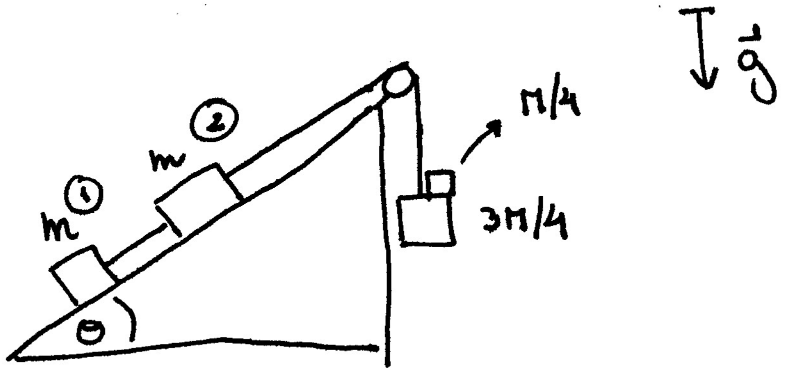
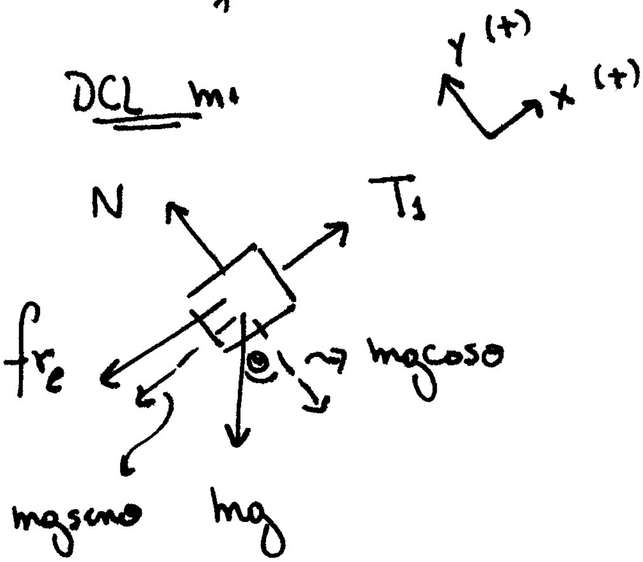
Pauta P1-C2



a) Para que comiencen a deslizar:

DCL m_1



(aún no deslizan.)

$$\sum F_x = m \cdot \cancel{a_x^1}^{0} = 0$$

$$\Rightarrow T_1 - fr_e - mg\operatorname{sen}\theta = 0$$

pero

$$fr_e \leq \mu_e \cdot N$$

en el límite:

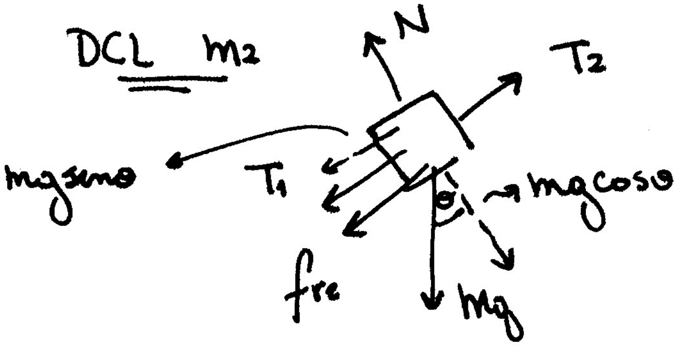
$$\Rightarrow \boxed{T_1 - \mu_e \cdot N - mg\operatorname{sen}\theta = 0}$$

$$\sum F_y = m a_y^1 = 0$$

$$\Rightarrow N - mg\cos\theta = 0 \Rightarrow N = mg\cos\theta$$

$$\Rightarrow \boxed{T_1 = mg(\operatorname{sen}\theta + \mu_e \cos\theta)} \quad (1)$$

DCL m_2



(aún no deslizan)

$$\sum F_x = m \cdot \cancel{a_2^x}^{0} = 0$$

$$\Rightarrow T_2 - T_1 - mg\operatorname{sen}\theta - fr_e = 0 \ (*)$$

$$y \quad \sum F_y = m \cdot a_y^2 = 0$$

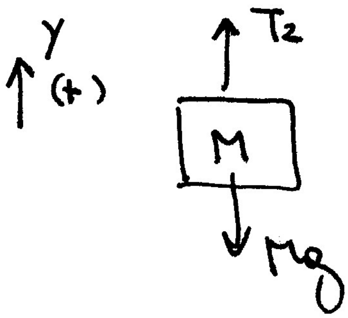
$$\Rightarrow \quad N - mg\cos\theta = 0 \quad \Rightarrow \quad N = mg\cos\theta$$

en (*)

$$\Rightarrow \quad T_2 - T_1 - mg\sin\theta - \mu_e mg\cos\theta = 0$$

$$\Rightarrow \quad \boxed{T_2 = 2mg(\sin\theta + \mu_e\cos\theta)}$$

<u>DCL</u> $(M/4 + 3M/4)$



$$\Rightarrow \quad \sum F_y = M\cdot \cancel{a_y^M}^{\,0}$$

(aún no hay mov.)

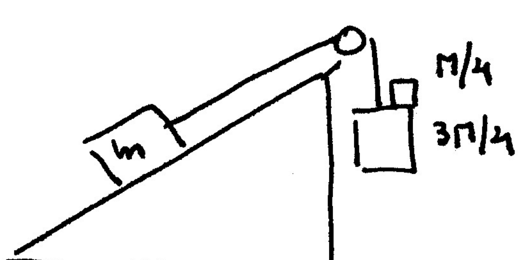
$$\Rightarrow \quad T_2 - Mg = 0$$

$$\Rightarrow \quad \boxed{T_2 = Mg}$$

Igualando:

$$\Rightarrow \quad \boxed{M = 2m(\sin\theta + \mu_e\cos\theta)}$$

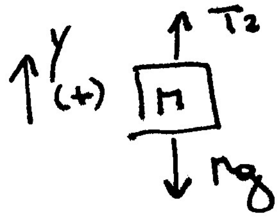
b) hay deslizamiento.



Dado que $3M/4$ cae con $|\text{aceleración}|$ menor a $|\vec{g}|$

$\Rightarrow$ $M/4$ y $3M/4$ caen juntos

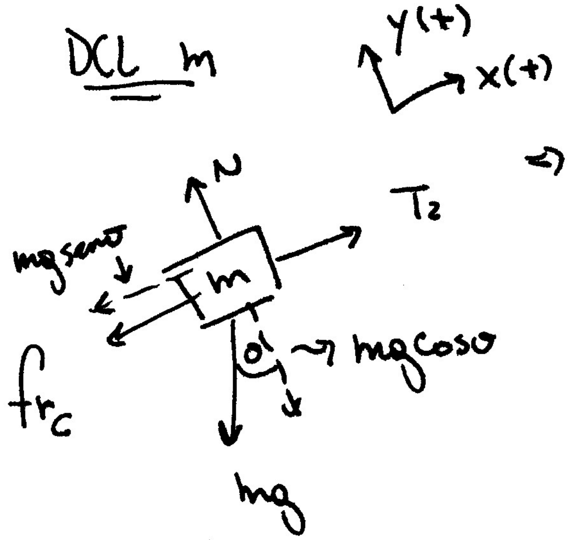
$\Rightarrow$ <u>DCL</u> $(M/4 + 3M/4)$



$$\Rightarrow \quad \sum F_y = M\cdot a_y^M$$

$$\Rightarrow \quad \boxed{T_2 - Mg = M a_y^M}$$

DCL m



$$\Rightarrow \sum F_x = m \cdot a_x^m$$

$$\Rightarrow T_2 - mg\,\text{sen}\theta - fr_c = m \cdot a_x^m$$

y $\sum F_y = 0$

$$\Rightarrow N = mg\cos\theta$$

$$\Rightarrow \boxed{T_2 - mg\,\text{sen}\theta - \mu_c\, mg\cos\theta = m \cdot a_x^m}$$

pero $a_y^M = -a_x^m = -a$

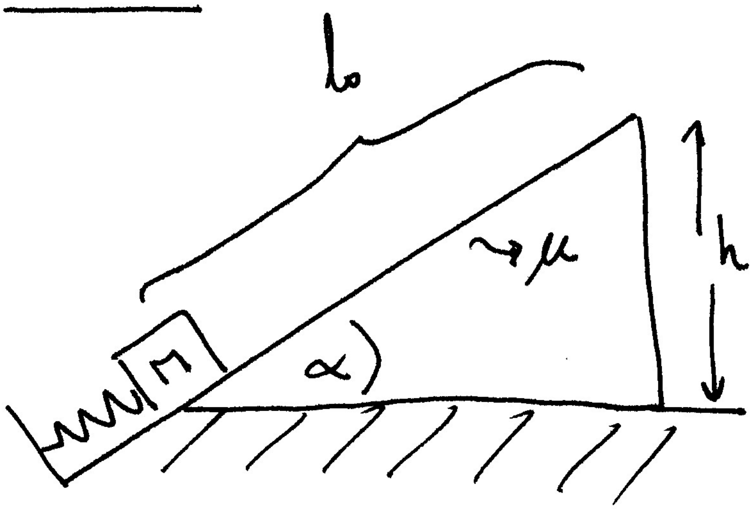
$$\Rightarrow T_2 - Mg = -Ma \qquad / \cdot -1$$

$$\underline{T_2 - mg(\text{sen}\theta + \mu_c\cos\theta) = ma}$$

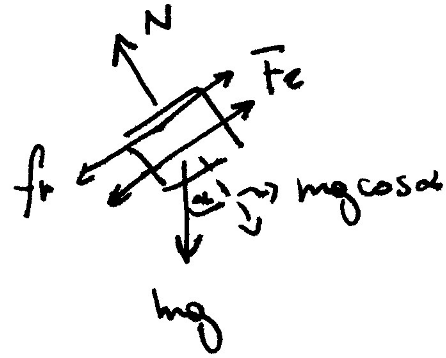
$$Mg - mg(\text{sen}\theta + \mu_c\cos\theta) = (m+M)a$$

$$\Rightarrow \boxed{a = \frac{Mg - mg(\text{sen}\theta + \mu_c\cos\theta)}{(m+M)}}$$

Pauta P2-C2



$$l_0 = \frac{h}{\operatorname{sen}\alpha}$$



a) $E_i = \frac{1}{2} K_0 l_0^2 = \frac{1}{2} K_0 \frac{h^2}{\operatorname{sen}^2\alpha}$

$E_f = Mgh$

pero hay roce:

$$\Rightarrow E_f - E_i = W_{FNC} = -|\vec{f_r}| \cdot |\Delta \vec{r}|$$

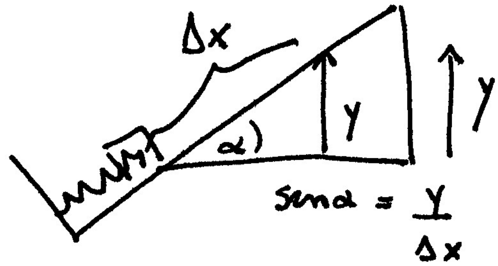
$$= -\mu_c \cdot N \cdot l_0$$

$$\Rightarrow Mg\cancel{h} - \frac{1}{2} K_0 \frac{h^{\cancel{2}}}{\operatorname{sen}^2\alpha} = -\mu_c \cdot Mg\cos\alpha \cdot \frac{\cancel{h}}{\operatorname{sen}\alpha}$$

$$\Rightarrow Mg + \mu_c Mg \frac{\cos\alpha}{\operatorname{sen}\alpha} = \frac{1}{2} K_0 \frac{h}{\operatorname{sen}^2\alpha}$$

$$\Rightarrow \boxed{\frac{2Mg}{h}\left(\operatorname{sen}^2\alpha + \mu_c \frac{\operatorname{sen}(2\alpha)}{2}\right) = K_0}$$

b) $E_i = \frac{1}{2} K l_0^2$



$\operatorname{sen}\alpha = \frac{y}{\Delta x}$

$E_f = Mg\,y$

pero :

$$Mgy - \frac{1}{2} K l_0^2 = W_{FNC} = -\mu_c Mg \cos\alpha \cdot \Delta x$$

$$\Rightarrow Mgy - \frac{1}{2} K l_0^2 = -\mu_c Mg \cos\alpha \cdot \frac{y}{\operatorname{sen}\alpha}$$

$$\Rightarrow Mg\left(1 + \mu_c \frac{\cos\alpha}{\operatorname{sen}\alpha}\right) y = \frac{1}{2} K l_0^2$$

$$= \frac{1}{2} K \frac{h^2}{\operatorname{sen}^2\alpha}$$

$$\Rightarrow Mg\left(\operatorname{sen}^2\alpha + \mu_c \frac{\operatorname{sen}(2\alpha)}{2}\right) y = \frac{1}{2} K h^2$$

$$\Rightarrow \boxed{y_{max} = \frac{K h^2}{2Mg\left(\operatorname{sen}^2\alpha + \mu_c \frac{\operatorname{sen}(2\alpha)}{2}\right)}}$$

c) $E_i = \frac{1}{2} K l_0^2$

$E_f = Mgh + \frac{1}{2} M V^2$

$\Rightarrow E_f - E_i = -\mu_c Mg\cos\alpha \cdot \frac{h}{\sin\alpha}$

$$Mgh + \frac{1}{2} M V^2 - \frac{1}{2} K l_0^2 = -\mu_c Mg \cos\alpha \cdot \frac{h}{\sin\alpha}$$

$$\Rightarrow \frac{1}{2} M V^2 = \frac{1}{2} K l_0^2 - Mgh - \mu_c Mg h \frac{\cos\alpha}{\sin\alpha}$$

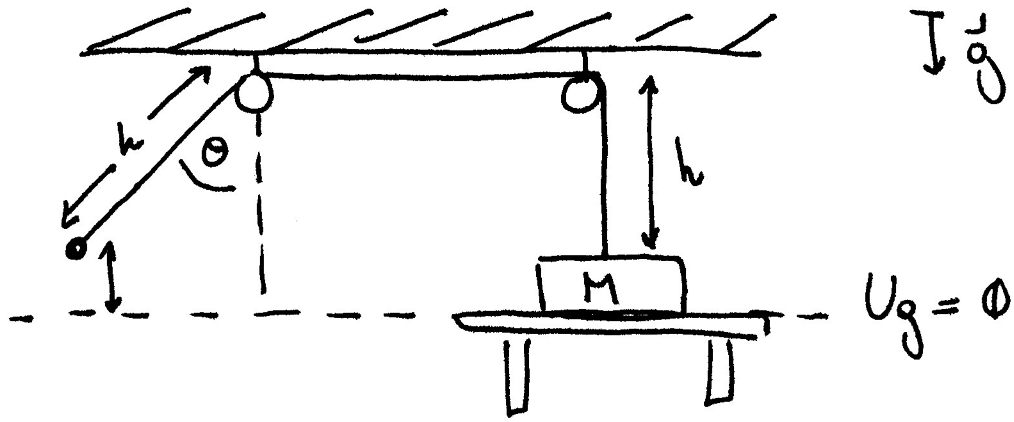
$$\Rightarrow V^2 = \frac{K l_0^2}{M} - 2gh - 2\mu_c g h \cot g\alpha$$

$$= \frac{K}{M} \frac{h^2}{\sin^2\alpha} - 2gh - 2\mu_c g h \cot g\alpha$$

$$\Rightarrow V^2 = \frac{K}{M} \frac{h^2}{\sin^2\alpha} - 2gh(1 + \mu_c \cot g\alpha)$$

$$\Rightarrow \boxed{V = \frac{K}{M} \frac{h^2}{\sin^2\alpha} - 2gh(1 + \mu_c \cot g\alpha)}$$

Pauta P3-C2



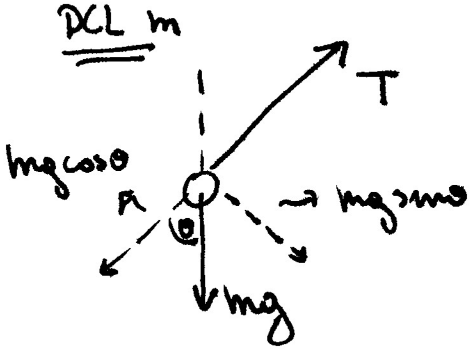
Calculamos el ángulo θ^*:

$$E_i = mgh(1-\cos\theta^*)$$

$$E = \frac{1}{2} m h^2 \dot{\theta}^2 + mgh(1-\cos\theta)$$ (M aún sobre la mesa)

Por otro lado:

DCL m



Entonces:

$$m\frac{V^2}{R} = T - mg\cos\theta$$

ie: $$m\frac{h^{\cancel{2}}\dot{\theta}^2}{\cancel{h}} = T - mg\cos\theta$$

$$\Rightarrow mh\dot{\theta}^2 = T - mg\cos\theta$$

$$\Rightarrow \boxed{mh\dot{\theta}^2 + mg\cos\theta = T}$$ (seguimos asumiendo que M pegado a la mesa)

Por consv. de la energía:

$$mgh(1-\cos\theta^*) = \frac{1}{2} m h^2 \dot{\theta}^2 + mgh(1-\cos\theta)$$

$$\Rightarrow mh\dot{\theta}^2 = 2mg(1-\cos\theta^*) - 2mg(1-\cos\theta)$$

Reemplazando en T:

$$\Rightarrow T = 2mg(1-\cos\theta^*) - 2mg(1-\cos\theta) + mg\cos\theta$$

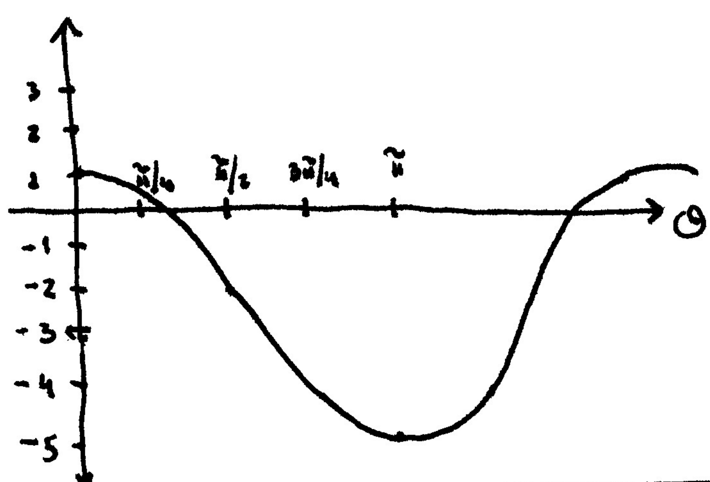
$$= \underbrace{2mg(1-\cos\theta^*)}_{\text{cte}} + \underbrace{3mg\cos\theta - 2mg}_{\text{variable}}$$

T es máxima cuando:

$3\cos\theta - 2$ es máximo.

$\Rightarrow \theta = 0$ ó $\theta = 2k\pi$

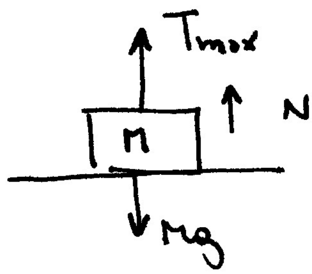
$\Rightarrow \cos\theta = 1.$



$$\Rightarrow \boxed{T_{max} = 2mg(1-\cos\theta^*) + mg}$$

Cuando $T = T_{máx}$ $\Rightarrow$ M se levanta.

DCL M



$\Rightarrow$ $T_{max} + N - Mg = 0$

$\Rightarrow$ $T_{max} = Mg - N$

Cuando se despega $\Rightarrow$ $N = 0$

$\Rightarrow$ $\boxed{T_{max} = Mg}$

$\Rightarrow$ $2mg(1-\cos\theta^*) + mg = Mg$

y si: $M = 3m - \sqrt{3}m$

$\Rightarrow$ $2\cancel{mg}(1-\cos\theta^*) + \cancel{mg} = 3\cancel{mg} - \sqrt{3}\cancel{mg}$

$\cancel{2} - 2\cos\theta^* + \cancel{1} = \cancel{3} - \sqrt{3}$

$\Rightarrow$ $\cos\theta^* = \dfrac{\sqrt{3}}{2}$

$\Rightarrow$ $\boxed{\theta^* = \dfrac{\pi}{6}}$