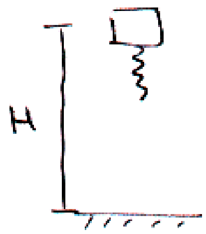
SOLUCIÓN CONTROL 2

P1



$$E_i = mgH$$

COMPRESIÓN MÁXIMA $\Rightarrow$ $v = 0$

$$\Rightarrow \quad E_f = \frac{1}{2} k \Delta^2 + mg(L_o - \Delta)$$

CONS. ENERGÍA $\quad E_i = E_f$

$$mgH = \frac{1}{2} k \Delta^2 + mg(L_o - \Delta)$$

$$\Delta^2 + \frac{2mg}{k}(L_o - \Delta) - \frac{2mg}{k} H = 0$$

(sólo sirve el signo +) $\quad \Delta = \frac{mg}{k}\left[1 \pm \sqrt{1 + \frac{k}{mg}(H - L_o)}\right]$

OTRA MANERA SIGUIENDO INDICACIONES DEL ENUNCIADO

$$E_f = \frac{1}{2} k \Delta^2 + \cancel{mg(L_o - \Delta)}^{\to 0}$$

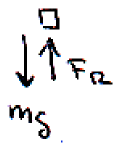
despreciable c/r a
Energía pot. en H

$$\therefore \quad \frac{1}{2} k \Delta^2 = mgH \Rightarrow \Delta = \sqrt{\frac{2mgH}{k}}$$

SOLUCIÓN CONTROL 2

ii)

DCL



$$ma = k(L_o - y) - mg$$

$$a = -\frac{k}{m}y + \frac{k}{m}\Big(L_o - \underbrace{\cancel{\frac{mg}{k}}}_{0}\Big)$$

$$\Rightarrow \quad \ddot{y} = -\frac{k}{m}y + \frac{kL_o}{m} = -\frac{k}{m}(y - L_o) \qquad \text{MAS}$$

SOLUCIÓN

$$x(t) \equiv y(t) - L_o = A\cos(\omega t + \varphi)$$

CONDICIONES INICIALES: $x_o = 0$ y $v_o = \sqrt{2g(H - L_o)} \underset{H \gg L_o}{\simeq} \sqrt{2gH}$

$$\Rightarrow \quad \tan\varphi = -\frac{v_o}{x_o\,\omega} \to \infty \quad \Rightarrow \quad \varphi = \frac{\pi}{2}$$

$$A = \sqrt{x_o^2 + \left(\frac{v_o}{\omega}\right)^2} = \frac{v_o}{\omega} \quad \Rightarrow \quad A = \sqrt{2gH}\,\sqrt{\frac{m}{k}}$$

POR LO TANTO

$$x(t) = y(t) - L_o = -\sqrt{2gH}\,\sqrt{\frac{m}{k}}\cos\left(\sqrt{\frac{k}{m}}\,t + \frac{\pi}{2}\right)$$

SOLUCIÓN CONTROL 2

PERO $\cos(\alpha + \pi/2) = \operatorname{sen} \alpha$

$$\therefore \quad y(t) = L_0 - \sqrt{2gH}\sqrt{\frac{m}{k}}\,\operatorname{sen}\left(\sqrt{\frac{k}{m}}\,t\right)$$

EL RESORTE DEJA DE ESTAR EN CONTACTO CON EL SUELO CUANDO $y(t) = L_0$

$$\Rightarrow \quad \operatorname{sen}\left(\sqrt{\frac{k}{m}}\,t^*\right) = 0 \Rightarrow \boxed{t^* = \pi\sqrt{\frac{m}{k}}}$$

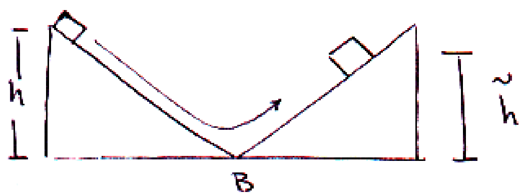
iii) FUERZA PROMEDIO $= \dfrac{\Delta p}{\Delta t} = \dfrac{2m\sqrt{2gH}}{t^*}$

$$\text{FZA. PROMEDIO} = \frac{2\sqrt{2}}{\pi}\sqrt{mgHk}$$

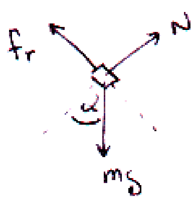
SOLUCIÓN CONTROL 2

P2

i)



DCL



$$-f_r + mg\,\text{sen}\,\alpha = ma$$

$$N - mg\cos\alpha = 0$$

$$\Rightarrow \quad N = mg\cos\alpha$$

$$a = g\,\text{sen}\,\alpha\left(1 - \frac{\mu}{\tan\alpha}\right)$$

DESDE EL PUNTO h HASTA LA BASE DEL PLANO INCLINADO

$$\left.\begin{aligned} E_i &= mgh \\ E_B &= \tfrac{1}{2} m v_B^2 \end{aligned}\right\} \quad \Delta E = \tfrac{1}{2} m v_B^2 - mgh = W_{roce}$$

$$\tfrac{1}{2} m v_B^2 - mgh = -\mu mg\cos\alpha \,\frac{h}{\text{sen}\,\alpha}$$

$$\tfrac{1}{2} m v_B^2 = mgh\left(1 - \frac{\mu}{\tan\alpha}\right)$$

DESDE LA BASE HASTA EL PUNTO $\tilde{h}$

$$E_i = E_B = \tfrac{1}{2} m v_B^2$$

$$E_f = mg\tilde{h}$$

SOLUCIÓN CONTROL 2

$$\Delta E = W_{ROCE} \Rightarrow \quad mg\tilde{h} - \frac{1}{2}mv_B^2 = -\mu mg\frac{\tilde{h}}{\tan\alpha}$$

$$\frac{1}{2}mv_B^2 = mg\tilde{h}\left(1 + \frac{\mu}{\tan\alpha}\right)$$

POR LO TANTO

$$mgh\left(1 - \frac{\mu}{\tan\alpha}\right) = mg\tilde{h}\left(1 + \frac{\mu}{\tan\alpha}\right)$$

$$\tilde{h} = h\left(\frac{\tan\alpha - \mu}{\tan\alpha + \mu}\right)$$

DESPUÉS DE UN CICLO

$$h_1 = \tilde{h}\left(\frac{\tan\alpha - \mu}{\tan\alpha + \mu}\right) = h\left(\frac{\tan\alpha - \mu}{\tan\alpha + \mu}\right)^2$$

PARA EL CICLO $\underline{n}$

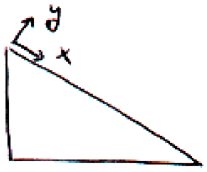
$$h_{n+1} = h_n\left(\frac{\tan\alpha - \mu}{\tan\alpha + \mu}\right)^2$$

$$\Rightarrow \boxed{h_n = h\left(\frac{\tan\alpha - \mu}{\tan\alpha + \mu}\right)^{2n}}$$

$$\therefore \quad \Delta h = h_{n+1} - h_n = h_n\left[\left(\frac{\tan\alpha - \mu}{\tan\alpha + \mu}\right)^2 - 1\right]$$

SOLUCIÓN CONTROL 2

ii)



TIEMPO DE BAJADA t_1

$$x = \frac{1}{2} a t^2 = \frac{1}{2} g \operatorname{sen}\alpha \left(1 - \frac{\mu}{\tan\alpha}\right) t^2$$

$$x = \frac{h}{\operatorname{sen}\alpha} \Rightarrow \quad t_1 = \frac{1}{\operatorname{sen}\alpha} \sqrt{\frac{2h}{g(1 - \mu/\tan\alpha)}}$$

TIEMPO DE SUBIDA t_2

DCL

$$-f_r - mg\operatorname{sen}\alpha = ma$$

$$N - mg\cos\alpha = 0$$

$$\Rightarrow \quad a = -g\operatorname{sen}\alpha \left(1 + \frac{\mu}{\tan\alpha}\right)$$

ENTONCES

$$v = v_B + at = v_B - g\operatorname{sen}\alpha\left(1 + \frac{\mu}{\tan\alpha}\right)t$$

$$v = 0 \Rightarrow \quad v_B = g\operatorname{sen}\alpha\left(1 + \frac{\mu}{\tan\alpha}\right) t_2$$

PERO

$$\frac{1}{2} m v_B^2 = mgh\left(1 - \frac{\mu}{\tan\alpha}\right)$$

$$v_B = \sqrt{2gh\left(1 - \frac{\mu}{\tan\alpha}\right)}$$

SOLUCIÓN CONTROL 2

$$\therefore \quad t_2 = \frac{\sqrt{2gh\,(1-\mu/\tan\alpha)}}{g\,\mathrm{sen}\,\alpha\,(1+\mu/\tan\alpha)}$$

TIEMPO PARA LLEGAR A $\tilde{h}$

$$\tilde{t} = t_1 + t_2 = \frac{1}{\mathrm{sen}\,\alpha}\sqrt{\frac{2h}{g}}\left[\frac{1}{\sqrt{1-\mu/\tan\alpha}} + \frac{\sqrt{1-\mu/\tan\alpha}}{1+\mu/\tan\alpha}\right]$$

TIEMPO PARA IR DE $\tilde{h}$ A h_1

$$\tilde{t}_1 = \frac{1}{\mathrm{sen}\,\alpha}\sqrt{\frac{2\tilde{h}}{g}}\left[\frac{1}{\sqrt{1-\mu/\tan\alpha}} + \frac{\sqrt{1-\mu/\tan\alpha}}{1+\mu/\tan\alpha}\right]$$

PERO
$$\tilde{h} = h\left(\frac{\tan\alpha-\mu}{\tan\alpha+\mu}\right) = h\left(\frac{1-\mu/\tan\alpha}{1+\mu/\tan\alpha}\right)$$

POR LO TANTO, EL TIEMPO PARA COMPLETAR EL CICLO ESTÁ DADO POR

$$T_1 = \tilde{t} + \tilde{t}_1 = \frac{1}{\mathrm{sen}\,\alpha}\sqrt{\frac{2h}{g}}\left[\frac{1}{\sqrt{1-\mu/\tan\alpha}} + \frac{\sqrt{1-\mu/\tan\alpha}}{1+\mu/\tan\alpha}\right] \times\left[1+\sqrt{\frac{1-\mu/\tan\alpha}{1+\mu/\tan\alpha}}\right]$$

$$\therefore \quad T_1 = \frac{1}{\mathrm{sen}\,\alpha}\sqrt{\frac{2h}{g}}\,\frac{2}{(1+\mu/\tan\alpha)}\left[\frac{1}{\sqrt{1-\mu/\tan\alpha}} + \frac{1}{\sqrt{1+\mu/\tan\alpha}}\right]$$

SOLUCIÓN CONTROL 2

PARA EL CICLO $\underline{n}$ SE TIENE

$$T_{n+1} = \frac{1}{\operatorname{sen}\alpha}\sqrt{\frac{2h_n}{g}}\ \frac{2}{(1+\mu/\tan\alpha)}\left[\frac{1}{\sqrt{1-\mu/\tan\alpha}} + \frac{1}{\sqrt{1+\mu/\tan\alpha}}\right]$$

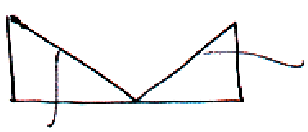
iii) DISTANCIA TOTAL USANDO TRABAJO FZA. ROCE

$$W_{\substack{total\\roce}} = -\mu m g \cos\alpha\, D$$

$$\Delta E = W \qquad \Rightarrow \qquad -mgh = -\mu m g\cos\alpha\, D$$

$$\boxed{D = \frac{h}{\mu\cos\alpha}}$$

USANDO ALTURAS h_n



$$d_2 = \frac{\tilde{h}}{\operatorname{sen}\alpha} \times 2 = \frac{2h}{\operatorname{sen}\alpha}\left(\frac{1-\mu/\tan\alpha}{1+\mu/\tan\alpha}\right)$$

$$d_1 = \frac{h}{\operatorname{sen}\alpha}$$

$$d_3 = \frac{h_1}{\operatorname{sen}\alpha} = \frac{h}{\operatorname{sen}\alpha}\left(\frac{1-\mu/\tan\alpha}{1+\mu/\tan\alpha}\right)^2$$

SOLUCIÓN CONTROL 2

$$D = \frac{h}{\operatorname{sen}\alpha} + \frac{2\tilde{h}}{\operatorname{sen}\alpha} + \frac{2h_1}{\operatorname{sen}\alpha} + \frac{2\tilde{h}_1}{\operatorname{sen}\alpha} + \frac{2h_2}{\operatorname{sen}\alpha} + \dots$$

con $h_n = h\left(\frac{\tan\alpha - \mu}{\tan\alpha + \mu}\right)^{2n}$ y $\tilde{h}_n = h\left(\frac{\tan\alpha - \mu}{\tan\alpha + \mu}\right)^{n}$

$$\therefore \quad D = \frac{h}{\operatorname{sen}\alpha} + \sum_{n=1}^{\infty} \frac{2h}{\operatorname{sen}\alpha}\left(\frac{\tan\alpha - \mu}{\tan\alpha + \mu}\right)^{n}$$

$$D = \frac{h}{\operatorname{sen}\alpha}\left[1 + 2\sum_{n=1}^{\infty}\left(\frac{1-\mu/\tan\alpha}{1+\mu/\tan\alpha}\right)^{n}\right]$$

$$D = \frac{h}{\operatorname{sen}\alpha}\left[1 + 2\left(\frac{1-\mu/\tan\alpha}{1+\mu/\tan\alpha}\right)\cdot\frac{1}{1-\left(\frac{1-\mu/\tan\alpha}{1+\mu/\tan\alpha}\right)}\right]$$

$$D = \frac{h}{\operatorname{sen}\alpha}\left[1 + \frac{\cancel{2}(1-\mu/\tan\alpha)}{\cancel{2}\mu/\tan\alpha}\right]$$

$$D = \frac{h}{\operatorname{sen}\alpha}\cdot\frac{1}{\mu/\tan\alpha} = \frac{h}{\mu\cos\alpha}$$

v) si $\mu > \tan\alpha$ EL BLOQUE NO SE MUEVE

iv) En clase auxiliar

SOLUCIÓN CONTROL 2

P3

MOV. CIRCULAR + GRAVITACIÓN

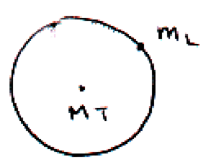
$$\frac{\cancel{m} v^2}{R_L} = \frac{G M_x \cancel{m}}{R_L^2} + \frac{G m^{\cancel{2}}}{(2R_L)^2}$$

PERO $v = \frac{2\pi R}{T}$ ENTONCES

$$\frac{4\pi^2}{T^2} R_L = \frac{G M_x}{R_L^2} + \frac{G m}{4 R_L^2}$$

$$\frac{4\pi^2}{T^2} = \frac{G M_x}{R_L^3}\left[1 + \frac{m}{4 M_x}\right] = \lambda \frac{G M_T}{R_L^3}\left[1 + \frac{m}{4\lambda M_T}\right]$$

PARA EL SISTEMA TIERRA-LUNA



$$T_L^2 = \frac{4\pi^2}{G M_T} R_L^3$$

ENTONCES

$$\frac{\cancel{4\pi^2}}{T^2} = \lambda \frac{\cancel{4\pi^2}}{T_L^2}\left[1 + \frac{m}{4\lambda M_T}\right]$$

$$\boxed{T = \frac{T_L}{\sqrt{\lambda\left(1 + \frac{m}{4\lambda M_T}\right)}}}$$