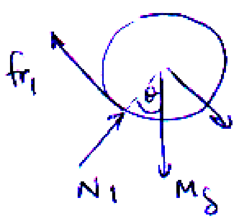
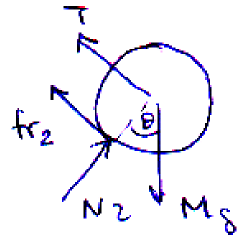
SOLUCIÓN CONTROL 3

P1 i)





$\Sigma \vec{\tau}$ c/r a puntos de contacto con el suelo

(0.5 puntos) $$TR + MgR\,\text{sen}\,\theta = (I_1 + MR^2)\,\alpha \qquad (1)$$

(0.5 puntos) $$-TR + MgR\,\text{sen}\,\theta = (I_2 + MR^2)\,\alpha \qquad (2)$$

$$\Rightarrow \qquad 2MgR\,\text{sen}\,\theta = (I_1 + I_2 + 2MR^2)\,\alpha$$

$$\boxed{\alpha = \frac{2MgR\,\text{sen}\,\theta}{I_1 + I_2 + 2MR^2}}$$ (1 pto)

reemplazando en (1) o (2)

$$T = \frac{(I_1 + MR^2)}{R}\,\alpha - Mg\,\text{sen}\,\theta$$

$$\boxed{T = \frac{(I_1 - I_2)}{I_1 + I_2 + 2MR^2} \cdot Mg\,\text{sen}\,\theta}$$ (1 pto)

SOLUCIÓN CONTROL 3

Otra manera de resolver el problema (similar asignación de puntaje)

$\sum \vec{\tau}$ c/r al centro de masa de c/u de los cilindros

$$f_{r_1} R = I_1 \alpha \quad (1)$$

$$f_{r_2} R = I_2 \alpha \quad (2)$$

$\sum \vec{F} \Rightarrow$

$$N_1 - Mg\cos\theta = 0$$

$$T + Mg\,\text{sen}\,\theta - f_{r_1} = Ma \quad (3)$$

$$N_2 - Mg\cos\theta = 0$$

$$Mg\,\text{sen}\,\theta - T - f_{r_2} = Ma \quad (4)$$

RSR

$\Rightarrow \alpha = \frac{a}{R}$

entonces de (1) y (3)

$$T + Mg\,\text{sen}\,\theta - \frac{I_1}{R^2} a = Ma$$

de (2) y (4)

$$Mg\,\text{sen}\,\theta - T - \frac{I_2}{R^2} a = Ma$$

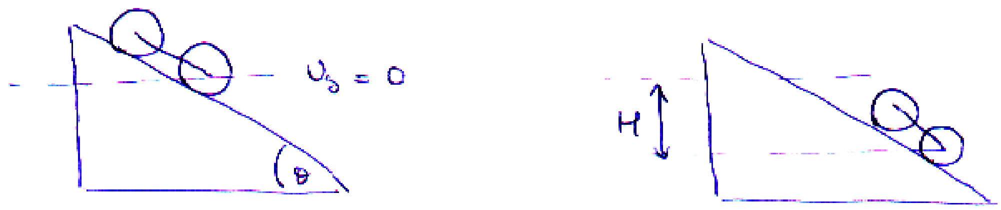
$$\Rightarrow \quad T + Mg\,\text{sen}\,\theta = (I_1 + MR^2)\frac{a}{R^2}$$

$$-T + Mg\,\text{sen}\,\theta = (I_2 + MR^2)\frac{a}{R^2}$$

etc...

SOLUCIÓN CONTROL 3

ii)



$$E_i = 0$$

$$E_f = -2MgH + \frac{1}{2}Mv^2 + \frac{1}{2}Mv^2 + \frac{1}{2}I_1\omega^2 + \frac{1}{2}I\omega^2$$

(0.5 ptos)

cons. de energía $E_i = E_f$ (0.5 ptos)

$$Mv^2 + \frac{1}{2}I_1\omega^2 + \frac{1}{2}I_2\omega^2 = 2MgH$$

$$\left(2M + \frac{I_1}{R^2} + \frac{I_2}{R^2}\right)v^2 = \frac{4MgH}{I_1 + I_2}$$ (0.5 ptos)

$$\boxed{v = 2R\sqrt{\frac{MgH}{2MR^2 + I_1 + I_2}}}$$ (0.5 ptos)

SOLUCIÓN CONTROL 3

P2

Ec. de Bernoulli: $\frac{1}{2}\rho v_A^2 + P_A = \frac{1}{2}\rho v_B^2 + P_B$

$$P_A - P_B = \frac{1}{2}\rho (v_B^2 - v_A^2) \quad \text{(1pto)}$$

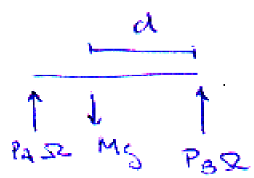
Cons. de masa: $Q = v_A S_A = v_B S_B$

entonces $P_A - P_B = \frac{1}{2}\rho Q^2 \left(\frac{1}{S_B^2} - \frac{1}{S_A^2} \right)$ (1)

(1pto)

Equilibrio de fzas. en la tabla

DCL



$$P_A + P_B = \frac{Mg}{\Omega} \quad (2)$$

(1pto)

$\Sigma \tau_B = 0 \Rightarrow P_A \Omega L - Mg\, d = 0$

$$d = \frac{P_A \Omega L}{Mg} \quad \text{(1pto)}$$

De (2) se tiene $P_B = \frac{Mg}{\Omega} - P_A$ reemplazando en (1)

$$P_A - \frac{Mg}{\Omega} + P_A = \frac{1}{2}\rho Q^2 \left(\frac{1}{S_B^2} - \frac{1}{S_A^2} \right) \quad \text{(1pto)}$$

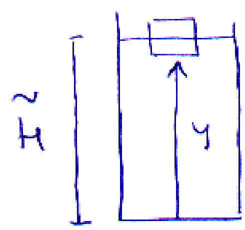
$$P_A = \frac{Mg}{2\Omega} + \frac{1}{4}\rho Q^2 \left(\frac{1}{S_B^2} - \frac{1}{S_A^2} \right)$$

Por lo tanto

$$\boxed{d = \frac{L}{2} + \frac{\Omega \rho Q^2}{4Mg} \left(\frac{1}{S_B^2} - \frac{1}{S_A^2} \right)} \quad \text{(1pto)}$$

SOLUCIÓN CONTROL 3

P3 |



DCL

(0.5 ptos)

$$F_e = \rho_a V_s g$$

densidad del agua

mg

volumen bajo la superficie

$$V_s = \pi R_2^2 (\tilde{H} - y) \quad \text{(0.5 ptos)}$$

Por otro lado, el volumen del agua + flotador es

$$\tilde{V} = \pi R_1^2 \tilde{H} = V_{agua} + V_s \quad \text{(0.5 ptos)}$$

$$\pi R_1^2 \tilde{H} = \pi R_1^2 H + \pi R_2^2 (\tilde{H} - y)$$

$$\tilde{H} = H + \left(\frac{R_2}{R_1}\right)^2 (\tilde{H} - y)$$

$$\tilde{H} = \frac{H - (R_2/R_1)^2 y}{1 - (R_2/R_1)^2} \quad (*) \quad \text{(0.5 ptos)}$$

Para el flotador en equilibrio

$$F_e = mg \quad \text{(0.5 ptos)}$$

$$\rho_a \cancel{\pi R_2^2} (\tilde{H} - y_{eq}) \cancel{g} = \rho_0 \cancel{\pi R_2^2} h \cancel{g}$$

$$\rho_a (\tilde{H} - y_{eq}) = \rho_0 h$$

SOLUCIÓN CONTROL 3

Usando (*)

$$\rho_a \left\{ \frac{H - (R_2/R_1)^2 y_{eq}}{1-(R_2/R_1)^2} - y_{eq} \right\} = \rho_o h$$

(0.5 ptos)

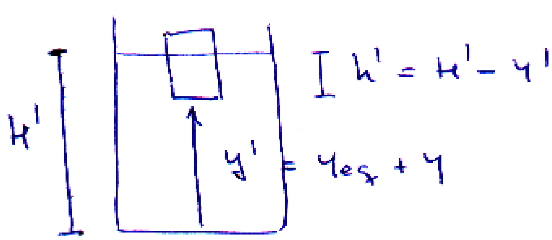
$$\frac{\rho_a}{1-(R_2/R_1)^2} \left\{ H - \cancel{\left(\frac{R_2}{R_1}\right)^2 y_{eq}} - y_{eq} + \cancel{\left(\frac{R_2}{R_1}\right)^2 y_{eq}} \right\} = \rho_o h$$

$$H - y_{eq} = \frac{\rho_o}{\rho_a} h \left[1 - \left(\frac{R_2}{R_1}\right)^2 \right]$$

$$\boxed{y_{eq} = H - \frac{\rho_o}{\rho_a} \left[1 - \left(\frac{R_2}{R_1}\right)^2 \right] h}$$

(1 pto)

ii) Desplazemos el flotador de su posición de equilibrio y_{eq}



El "nuevo" volumen H' está dado por

$$\pi R_1^2 H' = \pi R_1^2 H + \pi R_2^2 h'$$

(0.5 ptos)

pero $\quad H' = h' + y' = h' + y_{eq} + y$

SOLUCIÓN CONTROL 3

entonces $$H' = H + \left(\frac{R_2}{R_1}\right)^2 h'$$

$$h' + y_{eq} + y = H + \left(\frac{R_2}{R_1}\right)^2 h'$$

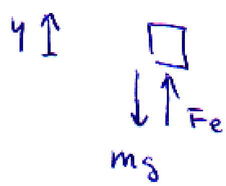
$$h'\left[1 - \left(\frac{R_2}{R_1}\right)^2\right] = H - y_{eq} - y$$

De i) se tiene que $$y_{eq} = H - \frac{\rho_o}{\rho_a}\left[1 - \left(\frac{R_2}{R_1}\right)^2\right]h$$

$$\Rightarrow \quad h'\left[1 - \left(\frac{R_2}{R_1}\right)^2\right] = \frac{\rho_o}{\rho_a} h \left[1 - \left(\frac{R_2}{R_1}\right)^2\right] - y$$

$$h' = \frac{\rho_o}{\rho_a} h - \frac{y}{1 - \left(\frac{R_2}{R_1}\right)^2}$$ (0.5 ptos)

DCL



$$Fe - mg = m\ddot{y}$$

donde $$Fe = \rho_a V_s g = \rho_a g \pi R_2^2 h'$$

$$\Rightarrow \quad \rho_a g \pi R_2^2 \left\{ \frac{\rho_o}{\rho_a} h - \frac{y}{1 - \left(\frac{R_2}{R_1}\right)^2} \right\} - mg = m\ddot{y}$$

$$\cancel{\frac{\rho_o \pi R_2^2 h g}{m}} - \frac{\rho_a \pi R_2^2 g}{1 - \left(\frac{R_2}{R_1}\right)^2} y - \cancel{mg} = m\ddot{y}$$

SOLUCIÓN CONTROL 3

$$-\frac{\rho_a \pi \cancel{R_2^2} g}{1-\left(\frac{R_2}{R_1}\right)^2} y = \rho_o \pi \cancel{R_2^2} h \ddot{y} \quad (0.5 \text{ ptos})$$

$$\underbrace{-\frac{\rho_a}{\rho_o}\frac{g}{h}\frac{1}{\left[1-\left(\frac{R_2}{R_1}\right)^2\right]}}_{\omega^2} y = \ddot{y} \quad (\text{MAS})$$

$$\Rightarrow \boxed{\omega^2 = \frac{\rho_a}{\rho_o}\frac{g}{h}\frac{1}{\left[1-\left(\frac{R_2}{R_1}\right)^2\right]}} \quad (0.5 \text{ ptos})$$