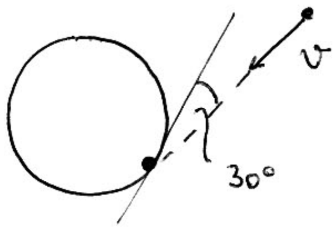
SOLUCIÓN CNONTROL 3

P1)



CONSERVACIÓN DE $\vec{L}$

$$\vec{L}_i = m_{pelota}\, v\, R \cos 30^\circ$$

$$\vec{L}_i = 24 \cos 30^\circ \left[\text{kg}\,\frac{\text{m}^2}{\text{s}}\right]$$

$$\vec{L}_f = I\omega \quad \text{con} \quad I = I_{disco} + I_{niño+pelota}$$

$$\Rightarrow \quad I = 150 \text{ kg m}^2 + 31 \text{ kg} \cdot 4 \text{ m}^2$$

$$I = 274 \ [\text{kg} \cdot \text{m}^2]$$

ENTONCES

$$\omega = \frac{L_f}{I} = \frac{L_i}{I}$$

$$\omega = \frac{24 \cos 30^\circ}{274} \left[\frac{1}{s}\right]$$

$$\omega = \frac{12\sqrt{3}}{274} \left[\frac{1}{s}\right]$$

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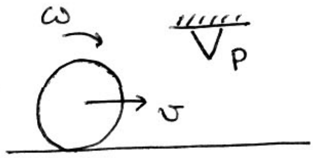
P2|

APLICANDO CONSERVACIÓN DE $\vec{L}$



$$\omega = \frac{\upsilon}{R} \qquad (R.S.R)$$

MOM. ANGULAR c/r A PUNTO P

$$L_i = RM\upsilon - I_{CM}\,\omega$$

$$L_i = RM\upsilon - \frac{1}{2}MR^2 \cdot \frac{\upsilon}{R} = \frac{1}{2}MR\upsilon$$

DESPUÉS DEL CHOQUE

$$L_f = I_P\,\omega' = (I_{CM} + MR^2)\,\omega'$$

$$L_f = \frac{3}{2}MR^2\omega' = L_i$$

$$\Rightarrow \qquad \omega' = \frac{\upsilon}{3R}$$

AHORA SÍ SE PUEDE APLICAR CONS. DE ENERGÍA

$$E_i = \frac{1}{2}I_P\,\omega'^2 = \frac{1}{12}M\upsilon^2$$

$$E_f = Mgh$$

SOLUCIÓN CNONTROL 3

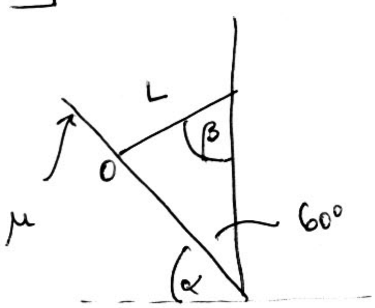
$$E_i = E_f \Rightarrow \frac{1}{12} M v^2 = M g h$$

$$h = \frac{v^2}{12 g}$$

ALTURA C/r AL SUELO

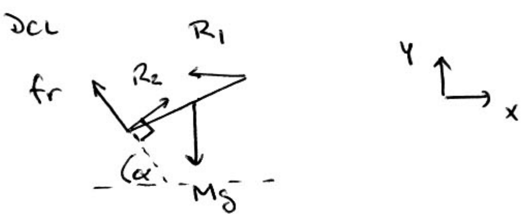
$$H = h + R = R + \frac{v^2}{12 g}$$

P3|



ESCALERA DESLIZA HACIA ABAJO

DCL



$$\sum \vec{F} = 0 \Rightarrow \quad x) \quad R_2 \cos\left(\frac{\pi}{2} - \alpha\right) - f_r \cos\alpha - R_1 = 0 \qquad (1)$$

$$y) \quad R_2 \operatorname{sen}\left(\frac{\pi}{2} - \alpha\right) + f_r \operatorname{sen}\alpha - Mg = 0 \qquad (2)$$

$$\sum \vec{\tau}_0 = 0 \Rightarrow \quad L R_1 \underbrace{\operatorname{sen}\left(\frac{\pi}{2} + \beta\right)}_{\cos\beta} - \frac{L}{2} M g \underbrace{\operatorname{sen}(\pi - \beta)}_{\operatorname{sen}\beta} = 0$$

SOLUCIÓN CNONTROL 3

$$\therefore \qquad L R_1 \cos\beta - \frac{L}{2} Mg \operatorname{sen}\beta = 0$$

$$\tan\beta = \frac{2R_1}{Mg} \qquad (3)$$

DE (1) Y (2) SE OBTIENE

$$(1) \rightarrow \qquad R_2 \operatorname{sen}\alpha - f_r \cos\alpha = R_1$$

$$(2) \rightarrow \qquad R_2 \cos\alpha + f_r \operatorname{sen}\alpha = Mg$$

HAGAMOS $(1) \cdot \operatorname{sen}\alpha + (2) \cos\alpha$ ENTONCES

$$R_2 = R_1 \operatorname{sen}\alpha + Mg \cos\alpha$$

REEMPLAZANDO EN (1)

$$(R_1 \operatorname{sen}\alpha + Mg\cos\alpha) \operatorname{sen}\alpha - f_r \cos\alpha = R_1$$

$$Mg \operatorname{sen}\alpha \cos\alpha - R_1 \cos^2\alpha = f_r \cos\alpha$$

PERO $f_r \leq \mu R_2$

$$\Rightarrow \qquad Mg \operatorname{sen}\alpha - R_1 \cos\alpha \leq \mu (R_1 \operatorname{sen}\alpha + Mg\cos\alpha)$$

$$Mg \operatorname{sen}\alpha - \mu Mg \cos\alpha \leq (\mu \operatorname{sen}\alpha + \cos\alpha) R_1$$

SOLUCIÓN CNONTROL 3

$$Mg\frac{(\operatorname{sen}\alpha - \mu\cos\alpha)}{(\mu\operatorname{sen}\alpha + \cos\alpha)} \leq R_1$$

POR LO TANTO, USANDO (3) SE OBTIENE

$$\tan\beta \geq \frac{2(\operatorname{sen}\alpha - \mu\cos\alpha)}{\mu\operatorname{sen}\alpha + \cos\alpha}$$

SI AHORA LA ESCALERA DESLIZA HACIA ABAJO SE TIENE

$$\sum\vec{F} = 0 \quad \Rightarrow \quad x)\ R_2\operatorname{sen}\alpha + f_r\cos\alpha = R_1$$

$$y)\ R_2\cos\alpha - f_r\operatorname{sen}\alpha = Mg$$

ENTONCES, SIGUIENDO PASOS ANALOGOS AL CASO ANTERIOR SE OBTIENE

$$\tan\beta \leq \frac{2(\operatorname{sen}\alpha + \mu\cos\alpha)}{\cos\alpha - \mu\operatorname{sen}\alpha}$$

SOLUCIÓN CNONTROL 3

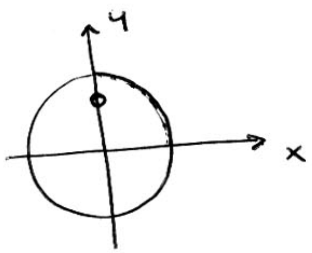
P4 I

POSICIÓN CM

MASA MONEDA SIN AGUJERO M_0

DENSIDAD MONEDA = $\frac{M_0}{\pi R^2} = \frac{m}{\pi (R/6)^2} \Rightarrow m = \frac{M_0}{36}$

CON m "MASA AGUJERO"



$\vec{x}_{AGUJERO} = (0, \frac{2}{3}R)$

$\vec{x}_{CM\ MONEDA\ SIN\ AGUJERO} = (0,0)$

POSICIÓN CM MONEDA CON AGUJERO

$$x_{CM} = 0$$

$$y_{CM} = \frac{1}{M}(M_0 \cdot 0 - m \frac{2}{3}R)$$

CON $M = M_0 - m = 35m \Rightarrow m = \frac{M}{35}$

$$\therefore \quad y_{CM} = -\frac{\frac{\cancel{M}}{35} \cdot \frac{2}{3}R}{\cancel{M}} = -\frac{2}{105}R$$

SOLUCIÓN CNONTROL 3

MOMENTO DE INERCIA :

$$I = \underbrace{I_{\text{MONEDA SIN AGUJERO}}}_{I_{\text{CIRCA}\,CM} + M d^2_{\text{EJE}}} - I_{\text{ORIFICIO}}$$

$$I = \frac{1}{2} M R^2 + M \left(\frac{2}{3} R\right)^2 - \frac{1}{2} m \left(\frac{R}{6}\right)^2$$

$$I = \frac{1}{2} M R^2 + \frac{4}{9} M R^2 - \frac{1}{70} \frac{1}{36} M R^2$$

$$I = \frac{793}{840} M R^2$$

PÉNDULO

$$\Sigma \tau = I \ddot{\theta}$$

$$-M g d \operatorname{sen} \theta = I \ddot{\theta}$$

DONDE d ES LA DISTANCIA DEL CM AL EJE

$$d = \frac{2}{3} R + \frac{2R}{105} = \frac{24}{35} R$$

SOLUCIÓN CNONTROL 3

ÁNGULOS PEQUEÑOS $\Rightarrow$ $-Mgd\theta = I\ddot{\theta}$ (MAS)

$$-\cancel{M}g\,\frac{24}{35}R\,\theta = \frac{793}{840}\cancel{M}R^2\,\ddot{\theta}$$

$$\underbrace{-\frac{576}{793}\frac{g}{R}}_{\omega^2}\,\theta = \ddot{\theta}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{793}{576}\frac{R}{g}}$$

P5

ANTES DEL CHOQUE

PLANETA : $v_{planeta} = \sqrt{\frac{GM_s}{R}}$ $\quad M_s$ = masa del Sol

COMETA :

$$L = mv_pR = mv_a4R \Rightarrow v_p = 4v_a \quad (1)$$

v_p = velocidad perihelio

v_a = velocidad afelio

$$E = \frac{1}{2}mv_p^2 - \frac{GM_sm}{R} = \frac{1}{2}mv_a^2 - \frac{GM_sm}{4R}$$

SOLUCIÓN CNONTROL 3

Usando (1) se tiene

$$\frac{1}{2} v_p^2 - \frac{GM_s}{R} = \frac{1}{2}\left(\frac{v_p}{4}\right)^2 - \frac{GM_s}{4R}$$

$$\left(1 - \frac{1}{16}\right) v_p^2 = \frac{1}{2}\frac{GM_s}{R}$$

$$\boxed{v_p = \sqrt{\frac{8}{15}\frac{GM_s}{R}}}$$ velocidad cometa en el perihelio

CHOQUE PLÁSTICO

$$M v_{planeta} + m v_p = (M+m) v_f$$

$$\Rightarrow \boxed{v_f = \sqrt{\frac{GM_s}{R}}\left(\frac{M + m\sqrt{\frac{8}{15}}}{M+m}\right)}$$ velocidad planeta + cometa después del choque

ORBITA SISTEMA PLANETA + COMETA

$$L = (M+m) v_f R = (M+m) v_a r_a \Rightarrow v_f R = v_a r_a$$

$$E = \frac{1}{2}(M+m) v_f^2 - \frac{GM_s(M+m)}{R} = \frac{1}{2}(M+m) v_a^2 - \frac{GM_s(M+m)}{r_a}$$

$$\Rightarrow \frac{1}{2} v_f^2 - \frac{1}{2} v_a^2 = \frac{GM_s}{R} - \frac{GM_s}{r_a}$$

$$v_f^2 - v_a^2 = \frac{2GM_s}{R}\left(1 - \frac{v_a}{v_f}\right)$$

SOLUCIÓN CNONTROL 3

$$\cancel{(v_f - v_a)}(v_f + v_a) = \frac{2GM_s}{R}\frac{\cancel{(v_f - v_a)}}{v_f}$$

$$v_a = \frac{2GM_s}{R}\frac{1}{v_f} - v_f$$

Entonces $\quad r_a = R\frac{v_f}{v_a} = \dfrac{R v_f}{\dfrac{2GM_s}{R v_f} - v_f}$

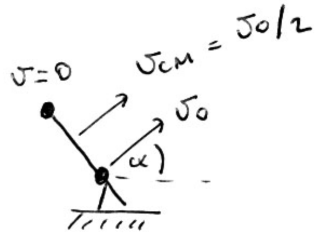
$$\Rightarrow \quad r_a = \frac{R}{\dfrac{2GM_s}{R v_f^2} - 1}$$

$$\therefore \quad \boxed{r_a = \frac{R}{2\left(\dfrac{M+m}{M+m\sqrt{\frac{8}{15}}}\right)^2 - 1}}$$

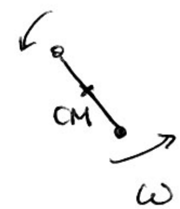
radio apohelio
planeta + cometa

SOLUCIÓN CNONTROL 3

P6



LA BARRA GIRA CON $\omega = cte$



$$\omega = \frac{v_0/2}{l/2} = \frac{v_0}{l}$$

MOV. PARABOLICO CM.

$$v_y = \frac{v_0}{2} \operatorname{sen} \alpha - g t$$

ALTURA MÁXIMA

$$v_y = 0 \Rightarrow \tilde{t} = \frac{v_0 \operatorname{sen} \alpha}{2 g}$$

$$\# \text{ VUELTAS} = \frac{1}{T} \cdot \tilde{t} = \frac{\omega}{2\pi} \frac{v_0 \operatorname{sen} \alpha}{2 g}$$

$$\# \text{ VUELTAS} = \frac{v_0^2 \operatorname{sen} \alpha}{4\pi l g}$$