Attitudes of Lines and Planes

This chapter is concerned with the orientations of lines and planes. The structural elements that we measure in the field are mostly lines and planes, and manipulating these elements on paper or on a computer screen helps us visualize and analyze geologic structures in three dimensions. In this chapter we will examine several graphical and mathematical techniques for solving apparent-dip problems. Each technique is appropriate in certain circumstances. The examination of various approaches to solving such problems serves as a good introduction to the techniques of solving structural problems in general. Finally, many of these problems are designed to help you visualize structural relations in three dimensions, a critical skill for the structural geologist.

The following terms are used to describe the orientations of lines and planes. All of these are measured in degrees, so values must be followed by the ° symbol.

Attitude   The orientation in space of a line or plane. By convention, the attitude of a plane is expressed as its *strike* and *dip*; the attitude of a line is expressed as *trend* and *plunge*.

Bearing   The horizontal angle between a line and a specified coordinate direction, usually true north or south; the compass direction or azimuth.

Strike   The bearing of a horizontal line contained within an inclined plane (Fig. 1.1). The strike is a line of equal elevation on a plane. There are an infinite number of parallel strike lines for any inclined plane.

Dip   The vertical angle between an inclined plane and a horizontal line perpendicular to its strike. The direction of dip can be thought of as the direction water would run down the plane (Fig. 1.1).

Trend   The bearing (compass direction) of a line (Fig. 1.2). Non-horizontal lines trend in the down-plunge direction.

Plunge   The vertical angle between a line and the horizontal (Fig. 1.2).

Pitch   The angle measured *within* an inclined plane between a horizontal line and the line in question (Fig. 1.3). Also called **rake**.
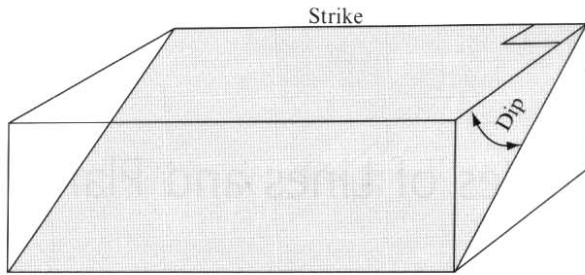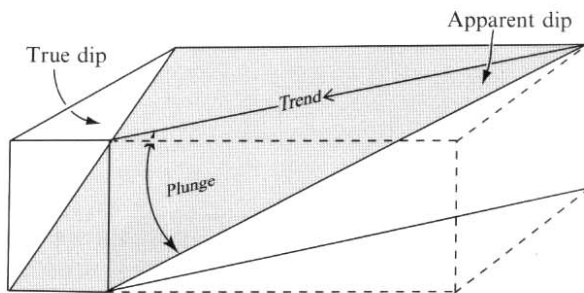
Fig. 1.1 Strike and dip of a plane.

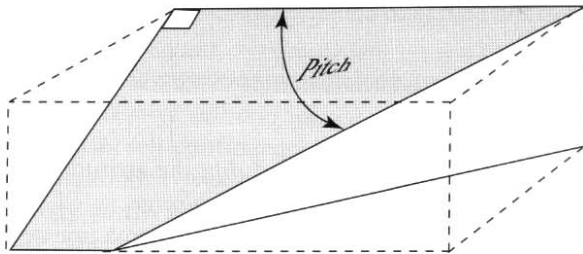**Fig. 1.2** Trend and plunge of an apparent dip.

Fig. 1.3 Pitch (or rake) of a line in an inclined plane.

Apparent dip The vertical angle between an inclined plane and a horizontal line that is not perpendicular to the strike of the plane (Fig. 1.2). For any inclined plane (except a vertical one), the true dip is always greater than any apparent dip. Note that an apparent dip may be defined by its trend and plunge *or* by its pitch within a plane.

There are two ways of expressing the strikes of planes and the trends of lines (Fig. 1.4). The *azimuth* method is based on a 360° clockwise circle; the *quadrant* method is based on four 90° quadrants. A plane that strikes northwest–southeast and dips 50° southwest could be described as 315°, 50°SW (azimuth) or N45°W, 50°SW (quadrant). Similarly, a line that trends due west and plunges 30° may be described as 30°, 270° (sometimes written as 30° → 270°) or 30°, N90°W. For azimuth notation, always use three digits (e.g., 008°, 065°, 255°) so that a bearing cannot be confused with a dip (one or two digits). In this book, the strike is given before the dip, and the plunge is given before the trend. To ensure that you become comfortable with both azimuth and quadrant notation, some examples and problems use azimuth and some use quadrant. However, we strongly recommend that you use the azimuth convention in your own work. It is much easier to make errors reading a bearing in quadrant notation (two letters and a number) than in azimuth notation (a single number). In addition, when entering orientation data into a computer program or spreadsheet file, it is much faster to enter azimuth notation because there are fewer characters to enter.

Notice that because the strike is a horizontal line, either direction may be used to describe it. Thus a strike of N45°W (315°) is exactly the same as S45°E (135°). In quadrant notation, the strike is
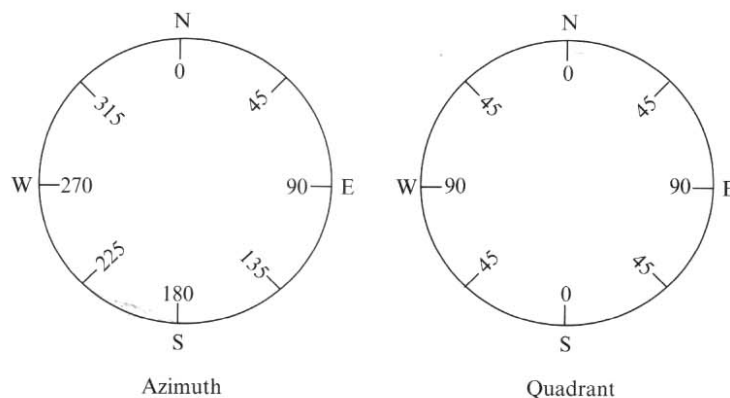

Fig. 1.4 Azimuth and quadrant methods of expressing compass directions.

commonly given in reference to north (N45°W rather than S45°E). In azimuth notation the "right-hand rule" is commonly followed. The right-hand rule states that you choose the strike azimuth such that the surface dips to your right. For example, the attitude of a plane expressed as 040°, 65°NW could be written as 220°, 65° using the right-hand rule convention because the 65°NW dip direction would lie to the right of the 220° strike bearing.

The dip, on the other hand, is usually not a horizontal line, so the down-dip direction must somehow be specified. The safest way is to record the compass direction of the dip. Because the direction of dip is always perpendicular to the strike, the exact bearing is not needed; the dip direction is approximated by giving the quadrant in which it lies or the cardinal point (north, south, east, or west) to which it most nearly points. If the right-hand rule is strictly followed, it is possible to specify the dip direction without actually writing down the direction of dip.

Solve Problems 1.1, 1.2, and 1.3.

Apparent-dip problems

There are many situations in which the true dip of a plane cannot be measured accurately in the field or cannot be drawn on a cross-section view. Any cross section not drawn perpendicular to strike displays an apparent dip rather than the true dip of a plane (except for horizontal and vertical planes).

Apparent-dip problems involve determining the attitude of a plane from the attitude of one or more apparent dips, or vice versa. The strike and dip of a plane may be determined from either: (1) the strike of the plane and the attitude of one apparent dip, or (2) the attitudes of two apparent dips.

There are four major techniques for solving apparent-dip problems. These are: (1) orthographic projection, (2) trigonometry, (3) alignment diagrams (nomograms), and (4) stereographic projection. Stereographic projection is described in Chapter 5. The other three techniques are discussed in this chapter.

Throughout this and subsequent chapters the following symbols will be used:

α (alpha) = plunge of apparent dip
β (beta) = angle between the strike of a plane and the trend of an apparent dip
δ (delta) = plunge of true dip
θ (theta) = direction (trend) of apparent dip.

Orthographic projection

One way to solve apparent-dip problems is to carefully draw a layout diagram of the situation. This technique, called orthographic projection, is more time-consuming than the other approaches, but it helps you to develop the ability to visualize in three dimensions and to draw precisely.

Example 1.1: Determine true dip from strike plus attitude of one apparent dip

Suppose that a quarry wall faces due north and exposes a quartzite bed with an apparent dip of 40°, N90°W. Near the quarry the quartzite can be seen to strike N25°E. What is the true dip?

Before attempting a solution, it is crucial that you visualize the problem. If you cannot draw it, then you probably do not understand it. Figure 1.5a shows the elements of this problem.
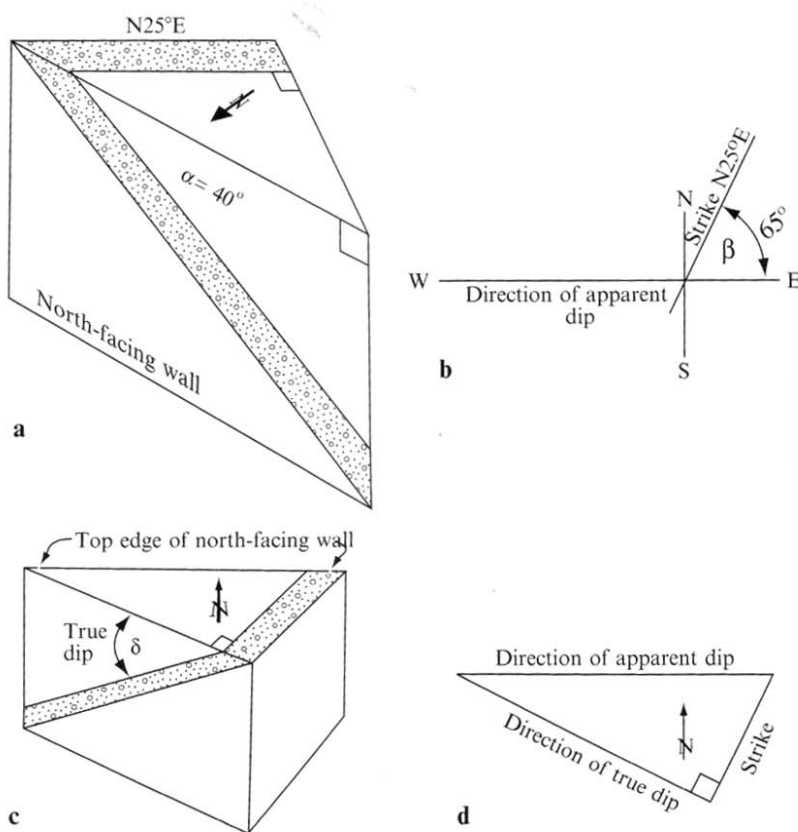
Fig. 1.5 Solution of Example 1.1. The dashed lines are fold lines. (a) Block diagram. (b) Step 1 of orthographic solution. (c) Block diagram looking north. (d) Orthographic projection of step 2. (e) Block diagram of step 3. (f) Orthographic projection of step 3. (g) Block diagram of step 4. (h) Orthographic projection of step 4.

Solution

1   Carefully draw the strike line and the direction of the apparent dip in plan (map) view (Fig. 1.5b).

2   Add a line for the direction of true dip. This can be drawn anywhere perpendicular to the strike line but not through the intersection between the strike and apparent-dip lines (Fig. 1.5c,d).

3   Now we have a right triangle, the hypotenuse of which is the apparent-dip direction. Imagine that you are looking down from space and that this hypotenuse is the top edge of the quarry wall. Now imagine folding the quarry wall up into the horizontal plane. This is done graphically by drawing another right triangle adjacent to the first (Fig. 1.5e,f). The apparent-dip angle, known to be 40° in this problem, is measured and drawn directly adjacent

to the direction of the apparent-dip line. Since the apparent dip is to the west, the angle opens to the west on the drawing. The line opposite angle α is of length d and represents the depth to the layer of interest at point P (Fig. 1.5f).

4   Finally, the direction of true dip is used as a fold line, and another line of length d is drawn perpendicular to it (Fig. 1.5g,h). The true dip angle δ is then formed by connecting the end of this new line to the strike line. Because the true dip is to the northwest, angle δ opens toward the northwest. Angle δ is measured directly off the drawing to be 43°.

If you have trouble visualizing this process, make a photocopy of Fig. 1.5h, fold the paper along the fold lines, and reread the solution to this problem.
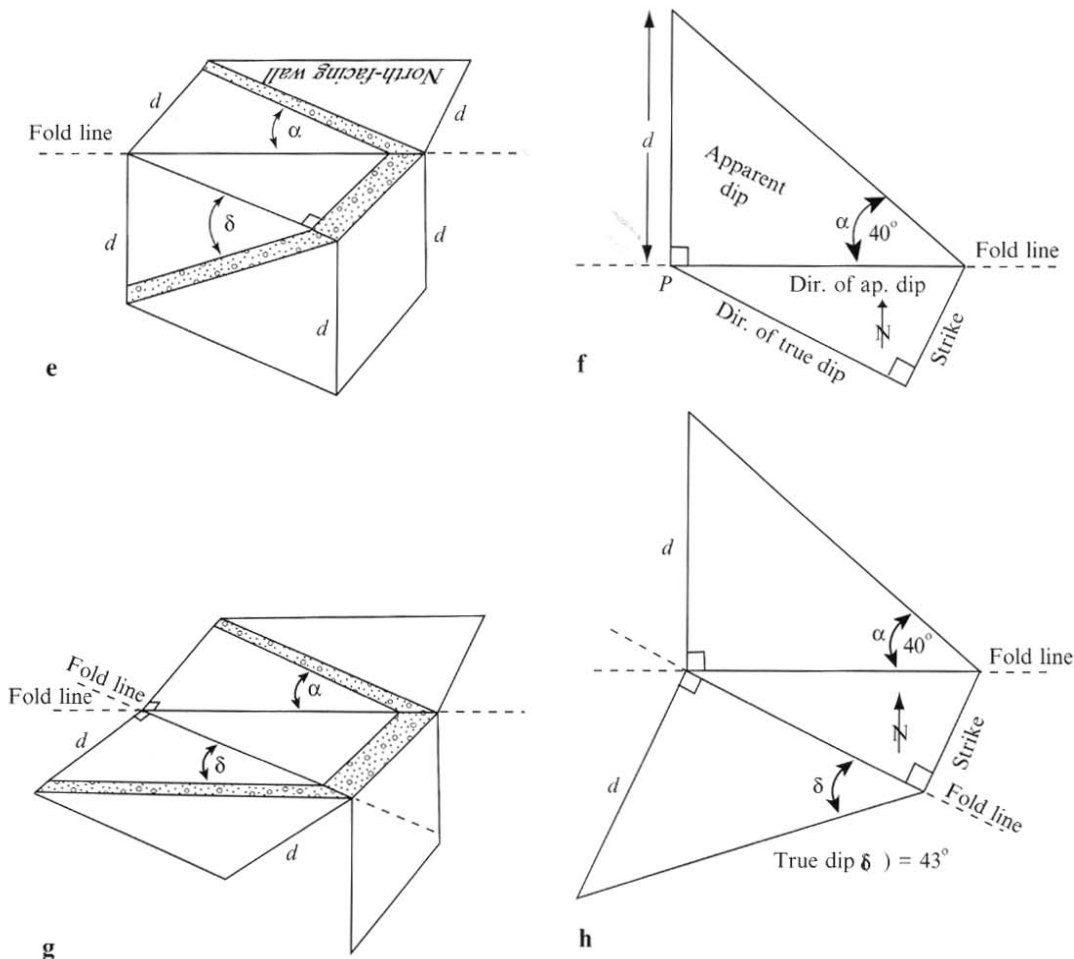
Solve Problems 1.4 and 1.5.

Fig. 1.5 (*Continued*)

Example 1.2: Determine strike and dip from two apparent dips

Suppose that a fault trace is exposed in two adjacent cliff faces. In one wall the apparent dip is 15°, S50°E, and in the other it is 28°, N45°E (Fig. 1.6a). What is the strike and dip of the fault plane?

Solution

1   Visualize the problem as shown in Fig. 1.6b. We will use the two trend lines, OA and OC, as fold lines. As in Example 1.1, we will use a vertical line of arbitrary length d. Draw the two trend lines in plan view (Fig. 1.6c).

2   From the junction of these two lines (point O) draw angles α_1 and α_2 (Fig. 1.6d). It does not
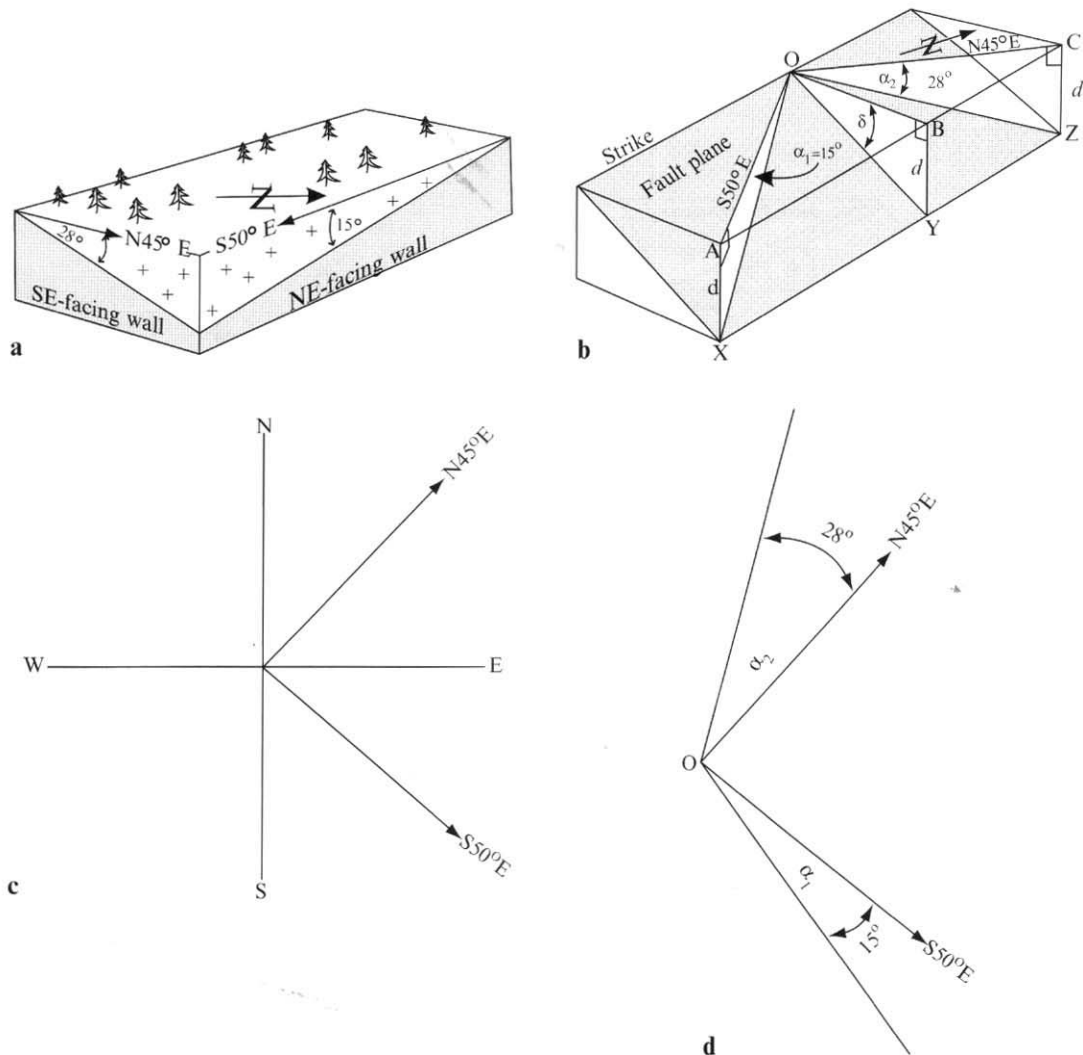

Fig. 1.6 Solution of Example 1.2. (a) Block diagram. (b) Block diagram showing triangles involved in orthographic projection and trigonometric solutions. (c) Step 1 of orthographic solution. (d) Step 2. (e) Step 3. (f) Step 4. (g) Steps 5 and 6.

really matter on which side of the trend lines you draw your angles, but drawing them outside the angle between the trend lines minimizes the clutter on your final diagram.

3 Draw a line of length d perpendicular to each of the trend lines to form the triangles COZ and AOX (Fig. 1.6e). Find these points on Fig. 1.6b. The value of d is not important, but it must always be drawn exactly the same length because it represents the depth to the layer along any strike line.

4 Figure 1.6e shows triangles COZ and AOX folded up into plan view with the two apparent-dip trend lines used as fold lines. As shown in Fig. 1.6b, line AC is horizontal and parallel to the fault plane; therefore it defines the fault's strike. We may therefore draw line AC on the diagram and measure its trend to determine the strike (Fig. 1.6f); it turns out to be N22°W.

5 Line OB is then added perpendicular to line AC; it represents the direction of true dip (Fig. 1.6g).

6 Using line OB as a fold line, triangle BOY (as shown in Fig. 1.6b) can be projected into the horizontal plane, again using length d to set the position of point Y (Fig. 1.6g). The true

dip δ can now be measured directly off the diagram to be 30°.
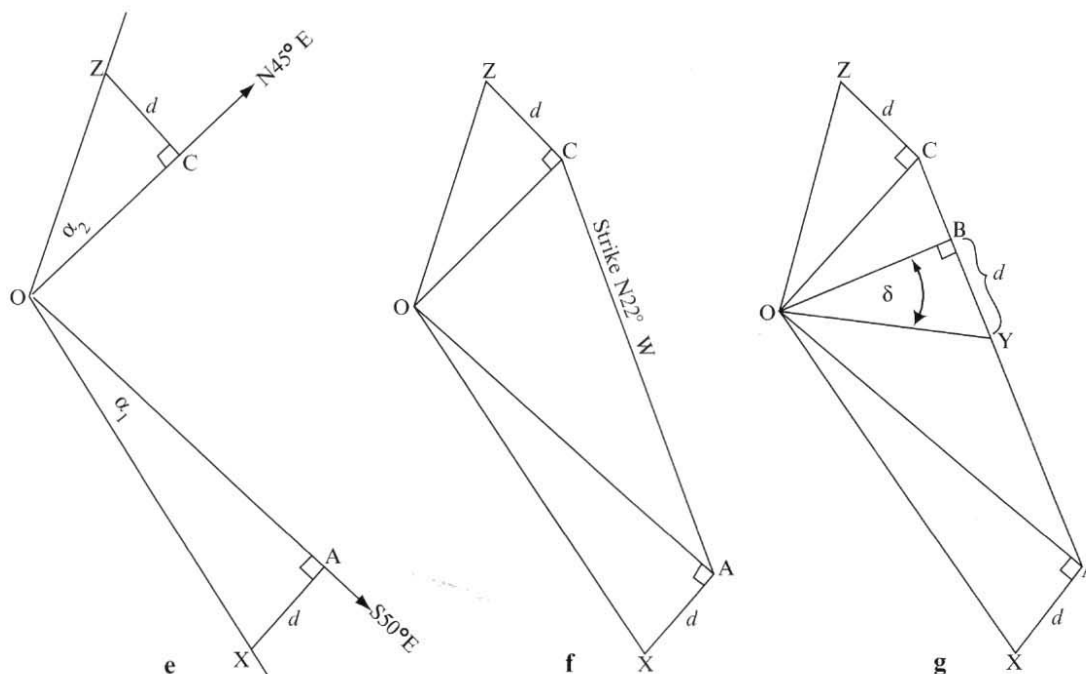
Solve Problems 1.6 and 1.7.

Fig. 1.6 (*Continued*)

Trigonometric solutions

Apparent-dip problems can be done much faster and more precisely trigonometrically, especially with a calculator. This method is particularly suitable when very small dip angles are involved. Even when the angles are not drawn orthographically, however, you should sketch a block diagram in order clearly to visualize the problem. Programs to solve apparent-dip problems on programmable calculators are discussed by De Jong (1975).

Refer to Fig. 1.6b for the following derivation:

$$AX = BY$$

$$\tan AOX = \frac{AX}{OA} = \frac{BY}{OA}$$

$$\tan AOX = \frac{BY}{OB(\sec AOB)}$$

$$\tan AOX = \frac{OB(\tan BOY)}{OB(\sec AOB)}$$

$$= \frac{\tan BOY}{\sec AOB}$$

$$= \tan BOY \cos AOB$$

or, using symbols,

$$\tan \alpha = (\tan \delta)(\cos \text{ angle between true-} \atop \text{and apparent-dip directions)} \quad (1.1)$$

or

$$\tan \delta = \frac{\tan \alpha}{\cos \text{ angle between true- and} \atop \text{apparent-dip directions}} \quad (1.2)$$

or

$$\tan \delta = \frac{\tan \alpha}{\sin \beta} \quad (1.3)$$

Example 1.3: Determine true dip from strike plus attitude of one apparent dip

Example 1.1 is a convenient problem of this type to solve trigonometrically. The strike of a bed is known to be 025° but we do not know the dip. An apparent dip is 40°, 270°, and angle β (between the strike of the bed and the trend of the apparent dip) is 65° (Fig. 1.5b).

$$\alpha = 40°, \tan 40° = 0.839$$
$$\beta = 65°, \sin 65° = 0.906$$

From equation 1.3,

$$\tan \delta = \frac{\tan \alpha}{\sin \beta} = \frac{0.839}{0.906} = 0.926$$
$$\delta = 42.8°$$

Example 1.4: Determine strike and dip from two apparent dips

Because two apparent dips with trend θ are involved, they will be labeled θ_1 and θ_2, which correspond to the two apparent-dip angles α_1 and α_2. θ_1 should represent the more gently dipping of the two apparent dips.

This type of problem has two steps. The first step is to determine the angle between the true-dip direction and θ_1. The relevant trigonometric relationships are as follows:

$$\frac{\tan \text{ angle between } \theta_1}{\text{and true-dip direction}} = \frac{(\csc \text{ angle between }}{\theta_1 \text{ and } \theta_2)}$$
$$\times \frac{[(\cot \alpha_1)(\tan \alpha_2) - (\cos}{\text{angle between } \theta_1 \text{ and } \theta_2)]} \quad (1.4)$$

Using Example 1.2, we have the situation shown in Fig. 1.6c,d.

$$\theta_1 = 130° (S50°E) \quad \alpha_1 = 15°$$
$$\theta_2 = 45° (N45°E) \quad \alpha_2 = 28°$$
$$\text{angle between } \theta_1 \text{ and } \theta_2 = 85°$$

Solution

From equation 1.4,

tan angle between θ_1 and true-dip direction
$$= (\csc 85°)[(\cot 15°)(\tan 28°) - (\cos 85°)]$$
$$= 1.004[(3.732)(0.532) - (0.087)]$$
$$= 1.004[1.985 - 0.087] = 1.91$$
angle between θ_1 and true-dip direction = 62.3°

This angle is measured from θ_1 in the direction of θ_2. In this case the computed angle (62.3°) is less than the angle between θ_1 and θ_2 (85°). The true-dip direction, therefore, lies between θ_1 and θ_2. θ_1 is 130° (S50°E) so the direction of true dip is 130° − 62° = 68° (N68°E). Examination of Fig. 1.6b shows that this is a reasonable dip

direction. A dip direction of N68°E corresponds to a strike of N22°W, which agrees with our orthographic projection solution.

If the angle between θ_1 and the true-dip direction is determined to be greater than the angle between θ_1 and θ_2, then the angle is measured from θ_1 toward and beyond θ_2.

Once the true-dip direction (and therefore the strike direction) has been determined, equation 1.3 is used to determine δ:

$$\tan \delta = \frac{\tan \alpha}{\sin \beta}$$

$$\alpha = 15°, \quad \tan \alpha = 0.268$$

$$\beta = \text{angle between } 130°(\text{S50°E})$$
$$\text{and } 158°(\text{S22°E}) = 28°$$

$$\sin \beta = 0.469$$

$$\tan \delta = \frac{0.268}{0.469} = 0.571$$

$$\delta = 30°$$

Alignment diagrams

Alignment diagrams (nomograms) usually involve three variables that have a simple mathematical relationship with one another. A straight line connects points on three scales. Figure 1.7 is an alignment diagram for δ, α, and β. If any two of these variables are known, the third may be quickly determined. This technique is particularly convenient for determining apparent-dip angles on geologic structure sections that are not perpendicular to strike, as discussed in Chapter 4.
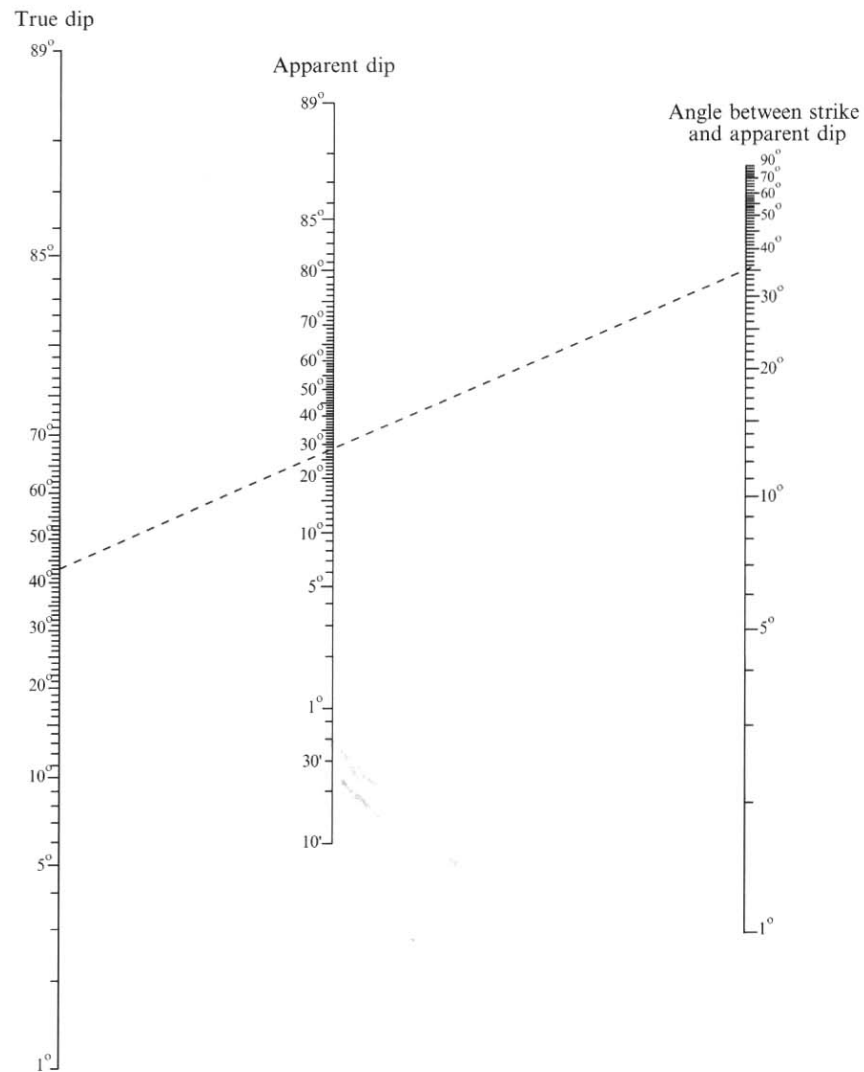
True dip

Apparent dip

Angle between strike
and apparent dip

Fig. 1.7 Alignment diagram (nomogram) for use in solving apparent-dip problems. Place a straight edge on the two known values to determine the unknown value. As shown on the diagram, if the true dip of a plane is 43° and the angle between the strike and the apparent dip direction is 35°, then the apparent dip is 28°. After Palmer (1918).