully developed. Although equations and nomograms exist to predict the parameters of developing waves for shorter fetch or duration conditions at different wind speeds, these either do not incorporate important variables such as the air and water temperature, or do not consider the combined effect of fetch and duration. Here, the wind conditions required for a fully developed sea are calculated from maximum wave heights as determined from the wind speed, together with a published growth law based on the friction velocity. This allows the parameters of developing waves to be estimated for any combination of wind velocity, fetch and duration, while also taking account of atmospheric conditions and water properties.

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## 1. Introduction

Estimation of long-term wave conditions is of vital importance in ocean and coastal engineering, whereas the forecast of short-term conditions is critical for maritime activities. While fully developed sea conditions (*FDS*) are often assumed in long-term forecasts, such a state is not necessarily reached in enclosed or semi-enclosed water bodies such as lakes or bays, where a short fetch will limit wave development. This is particularly true of storms, because higher wind speeds require a longer fetch to produce *FDS* waves. For short-term forecasts, the wind duration also plays an important role both on the open sea and in smaller water bodies. Developing waves may be more hazardous than in their fully developed state, because in spite of being lower they are normally much steeper, which poses a danger especially for smaller vessels.

Although much work has been done on wave growth as a function of wind speed, fetch or duration (e.g. Inoue, 1967; Barnett, 1968; Bunting, 1970; Hasselmann et al., 1973; Toba, 1978; Forristall et al., 1978; Resio and Vincent, 1979; Resio, 1981, 1987, 1988; Kahma, 1981; Kitaigorodskii, 1983; Hasselmann et al., 1985; Resio and Perrie, 1989; Cardone, 1992; Van Vledder and Holthuisjen, 1993; Demirbilek et al., 1993), a simple model taking account of all three variables at the same time has been lacking. In this paper, previous work on fully developed waves (Le Roux, 2007a,b, in press) is used together with a growth law based on the JONSWAP (Joint North Sea Wave Project) spectrum (Hasselmann et al., 1973; Demirbilek et al., 1993; Resio et al., 2003) to develop such a method.

## 2. Atmospheric conditions and water properties

The characteristics of gravity waves are not only a function of the wind conditions, but also depend on the physical properties of the water and air, in particular the difference between the water and air temperature (Geernaert et al., 1986). This difference ( $\Delta^\circ\text{C}$ ) has a direct influence on the drag force generated by wind friction on the water surface, so that it is necessary to determine the drag for different atmospheric conditions. The wind friction velocity  $U_{a^*}$  (the subscripts <sub>a</sub> and <sub>w</sub> referring to air and water, respectively) is estimated by Demirbilek et al. (1993) as follows:

$$U_{a^*} = \sqrt{(C_{da} U_a^2)} \quad (1)$$

where the dimensionless wind drag coefficient  $C_{da}$  is given by

$$C_{da} = 0.001(1.1 + 0.035U_a) \quad (2)$$

Eq. (2) gives an approximation of the drag coefficient for “normal” weather conditions, but does not take the air–water temperature difference ( $\Delta^\circ\text{C} = ^\circ\text{C}_a - ^\circ\text{C}_w$ ) into account. A more rigorous solution is given by a 3-D graph in Resio et al. (2003, Fig. II-2-5) that plots  $C_{da}$  as a series of curves against  $U_a$  for different values of  $\Delta^\circ\text{C}$  (Geernaert et al., 1986; Smith, 1988). This graph can be recast into the single equation:

$$C_{da} = \left( -1.7 \times 10^{-8} \Delta^\circ\text{C}^3 - 1.4 \times 10^{-6} \Delta^\circ\text{C}^2 - 3 \times 10^{-5} \Delta^\circ\text{C} + 0.001 \right) \exp \left[ U_a \left( -1.6 \times 10^{-6} \Delta^\circ\text{C}^3 + 2 \times 10^{-5} \Delta^\circ\text{C}^2 + 0.001 \Delta^\circ\text{C} + 0.0324 \right) \right] \quad (3)$$

At a wind speed of  $10 \text{ m s}^{-1}$ , an atmospheric pressure of 1010 mb, a relative humidity of 80%, and air and water temperatures of 20 and

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23 °C, respectively, the values from Eqs. (2) and (3) inserted into Eq. (1) coincide at 0.3808 m s<sup>-1</sup>. For practical reasons, this is taken here as the “normal condition” or NC.

Another factor probably playing a role in wave formation is the water and air density, as discussed below. The density of seawater depends on its temperature and salinity. For a normal seawater salinity of 35‰

$$\rho_w = 1000 + (-0.0051^\circ C_w^2 - 0.064^\circ C_w + 28.109) \text{ kg m}^{-3} \quad (4)$$

which yields 1023.9391 kg m<sup>-3</sup> for sea water at 23 °C.

Air density  $\rho_a$ , on the other hand, is a function of its temperature and relative humidity  $\Gamma$ , as well as the isobaric pressure  $P_a$ , vapor pressure  $P_v$  and saturated vapor pressure  $P_{vs}$  in millibar (mb).

$$\rho_a = 1000\{P_a/[2870.5(273.15 + ^\circ C_a)] - P_v/[4614.95(273.15 + ^\circ C_a)]\} \text{ kg m}^{-3} \quad (5)$$

where

$$P_v = (\Gamma/100)P_{vs} \text{ and } P_{vs} = 6.1078 \times 10^{7.5^\circ C_a / (237.3 + ^\circ C_a)} \quad (6)$$

For the NC,  $\rho_a$  works out at 1.18643 kg m<sup>-3</sup>.

### 3. Wave height as related to wind speed and the air/water density ratio

Air density probably plays a role in wave height in that the wind energy ( $E_a$ ) would increase with a higher  $\rho_a$  given the same wind speed  $U_a$ , because the total energy of any moving fluid according to the Bernoulli equation is given by

$$E_a = 0.5\rho_a U_a^2 + \rho_a gh + P_a \quad (7)$$

where the first and second terms in Eq. (7) refer to the kinetic and potential energies, respectively, and  $h$  is a length term. Both the kinematic and potential energies of the wind must therefore increase with the air density, whereas Eq. (5) shows that  $P_a$  is also directly proportional to  $\rho_a$ .

The total wave energy  $E_w$  per unit length of wave crest, on the other hand, is the sum of the kinetic energy and potential energy given by (Demirbilek and Vincent, 2002)

$$E_w = \rho_w g H_o^2 L_o / 16 + \rho_w g H_o^2 L_o / 16 = \rho_w g H_o^2 L_o / 8 \quad (8)$$

where  $H$  is the wave height and  $L$  the wavelength, the subscript  $o$  referring to the deepwater condition. The potential energy results from that part of the fluid mass being above the still water level (SWL), thus being directly related to the wave height, whereas the kinetic energy is due to water particle velocities associated with wave motion (Demirbilek and Vincent, 2002). Eq. (8) shows that the total wave

energy increases with the wave height and length as well as the water density. It follows that, in the case of wind energy being transferred to the waves, more energy would be required to reach the same wavelength and height in water with a higher density than the other way round. Considering Eq. (7), the wave height should therefore be directly proportional to the ratio  $\rho_a U_a^2 / \rho_w g$  (the gravity acceleration being added to maintain the dimensional correctness of this ratio).

Le Roux (in press), based on Demirbilek et al. (1993) and Resio et al. (2003), showed that the fully developed deepwater wave height is given by

$$H_o = g T_w^2 / 18\pi^2 \quad (9)$$

where  $T_w$  is the fully developed wave period obtained from

$$T_w = 2\pi U_a / g \quad (10)$$

Setting  $C_1 \rho_a U_a^2 / \rho_w g = g T_w^2 / 18\pi^2$  (where  $C_1$  is a dimensionless proportionality constant) and replacing  $T_w$  by  $2\pi U_a / g$ , this yields

$$H_o = 2C_1 \rho_a U_a^2 / 9g\rho_w \quad (11)$$

For the “normal condition” or NC,  $C_1$  has a value of 863.042. Rearranged,  $C_1(\rho_a / \rho_w) = 9gH_o / 2U_a^2 = 1$ , where the second term is an inverted wave Froude number.

Table 1 compares the values of  $H_o$  and  $T_w$  as obtained from Eqs. (11) and (10) with the nomograms of Resio et al. (2003) and an equation of Demirbilek et al. (1993).  $T_w$  as calculated here corresponds very well with the average between the nomogram values of Resio et al. (2003) and the equation of Demirbilek et al. (1993), whereas  $H_o$  agrees exactly with the equation of Demirbilek et al. (1993), but somewhat underestimates the nomogram values of Resio et al. (2003).

### 4. Fetch and duration as a function of the fully developed wave height

Waves grow in height and length not only in relation to the velocity  $U_a$  of the wind, but also to its duration  $T_a$  and fetch  $F$ , the latter being defined as the distance that the wind blows over open water without a significant change in direction (<15°) or sustained speed (<2.5 m s<sup>-1</sup>).

Demirbilek et al. (1993) proposed an equation modeling wave growth with fetch based on the JONSWAP data:

$$gH_{FL} / U_a^{2*} = 0.0413 (gF / U_a^{2*})^{1/2} \quad (12)$$

where  $H_{FL}$  is a fetch-limited, energy-based significant wave height.

**Table 1**

Comparison of wave heights and periods under fully developed sea conditions at different wind velocities as predicted Eqs. (9) and (11), nomograms in Resio et al. (2003) (DBT), and Eqs. (12) of Demirbilek et al. (1993) (DBT) and (14) of Resio et al. (2003) (RBT)

$U_a$ m s <sup>-1</sup>	$U_a^*$ m s <sup>-1</sup>	$T_w$ s	$T_w$ s	$T_w$ s	$H_o$ m	$H_o$ m	$H_o$ m	$F_{FDS}$ m	$T_{aFDS}$ s
		This paper	Nomogram RBT	Eq. (18) DBT	This paper	Eq. (12) DBT	Nomogram RBT	This paper	This paper
2.5	0.0851	1.6	1.6	1.6	0.14	0.14	0.14	15,566	21,974
5	0.1768	3.2	3.3	3.1	0.57	0.57	0.60	59,780	42,233
7.5	0.2756	4.8	5.1	4.6	1.27	1.27	1.35	122,129	58,645
10	0.3808	6.4	6.6	6.1	2.27	2.27	2.45	204,375	74,216
12.5	0.4937	8.0	8.4	7.5	3.54	3.54	4.13	295,699	87,067
15	0.6148	9.6	11.1	8.9	5.10	5.10	6.14	395,769	98,286
17.5	0.7445	11.2	11.8	10.2	6.94	6.94	8.25	499,756	107,727
20	0.8832	12.8	13.0	11.5	9.06	9.06	11.51	605,212	115,619

$U_a^*$  was calculated for both sets of equations using Eqs. (1), (3) and (4)–(6) with water and air temperatures of 23 and 20 °C, respectively, a water salinity of 35‰, a relative air humidity of 80% and a barometric pressure of 1010 mb.

Eq. (12) can be rearranged to yield  $F$  for  $FDS$  conditions ( $F_{FDS}$ ) at any particular wind speed, because  $H_{FL}$  can be replaced by  $H_o$ , whereas the latter can be calculated by Eq. (11). This gives

$$F_{FDS} = gH_o^2 / 1.70569 \times 10^{-3} U_{a*}^2 \quad (13)$$

where  $U_{a*}$  is obtained from Eqs. (1) and (3). For the  $NC$ ,  $H_o = 2.27$  m and  $F_{FDS}$  would be 204,375 m.

For any combination of wind speed and fetch, the time required for waves to become fetch-limited ( $T_{aFL}$ ) is given in Resio et al. (2003) as

$$T_{aFL} = [(gF) / (0.00523 U_{a*}^2)]^{2/3} (U_{a*} / g) \quad (14)$$

Eq. (14) can be used to determine the limiting wind duration at  $FDS$  conditions ( $T_{aFDS}$ ) by replacing  $F$  with  $F_{FDS}$  as obtained from Eq. (13), which would give 74,216 s or 20.6 h for the  $NC$ .

For a fetch or wind duration shorter than the  $F_{FDS}$  or  $T_{aFDS}$  corresponding to any particular wind speed, the waves would still be growing towards their  $FDS$  condition and the wave celerity  $C_o$  would be less than  $U_a$ , so that there would be excess wind shear and the wind velocity at 10 m would be higher than at the  $SWL$ . It can also be expected that at higher wind velocities, where an increasing proportion of the wind energy is expended in breaking wave crests and spray production, the wind velocity would always exceed the wave celerity. When this happens, Eqs. (13) and (14) probably become invalid and the maximum required fetch and wind duration for  $FDS$  conditions may level off. Gross and Gross (1996) noted, for example, that after about 30 h there is very little increase in wave growth for any wind condition regardless of fetch.

The condition at which wind energy becomes increasingly expended in breaking wave crests and spray at the expense of wave growth, is approximated by observations and theoretical modeling. According to the Beaufort wind scale (see, e.g., internet site [http://www.wikipedia.org/wiki/Beaufort\\_scale](http://www.wikipedia.org/wiki/Beaufort_scale)), foam begins to streak at wind speeds exceeding  $17.2 \text{ m s}^{-1}$ , but considerable spray only develops at the transition from gales to strong gales at  $20.8 \text{ m s}^{-1}$ , where the wave crests also begin to roll over. This is supported by Bye and Jenkins (2006), who theoretically modeled the drag coefficient for momentum transfer ( $C_{da}$ ) of an air–sea system with shear layers in both fluids. They concluded that the maximum reached by  $C_{da}$  under any wind speed is 0.002. According to Eqs. (1) and (3) at the  $NC$ , this value coincides with a wind speed of  $20.9 \text{ m s}^{-1}$ , which is considered here to be the maximum wind speed for which the equations in this paper are still valid. Inserted into Eqs. (8) and (9), respectively, together with the calculated friction velocity of  $0.9347 \text{ m s}^{-1}$ , this corresponds to a wave period of 13.39 s and wave height  $H_o$  of 9.90 m, which used with Eqs. (13) and (14) yield a maximum fetch and duration of 645,201 m and 118,399 s (32.89 h), respectively.

### 5. Characteristics of developing waves

The considerations above are valid for  $FDS$  conditions where the wind blows for an unlimited time over an unlimited fetch. However, there are many situations where the fetch is limited (denoted by the subscript  $FL$ ), for example in the case of lakes or narrow sea straits. Similarly, the wind may blow for only a few hours (time-limited conditions, denoted by  $TL$ ), in which case the resultant waves will not reach their full height even with an unlimited fetch.

#### 5.1. Height of developing waves

To estimate the height of developing waves, the actual fetch and duration can be added as ratios of the maximum fetch and duration as calculated by Eqs. (13) and (14) for the particular wind speed, thus maintaining the dimensional correctness of Eq. (11). Comparing this approach with the wave heights given by Eq. (12) of Demirbilek et al.

**Table 2**

Comparison of wave heights at different wind velocities for fetch- or duration-limited conditions as predicted by the equations proposed here, Eq. (12) of Demirbilek et al. (1993) (DBT) and a nomogram in Resio et al. (2003, Fig. II-2-25) (RBT)

$U_a$ m s <sup>-1</sup>	$F$ (km)	$H_{FL}$ m This paper	$H_{FL}$ m Eq. (12) DBT	$U_a$ m s <sup>-1</sup>	$T_a$ (h)	$H_{TL}$ m This paper	$H_{TL}$ m Nomogram RBT
2.5	5	0.08	0.08	5	5	0.30	0.30
5	7	0.19	0.20	5	10	0.50	0.50
7.5	3	0.20	0.20	7.5	9	0.82	0.80
7.5	60	0.89	0.89	10	7	1.01	1.00
10	40	1.00	1.00	12.5	4	0.92	0.90
12.5	6	0.50	0.50	15	8	2.03	2.00
12.5	100	2.06	2.06	15	20	4.04	4.00
15	10	0.81	0.81	17.5	6	2.08	2.00
17.5	5	0.69	0.69	17.5	10	3.05	3.00
20	70	3.08	3.08	20.0	8	3.19	3.00

$U_{a*}$  was calculated in the same way as for Table 1.

(1993), it turns out that if the wave height as obtained from Eq. (11) is multiplied by the square root of the ratio between the actual fetch and the limiting fetch for the specific wind speed, the resulting wave heights correspond almost exactly to that given by Eq. (12) for any wind velocity (Table 2). Therefore the fetch-limited wave height is given by

$$H_{FL} = 863.042 (\rho_a / \rho_w) (2U_a^2 / 9g) (F / F_{FDS})^{1/2} \quad (15)$$

Similarly, evaluating the effect of wind duration on the wave height against the nomogram in Resio et al. (2003, Fig. II-2-25) shows that the duration-limited wave height can be obtained by

$$H_{TL} = 863.042 (\rho_a / \rho_w) (2U_a^2 / 9g) (T_a / T_{aFDS})^{3/4} \quad (16)$$

Table 2 compares the values of Eq. (12) and the nomogram with the results of Eqs. (15) and (16).

The wave height for any combination of fetch and wind duration can therefore be calculated by

$$H_{FTL} = 863.042 (\rho_a / \rho_w) (2U_{a10}^2 / 9g) (F / F_{FDS})^{1/2} (T_a / T_{aFDS})^{3/4} \quad (17)$$

where the subscript  $FTL$  indicates both fetch- and time-limited conditions.

The advantage of Eq. (17), in comparison with Eqs. (12) and (14), is that it represents a combination of fetch and duration, whereas the latter two equations cannot be combined. For example, for the  $NC$ , a fetch of 100 km and wind duration of 10 h, Eq. (12) would yield a wave height of 1.59 m, whereas Eq. (14) would indicate that 12.8 h are required to reach this fetch-limited height. However, it cannot compute the actual fetch- and duration-limited wave height ( $H_{FTL}$ ) after 10 h. Eq. (17) in this case yields a wave height of 0.92 m. A second advantage is that the maximum wave height is limited by Eq. (17) to that given by Eqs. (9) or (11), whereas Eq. (12) simply increases with the fetch. For a fetch of 1000 km it would give a wave height of 5.02 m for the  $NC$ , for example, more than twice the maximum height of 2.27 m for  $FDS$  conditions. Demirbilek et al. (1993) calculate the maximum wave height for a  $FDS$  by  $H_o = 211.5 U_{a*}^2 / g$ , which in this case would be 3.13 m. This still exceeds the value of 2.27 m given by Eq. (11), for example, as well as the value shown by the nomogram (Fig. II-2-25) of Resio et al. (2003).

#### 5.2. Period of developing waves

Demirbilek et al. (1993) developed an equation to determine the wave period with growing fetch, given by:

$$T_{wFL} = 0.651 (gF / U_{a*}^2)^{1/3} (U_{a*} / g) \quad (18)$$

However, Eq. (18) does not correlate very well with Eq. (10). For the  $NC$ , it would indicate a wave period of 6.07 s for a  $FDS$ , with a

corresponding wave height of 2.03 m according to Eq. (9), which is lower than the maximum *FDS* height of 2.27 m (a difference of 10.6%).

An alternative method is to express  $T_{wFL}$  as a function of the fully developed period  $T_w$ , the wave height  $H_o$ , as well as the fetch and duration ratios. This can be done by first rearranging Eq. (9):

$$T_w = (18\pi^2 H_o/g)^{1/2}. \quad (19)$$

Eq. (19) gives exactly the same values as Eq. (10) for *FDS* conditions and compares reasonably well with those given by Eq. (18) if the same fetch is used for wind velocities up to  $20 \text{ m s}^{-1}$  (Table 1). At a wind speed of  $20.9 \text{ m s}^{-1}$ , Eqs. (9) and (19) yield a wave height and period of 9.90 m and 13.39 s, respectively, which agree well with observed 5-year maximum wave heights of 10.4 m and peak periods of 14.1 s in the Pacific Ocean (Resio et al., 2003, Table I-2-5). Eq. (18) in this case would underestimate the wave period somewhat at 12.0 s ( $F_{FDS}=645,201 \text{ m}$ ,  $U_{a^*}=0.9347 \text{ m s}^{-1}$ ).

The wave period can also be evaluated for both fetch- and duration-limited conditions by comparison with Figs. II-2-24 and II-2-26 of Resio et al. (2003). This yields:

$$T_{wFTL} = (18\pi^2 H_o/g)^{1/2} (F/F_{FDS})^{2/5} (T_a/T_{aFDS})^{5/9}. \quad (20)$$

Table 3 shows an excellent correlation, with the difference never exceeding 0.4 s. However, the advantage of Eq. (20) is again that a combination of both fetch and duration can be used, whereas the nomograms in Resio et al. (2003, Figs. II-2-24 and II-2-26) or Eq. (18) can be used for either of the two, but not both.

### 5.3. Length of developing waves

The wavelength of fully developed deepwater waves is given by the standard Airy equation  $L_o = gT_w^2/2\pi$ . Le Roux (2007a) demonstrated that the fully developed wavelength is also related to the fully developed wave height by  $L_o = 9\pi H_o$ . Multiplying these two terms expresses  $L_o$  in terms of the wave period and height:  $L_o^2 = (gT_w^2/2\pi)(9\pi H_o)$ . Because both the period and height of developing waves can be calculated by the equations proposed above, the wavelength should therefore be related to these parameters in the form

$$L_{FTL} = 3\sqrt{(H_{FTL}gT_{wFTL}^2/2)}. \quad (21)$$

Eq. (21) yields exactly the same wavelength in deepwater conditions for fully developed waves as the standard Airy equation. However, for developing waves, it gives longer wavelengths for the same wave period. A wind speed of  $7.8 \text{ m s}^{-1}$ , for example, would correspond to a fully developed wave period of 5 s, with  $H_o = 1.38 \text{ m}$  and  $L_o = 39.03 \text{ m}$ . For a  $15 \text{ m s}^{-1}$  wind speed and a limited duration of

**Table 3**

Comparison of wave periods at different wind velocities for fetch- or duration-limited conditions as predicted by the equations proposed here and nomograms in Resio et al. (2003, Figs. II-224 and II-2-26) (RBT)

$U_a$ $\text{m s}^{-1}$	$F$ (km)	$T_{wFL}$ s		$U_a$ $\text{m s}^{-1}$	$T_a$ (h)	$T_{wTL}$ s	
		This paper	Nomogram RBT			This paper	Nomogram RBT
7.5	10	1.8	2.0	5	10	2.9	3.0
7.5	80	4.0	4.0	7.5	3	1.9	2.0
10	60	3.9	4.0	7.5	7	3.0	3.0
10	200	6.4	6.0	10	5	2.9	3.0
12.5	20	2.7	3.0	10	9	4.0	4.0
12.5	90	5.0	5.0	12.5	4	2.9	3.0
15	200	7.3	7.0	12.5	7	4.0	4.0
17.5	30	3.6	4.0	15	9	5.2	5.0
17.5	60	4.8	5.0	17.5	6	4.6	5.0
20	50	4.7	5.0	20.0	4	4.0	4.0

$U_{a^*}$  was calculated in the same way as for Table 1.

**Table 4**

Development of wave parameters with fetch under NC (see text for details)

$F$ (km)	$H_{FL}$ (m)	$L_{FL}$ (m)	$T_{wFL}$ (s)	$C_{wFL}$ ( $\text{m s}^{-1}$ )	Steepness ( $H_{FL}/L_{FL}$ )
20	0.71	14.16	2.53	5.6	0.0501
40	1	22.19	3.34	6.64	0.0451
60	1.23	28.95	3.93	7.37	0.0425
80	1.42	34.91	4.41	7.92	0.0407
100	1.59	40.38	4.82	8.38	0.0394
120	1.74	45.40	5.18	8.76	0.0383
140	1.88	50.20	5.51	9.11	0.0374
160	2.01	54.82	5.82	9.42	0.0367
180	2.13	59.15	6.1	9.7	0.0360
200	2.25	63.39	6.36	9.97	0.0355
204,375	2.27	64.05	6.40	10	0.0354

**Table 5**

Development of wave parameters with wind duration under NC (see text for details)

$T_a$ (h)	$H_{TL}$ (m)	$L_{TL}$ (m)	$T_{wTL}$ (s)	$C_{wTL}$ ( $\text{m s}^{-1}$ )	Steepness ( $H_{TL}/L_{TL}$ )
2	0.39	7.26	1.75	4.15	0.0537
4	0.66	13.93	2.58	5.41	0.0474
6	0.9	20.36	3.23	6.3	0.0446
8	1.12	26.65	3.79	7.02	0.0419
10	1.32	32.75	4.29	7.63	0.0403
12	1.51	38.78	4.75	8.17	0.0389
14	1.70	44.80	5.17	8.66	0.0379
16	1.88	50.74	5.57	9.10	0.0371
18	2.05	56.6	5.95	9.51	0.0362
20	2.22	62.37	6.3	9.90	0.0356
20.62	2.27	64.05	6.40	10	0.0354

8.4 h, the wave period and height according to Eqs. (20) and (17) would be 5 s and 2.11 m, respectively, corresponding to a deepwater wavelength of 48.26 m, which is considerably longer than for fully developed 5 s waves. This is to be expected, because the higher wind speed would impart more energy to the waves, which is manifested in an increased wavelength and height. The wave steepness given by  $H_o/L_o$ , however, is 0.0354 for a fully developed 5 s wave and 0.0437 for the developing 5 s wave. The wave steepness thus increases with a decrease in fetch and duration, until it reaches a maximum value when the wave will presumably break. This is in accordance with observations at sea that developing waves are shorter and steeper than fully developed waves (Resio et al., 2003).

Tables 4 and 5 show the growth in NC wave parameters with fetch and time, respectively, whereas Table 6 illustrates wave development under different combinations of fetch and time. In all cases, a potential curve of the form  $y = ax^b$  can be fitted to the data. From the tables it is clear that, for similar fetch and time increments, fetch-limited waves are higher during the initial stages of wave development than time-limited waves and also have higher initial wavelengths and celerities. Waves that are both fetch- and time-limited are significantly steeper for the same wave wind speed than either fetch- or time-limited

**Table 6**

Development of wave parameters with wind fetch and duration under NC (see text for details)

$F$ (km)	$T_a$ (h)	$H_{FTL}$ (m)	$L_{FTL}$ (m)	$T_{wFTL}$ (s)	$C_{wFTL}$ ( $\text{m s}^{-1}$ )	Steepness ( $H_{FTL}/L_{FTL}$ )
20	2	0.12	1.59	0.69	2.3	0.0755
40	4	0.29	4.83	1.35	3.58	0.06
60	6	0.49	9.20	1.98	4.64	0.0533
80	8	0.7	14.50	2.61	5.56	0.0483
100	10	0.92	20.58	3.23	6.37	0.0446
120	12	1.16	27.48	3.84	7.16	0.0421
140	14	1.41	35.12	4.45	7.89	0.0401
160	16	1.66	43.32	5.06	8.57	0.0383
180	18	1.93	52.25	5.66	9.23	0.0369
200	20	2.20	61.69	6.26	9.85	0.0357
204,375	20.62	2.27	64.05	6.40	10.00	0.0354



waves. For example, for a fetch of 50 km and wind duration of 12 h, NC waves have a steepness of 0.0455, decreasing to 0.039 over an unlimited fetch for the same wind duration.

In Table 6, wave steepness values of 0.0755 and 0.06 are indicated after 2 and 4 h on fetch distances of 20 and 40 km, respectively, but such waves will probably be breaking (whitecaps), so that the wave heights will be restricted to less than the calculated values.

## 6. Conclusions

The method developed here provides a convenient way to estimate the wave height, length and period for any combination of wind velocity, fetch, and duration. It incorporates atmospheric conditions such as air temperature, pressure, relative humidity and density, as well as the water temperature, salinity and density.

Applying the proposed equations to calculate the wave climate under different atmospheric conditions, leads to the following general conclusions:

- 1) Waves should be higher when the atmospheric pressure increases because of an increase in the total energy (Eq. (7)). For example, for a wind speed of  $10 \text{ m s}^{-1}$  and an atmospheric pressure of 1000 mb, water and air temperatures both being  $20^\circ\text{C}$  and the air humidity 80%, waves would be 2.24 m high as compared to 2.31 m at 1030 mb, a difference of 3.1%. This means that waves associated with anticyclones should be higher than those of cyclones, given the same wind speed. By the same reasoning, high-altitude water bodies such as Lake Titicaca in Bolivia should have lower waves because of the decrease in atmospheric pressure and air density, given the same temperature.
- 2) On fresh water lakes, waves should be higher for the same duration- and fetch-limited conditions than on the ocean, because of the lower water salinity and density. For the NC and a FDS, waves would be 2.27 m high in seawater and 2.33 m in fresh water, an increase of 2.6%. The opposite effect will be caused by the presence of a substantial amount of sediment in suspension, for example in large effluent plumes opposite river mouths such as the Amazon, where wave heights should decrease because of an increase in water density. This is supported by the work of Camfield (1977) on the large dissipation rate of wave energy in the presence of extreme amounts of sediment or plant material in the water column.
- 3) Developing waves are shorter and steeper than fully developed waves for the same wind speed, which suggests that the most hazardous conditions for small vessels will be found on lakes or in narrow sea straits after the first hours of stormy conditions. The procedure outlined above allows such conditions to be calculated, so that certain maritime activities can be restricted during these periods.

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